The Predicted-Updates Dynamic Model

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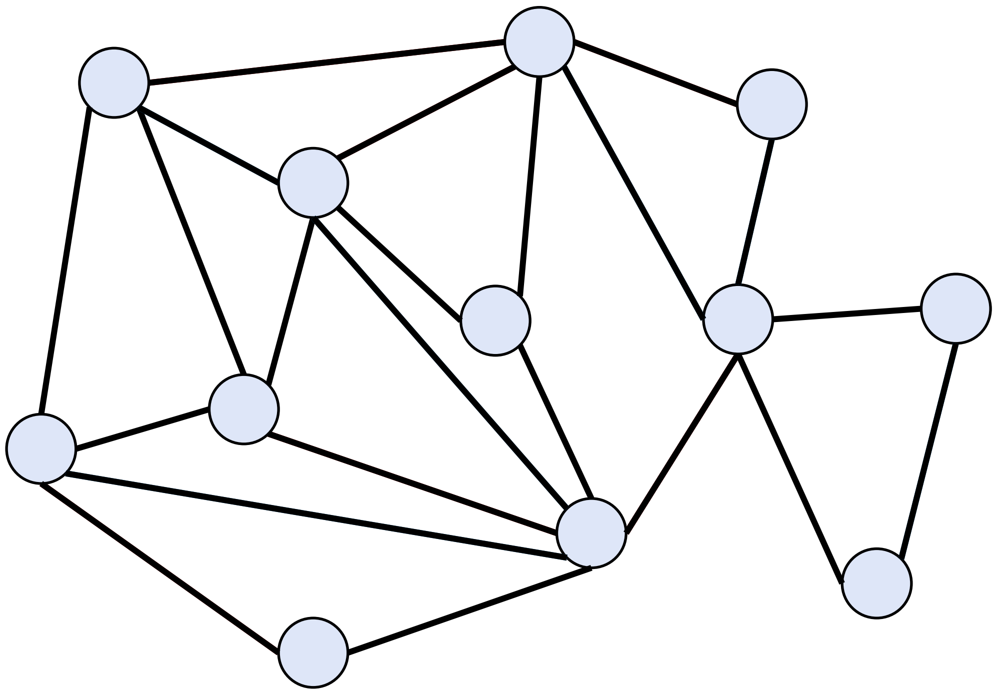
Dynamic Algorithms

- Updates to the dataset (e.g. graph) occurs where elements are added and deleted from the dataset

Edge **insertions/deletions** arrive **sequentially**

Maintain **graph property** after each update



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- Sometimes have **queries** (e.g. connectivity queries)

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- Dynamic maximum matching (find a matching of maximum size):
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Other algorithms have significantly larger runtimes!

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- Sometimes **large gap in runtimes**
  - **Polynomial** or **exponential gaps** in runtimes

[**L**, Srinivas COLT '24]

Types of Dynamic Algorithms

	Best Fully Dynamic		Best Offline/Partially Dynamic	
Planar Digraph APSP	$\widetilde{O}\left(n^{2/3}\right)$	[FR06, Kle05]	$\widetilde{O}(\sqrt{n})$	[DGWN22]
Triconnectivity	$O(n^{2/3})$	[GIS99]	$\widetilde{O}(1)$	[HR20, PSS17]
k-Edge Connectivity	$n^{o(1)}$	[JS22]	$\widetilde{O}(1)$	[CDK+21]
APSP	$\left(\frac{256}{k^2}\right)^{4/k}$-Approx $\widetilde{O}\left(n^k\right)$ update $\widetilde{O}(n^{k/8})$ query	[FGNS23]	$(2r-1)^k$-Approx $\widetilde{O}\left(m^{1/(k+1)}n^{k/r}\right)$	[CGH+20]
AP Maxflow/Mincut	$O(\log(n)\log\log n)$-Approx $\widetilde{O}\left(n^{2/3+o(1)}\right)$	[CGH+20]	$O\left(\log^{8k}(n)\right)$-Approx. $\widetilde{O}\left(n^{2/(k+1)}\right)$	[Gor19, GHS19]
MCF	$(1+\varepsilon)$-Approx $\widetilde{O}(1)$ update $\widetilde{O}(n)$ query	[CGH+20]	$O(\log^{8k}(n))$-Approx. $\widetilde{O}\left(n^{2/(k+1)}\right)$ update $\widetilde{O}(P^2)$ query	[Gor19, GHS19]
Uniform Sparsest Cut	$2^{O(\log^{5/6}(n))}$-Approx $2^{O(\log^{5/6}(n))}$ update $O(\log^{1/6}(n))$ query	[GRST21]	$O\left(\log^{8k}(n)\right)$-Approx $\widetilde{O}\left(n^{2/(k+1)}\right)$ $O(1)$ query	[Gor19, GHS19]
Submodular Max	$1/4$-Approx $\widetilde{O}(k^2)$	[DFL+23]	0.3178-Approx $\widetilde{O}(\text{poly}(k))$	[FLN+22]

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time for T updates

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$update$ is **worst-case update time** of **incremental/decremental** algorithm

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- [Henzinger-Saha-Seybold-Ye ITCS '24]:
  - Various lower bounds for different models of learning-augmented dynamic algorithms

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Degrades as a **linear function of the** L_1 **error**

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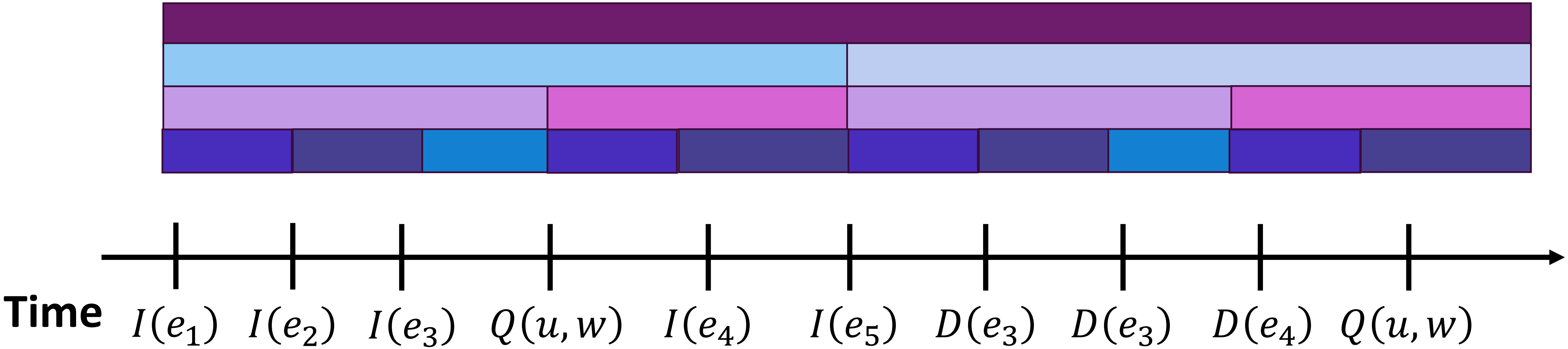
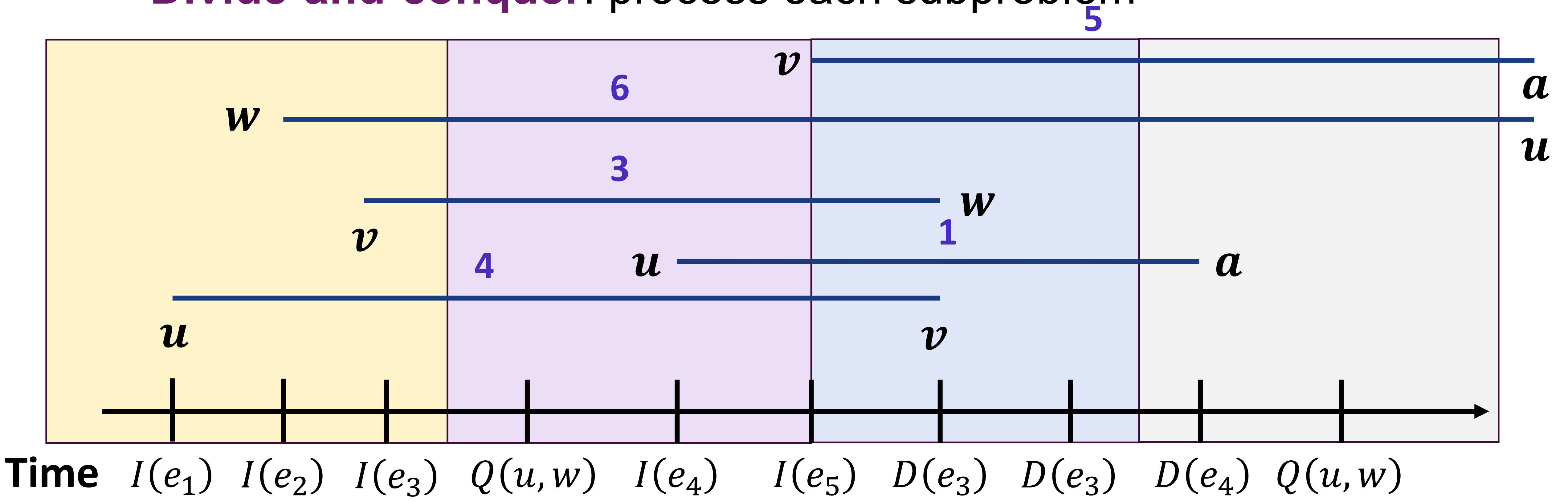


Solution for each update obtained from divide-and-conquer over update timestamps

Offline-Dynamic Minimum Spanning Tree

- Geometric representation of the problem
- **Divide-and-conquer**: process each subproblem



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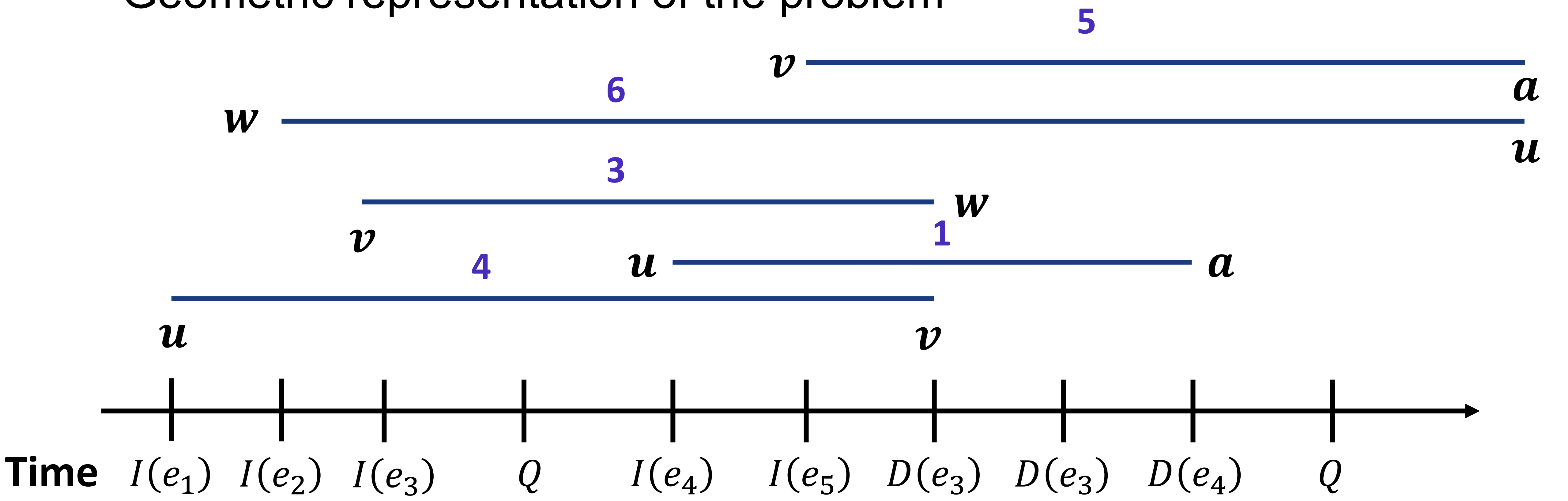

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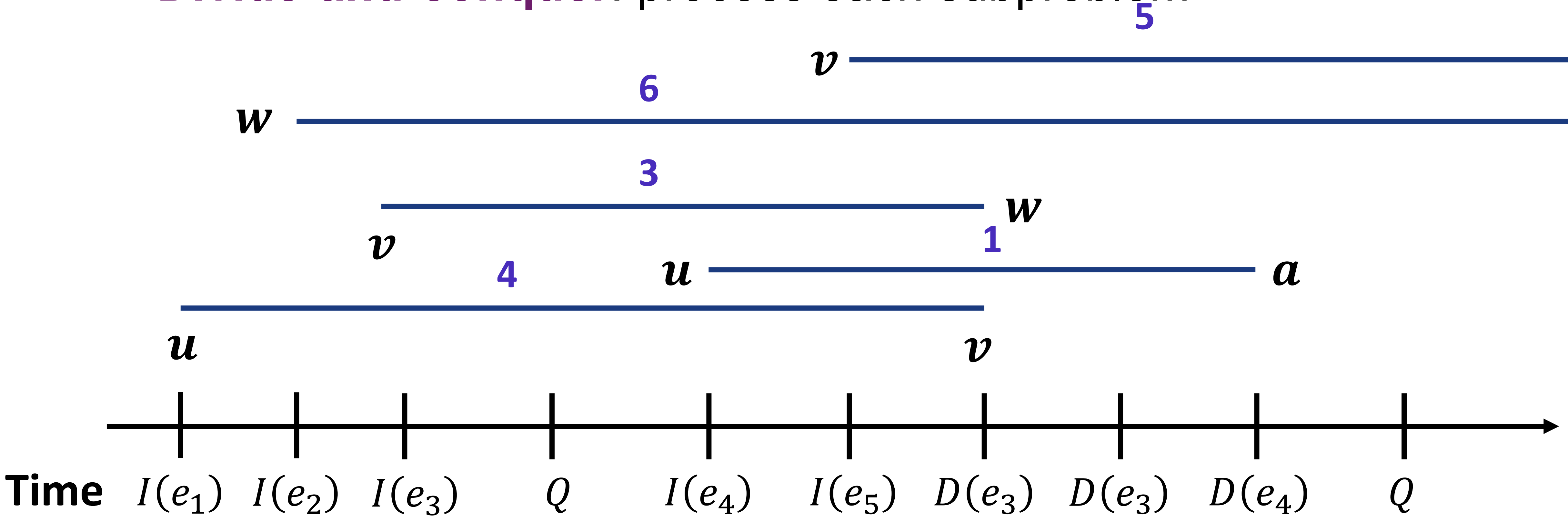

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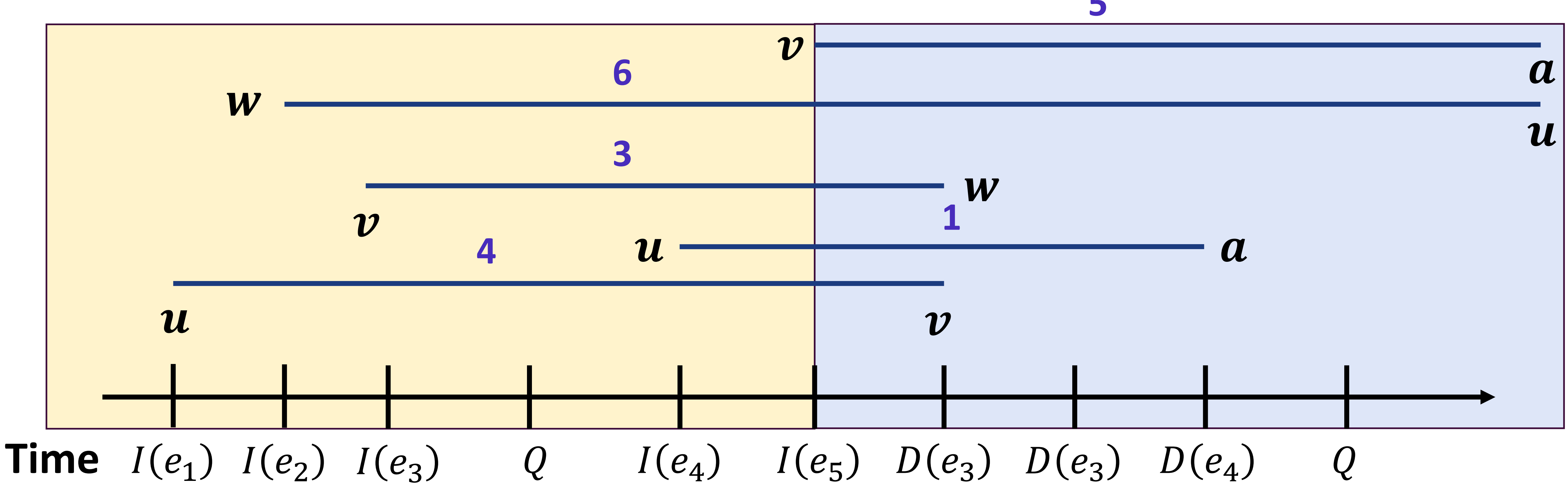

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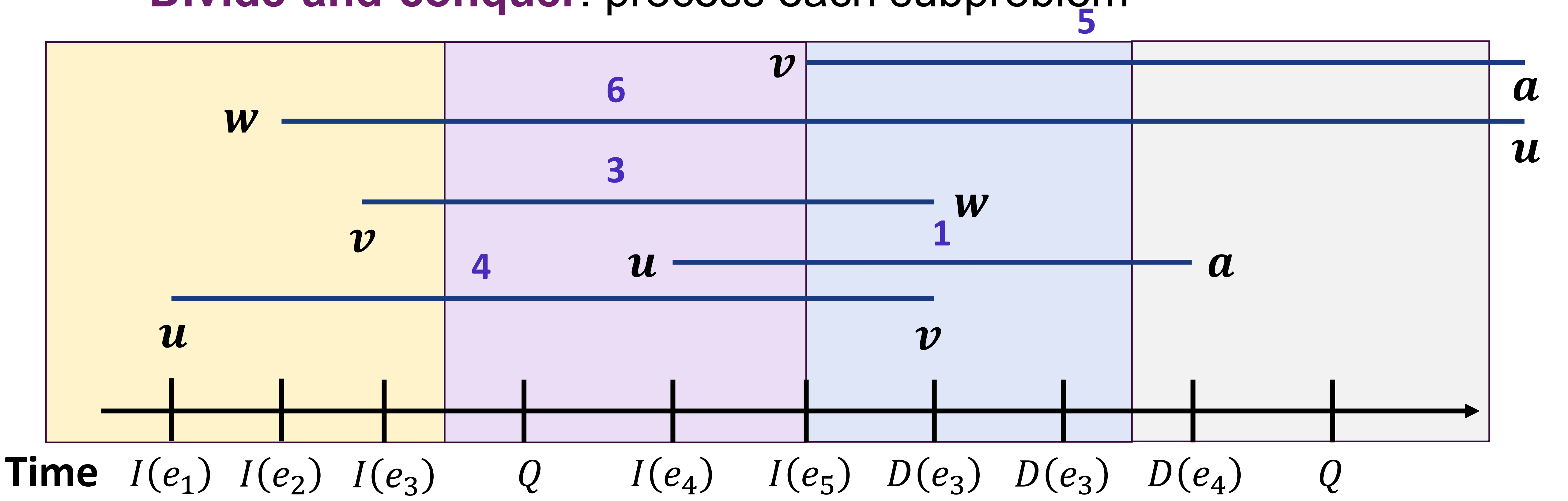

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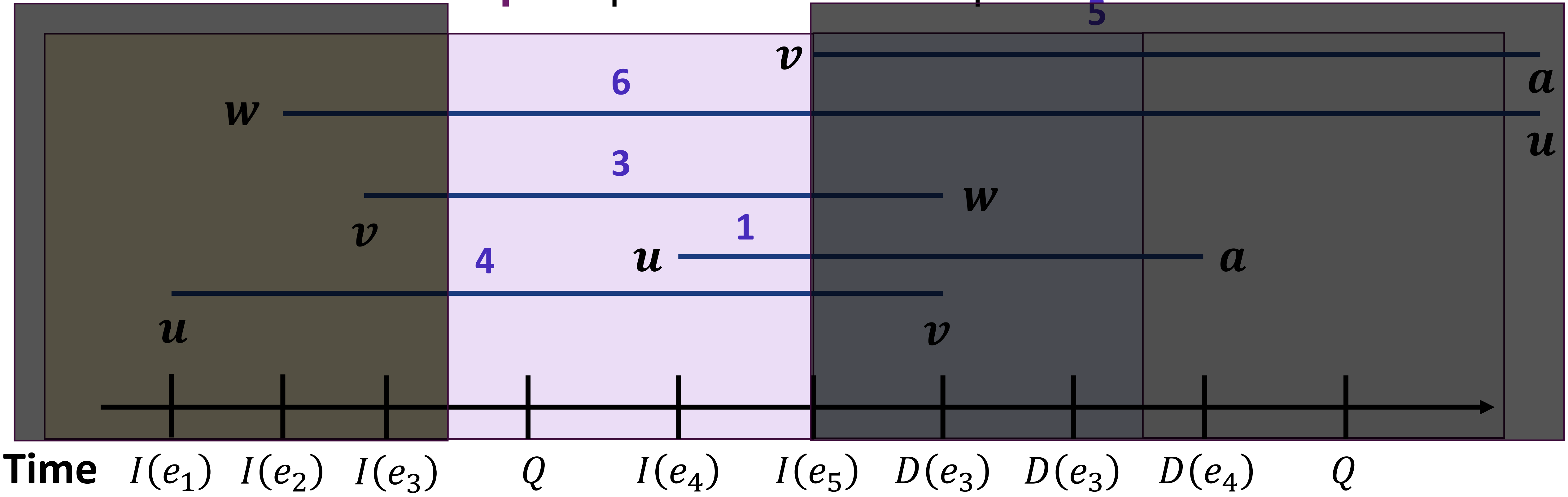

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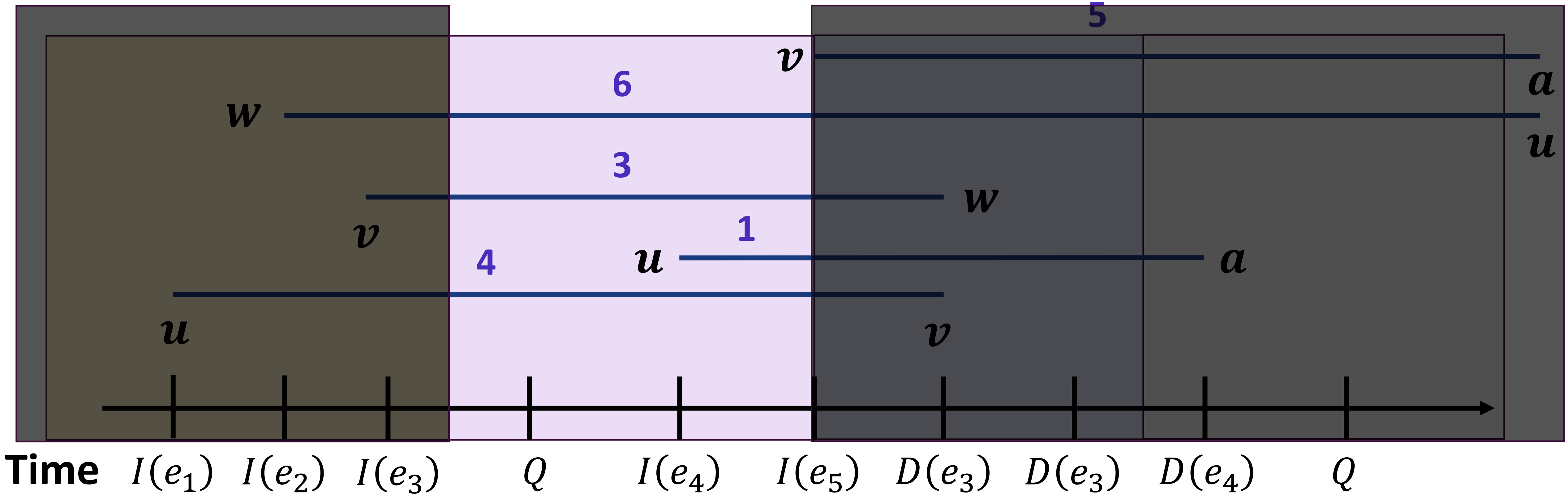

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- First, consider all **permanent edges** (edges that **go across the subproblem**)
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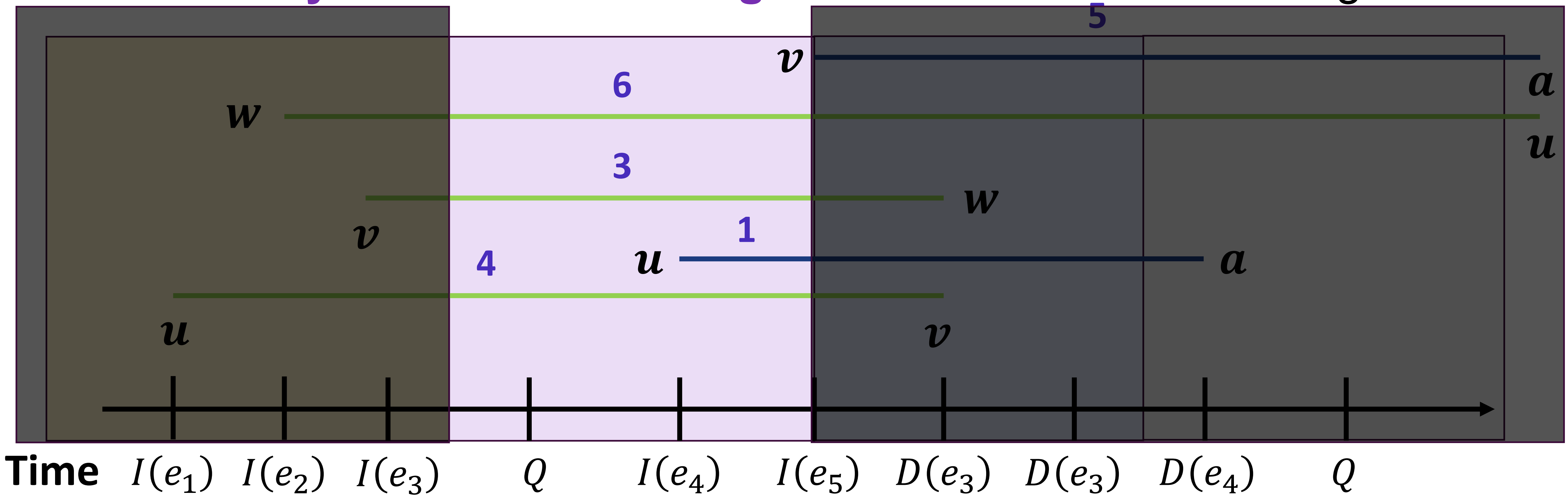

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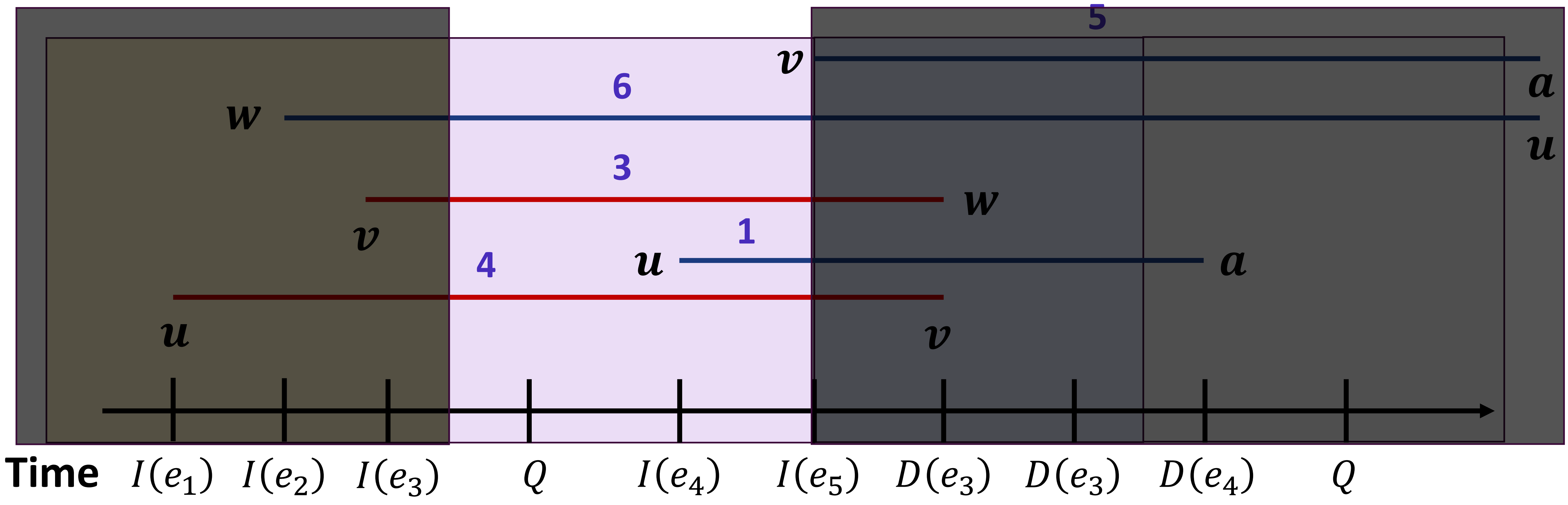

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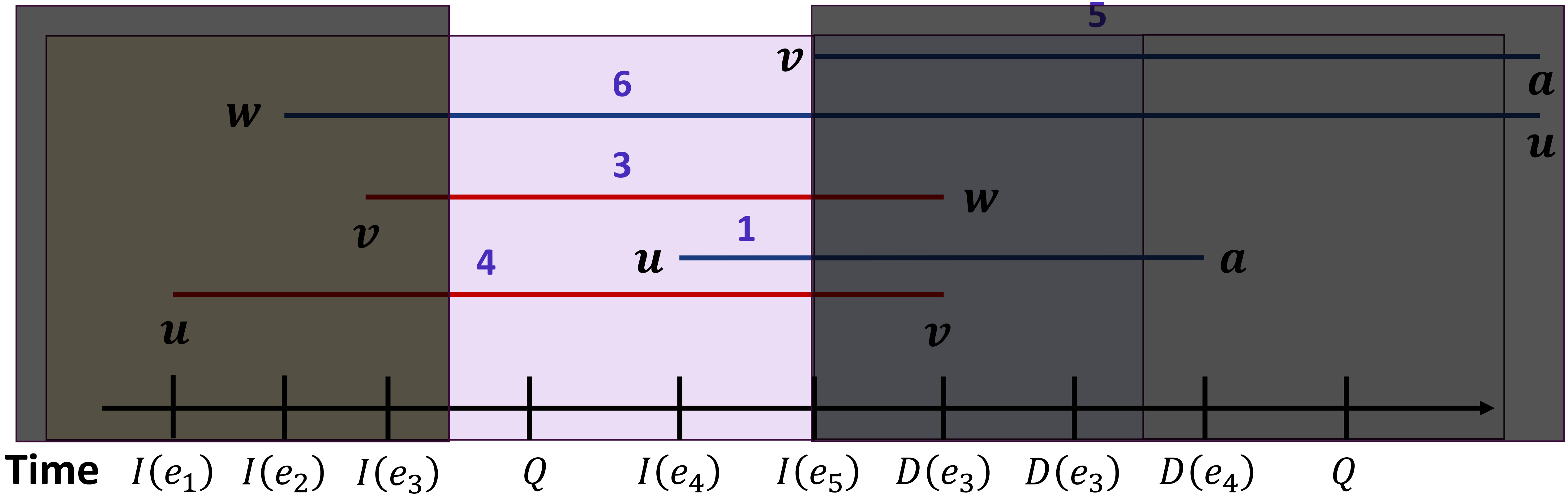

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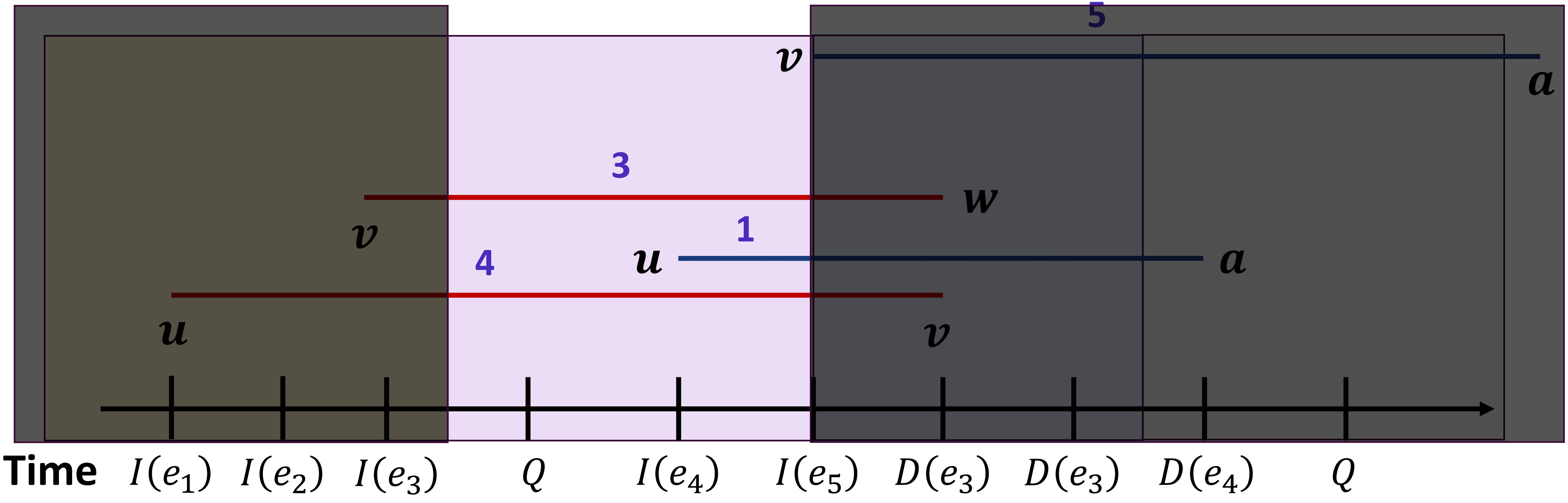

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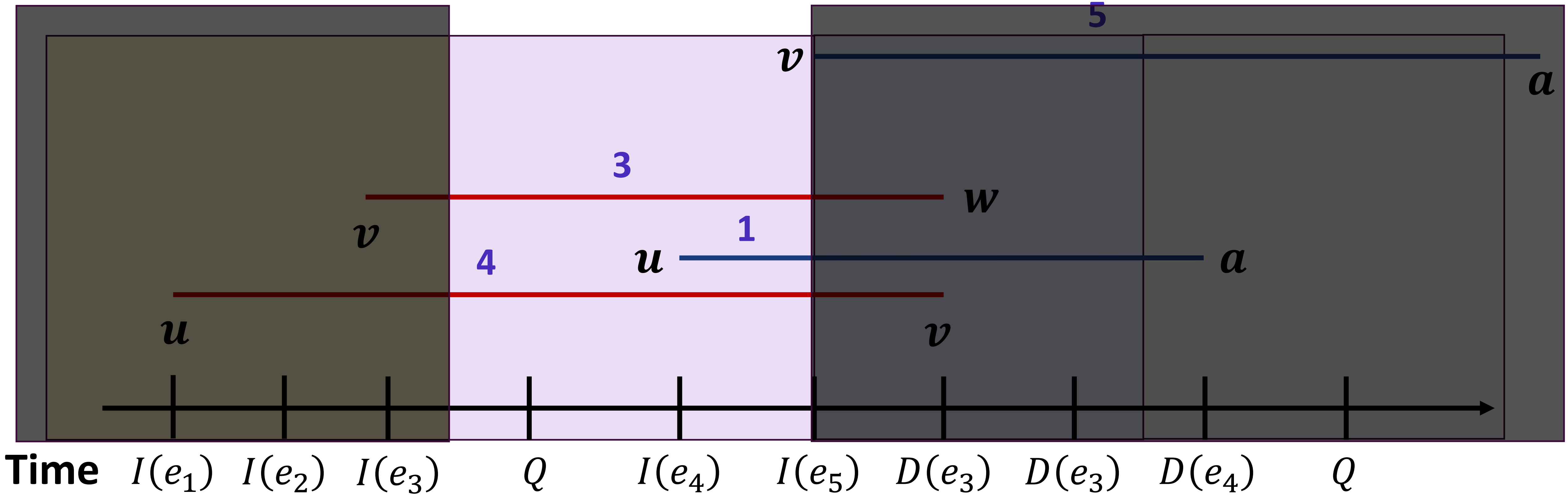

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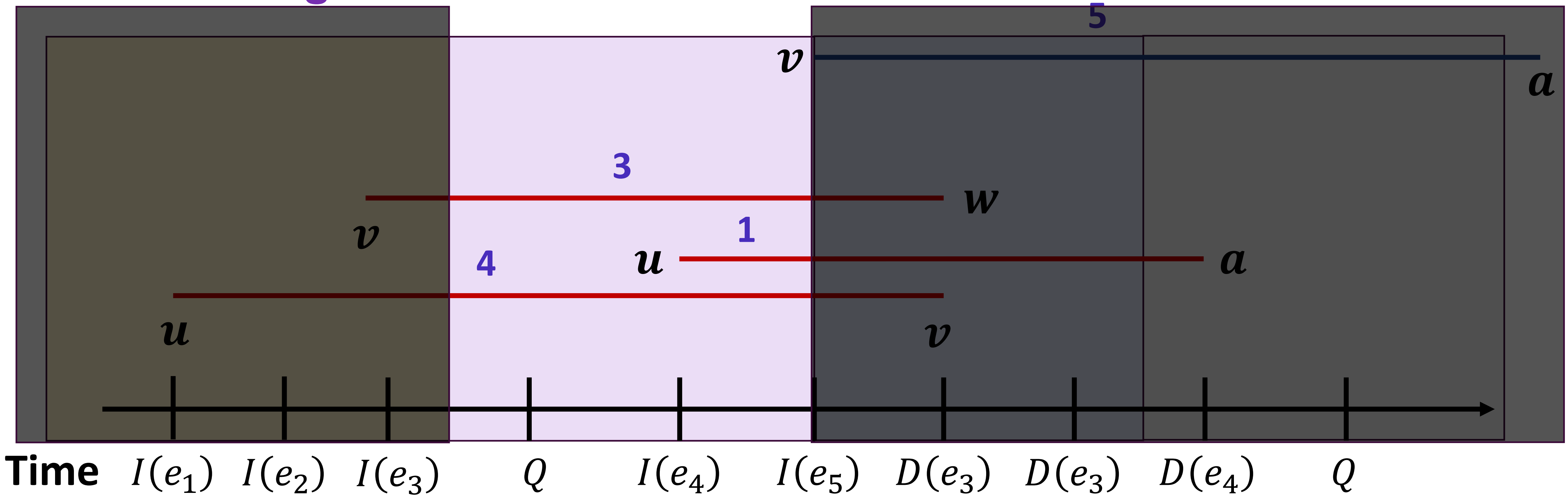
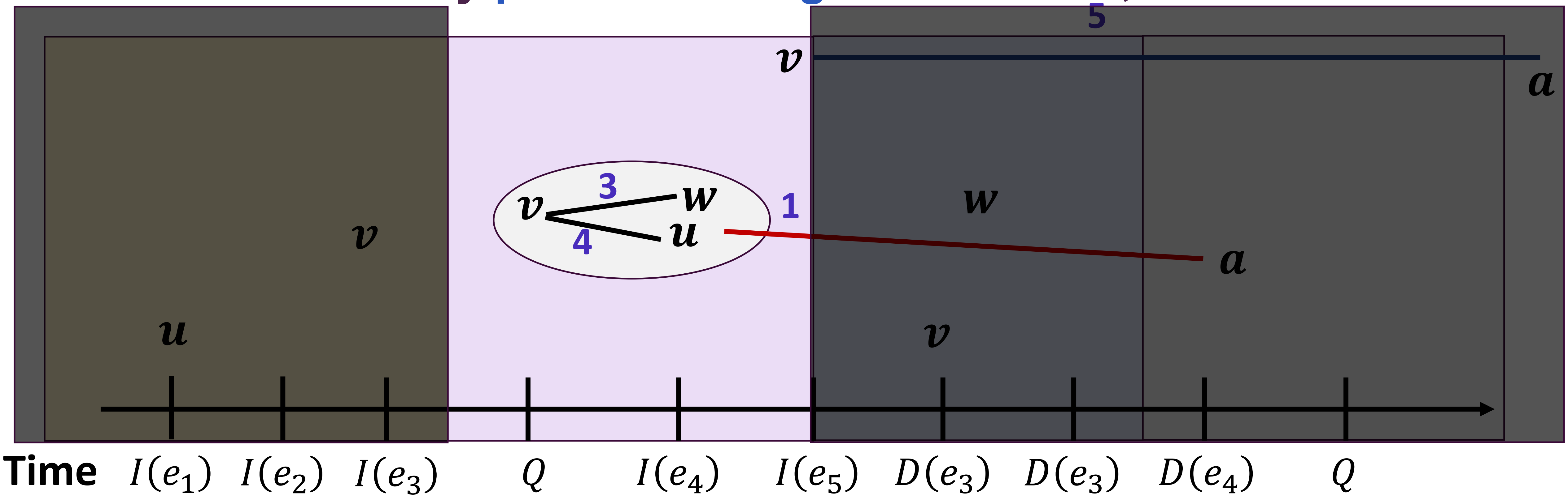


Offline-Dynamic Minimum Spanning Tree

- **Now consider all edges in subproblem**. Run **any linear time MST algorithm**
- **"Contract" any permanent edges in the MST;** link-cut tree



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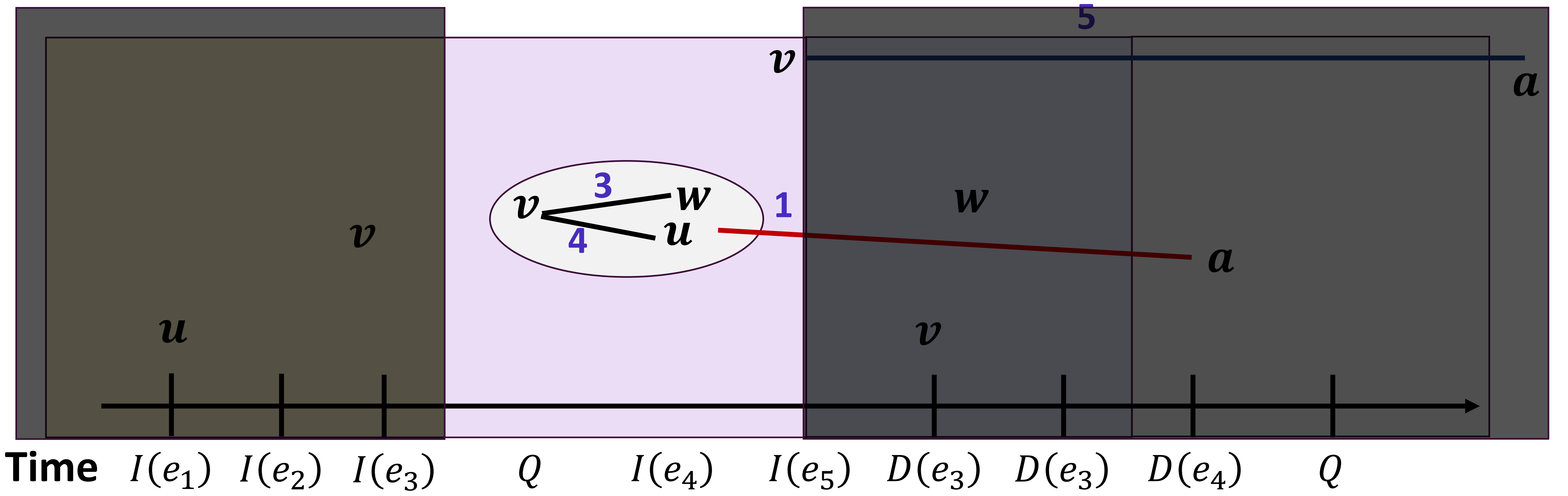

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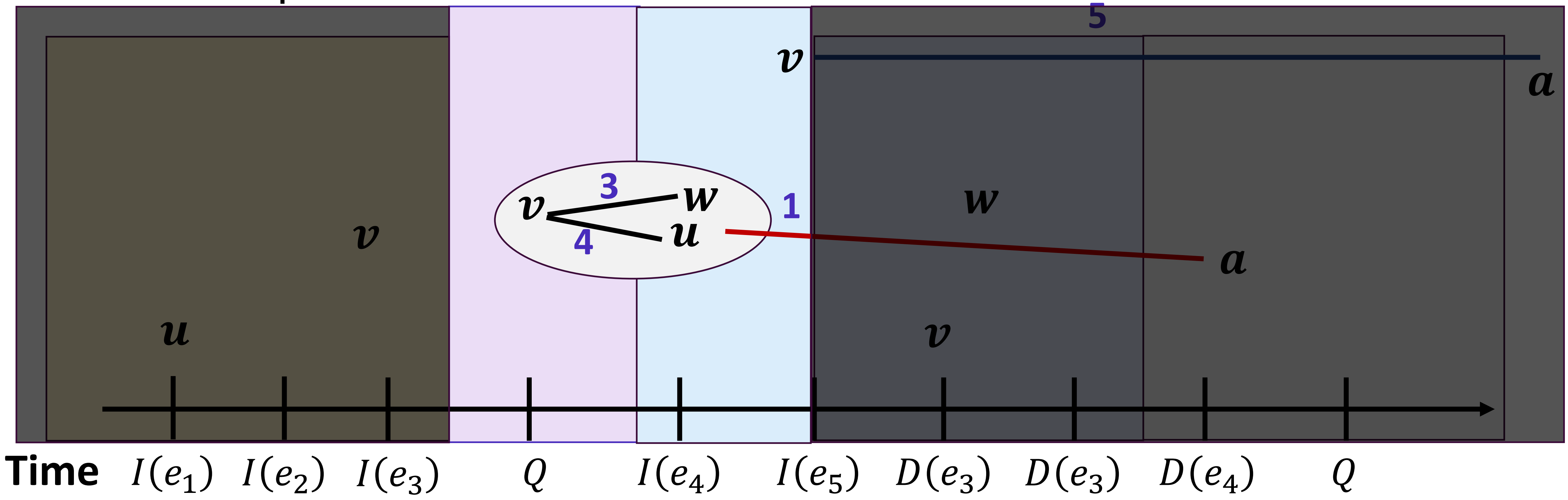
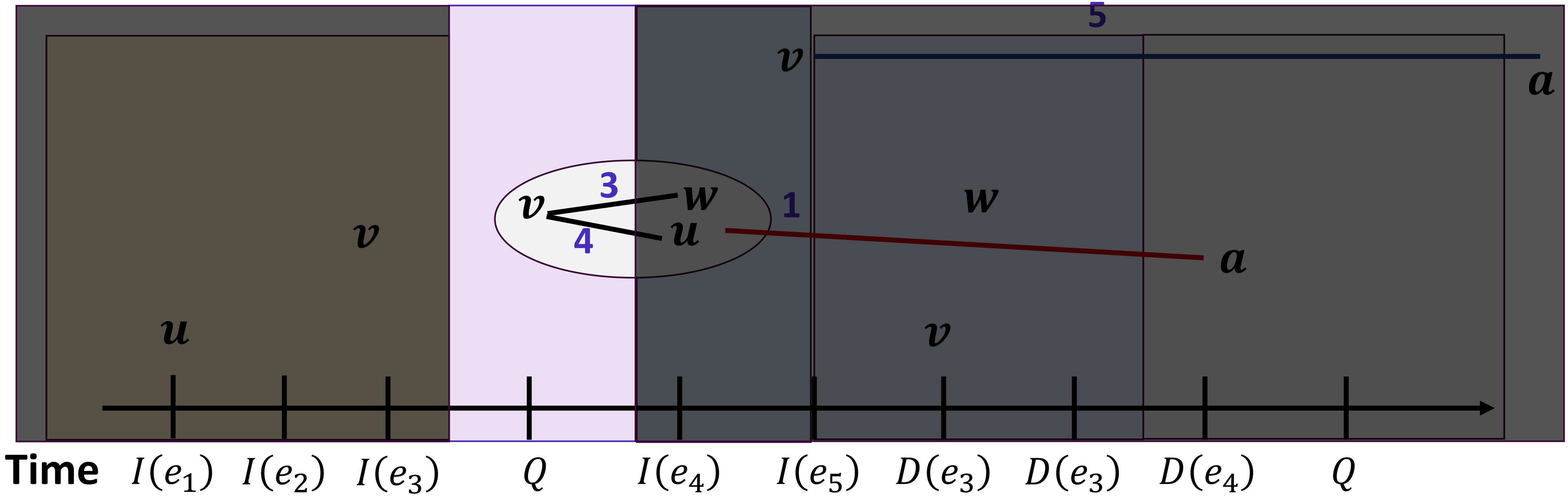
- Pass data structure to next smaller subproblem (persistence)
- Consider **non-contracted** and **not deleted edges permanent** in subproblem



Offline-Dynamic Minimum Spanning Tree

- **Queries**: consider tree at the **smallest subproblem containing the query Q**



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Total Runtime: $\tilde{O}(T \log(T))$ by Master Theorem

Offline-to-Fully Dynamic

- Learning-augmented minimum spanning tree:

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    - All updates and query **times** are given as **predictions**

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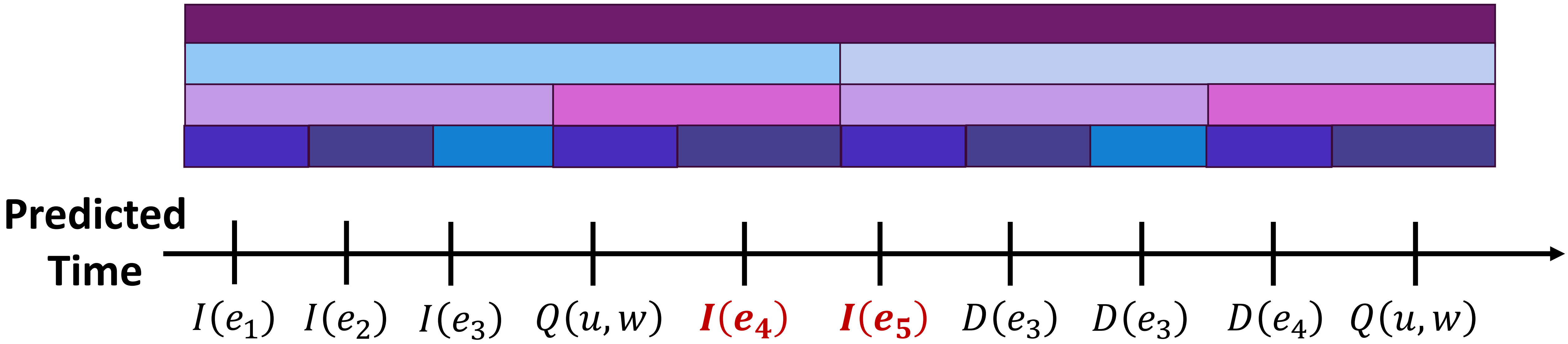
Offline-to-Fully Dynamic

- Learning-augmented minimum spanning tree:
  - **Predicted sequence**:
    - All updates and query **times** are given as **predictions**
    - Can give the predictions in batches
      - Will not discuss today
  - **Key Problem:** deterministic division of divide-and-conquer tree can lead to **arbitrarily bad runtime**

Offline to Online **First Attempt**

Use Offline Divide-and-Conquer Algorithm on Predicted Sequence



Offline to Online **First Attempt**

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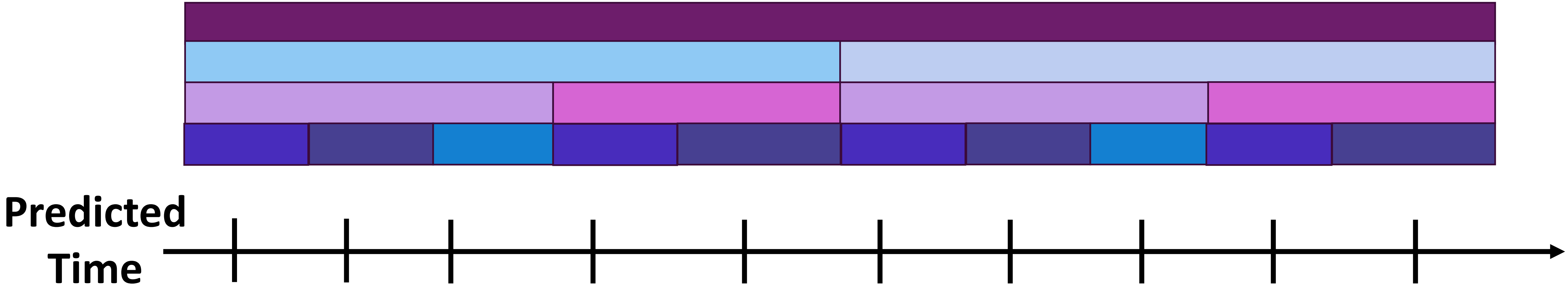


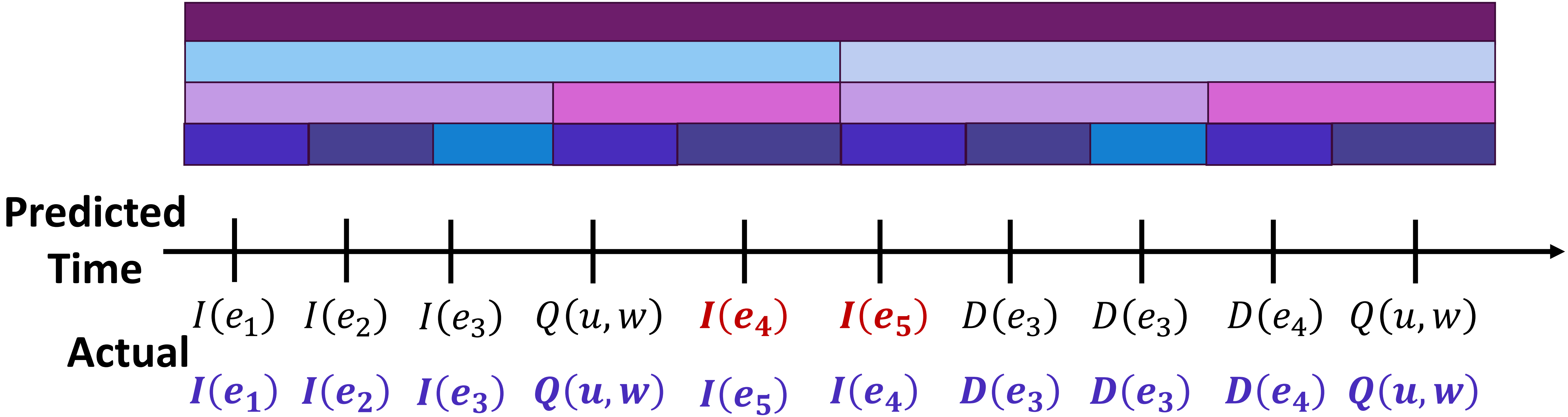
Predicted: $I(e_1)$ $I(e_2)$ $I(e_3)$ $Q(u,w)$ $\boldsymbol{I(e_4)}$ $\boldsymbol{I(e_5)}$ $D(e_3)$ $D(e_3)$ $D(e_4)$ $Q(u,w)$

Actual

$\boldsymbol{I(e_1)}$ $\boldsymbol{I(e_2)}$ $\boldsymbol{I(e_3)}$ $\boldsymbol{Q(u,w)}$ $\boldsymbol{I(e_5)}$ $\boldsymbol{I(e_4)}$ $\boldsymbol{D(e_3)}$ $\boldsymbol{D(e_3)}$ $\boldsymbol{D(e_4)}$ $\boldsymbol{Q(u,w)}$

Offline to Online **First Attempt**

Fix all subtrees up to largest subtree containing error as (i.e. redo subtrees containing $\boldsymbol{I(e_5)}$ as permanent edge; it becomes non-permanent)



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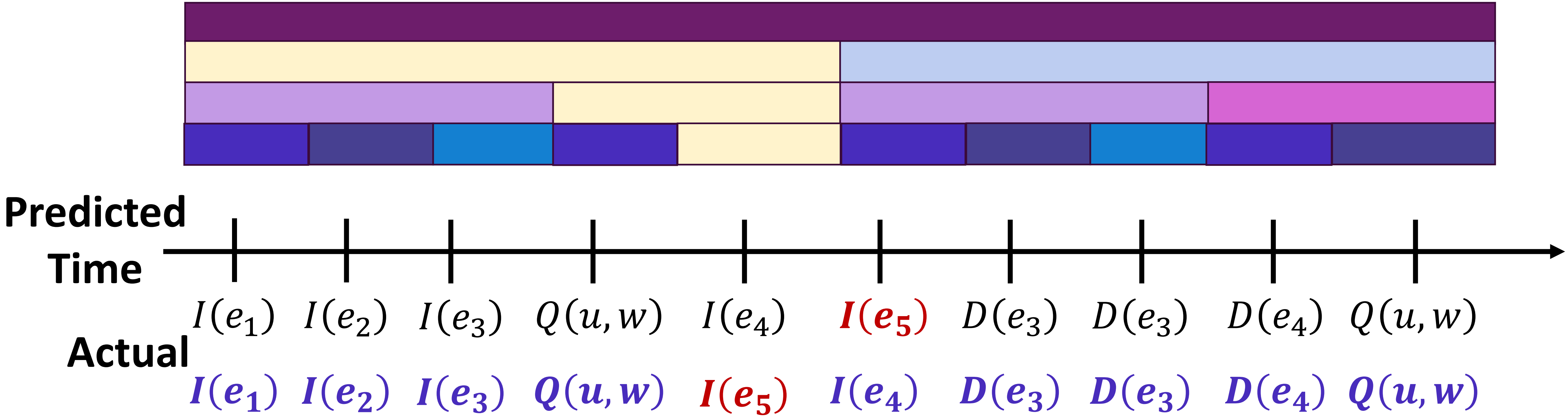
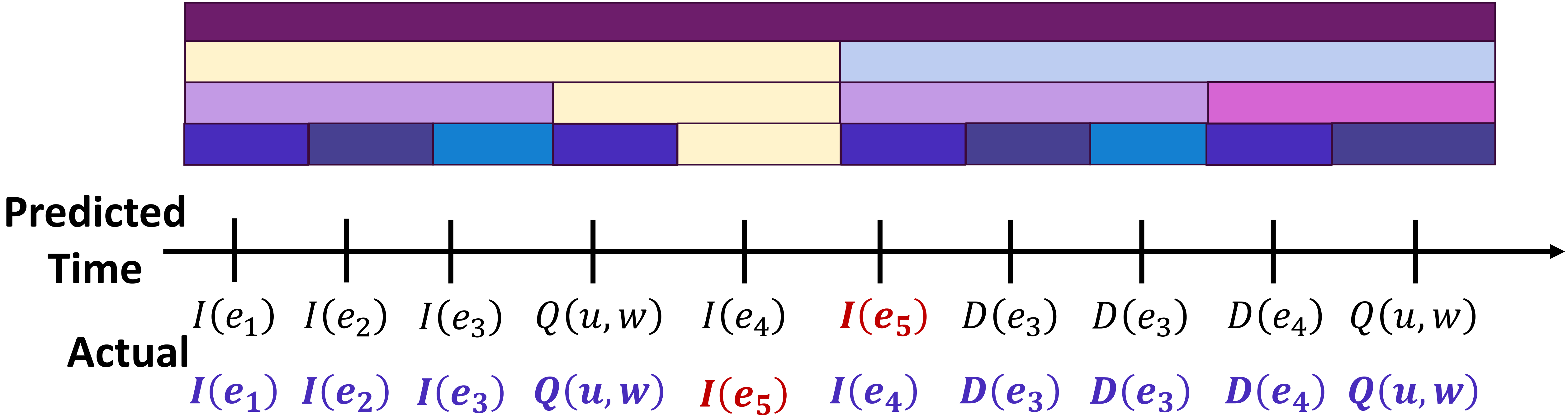


Offline to Online **First Attempt**

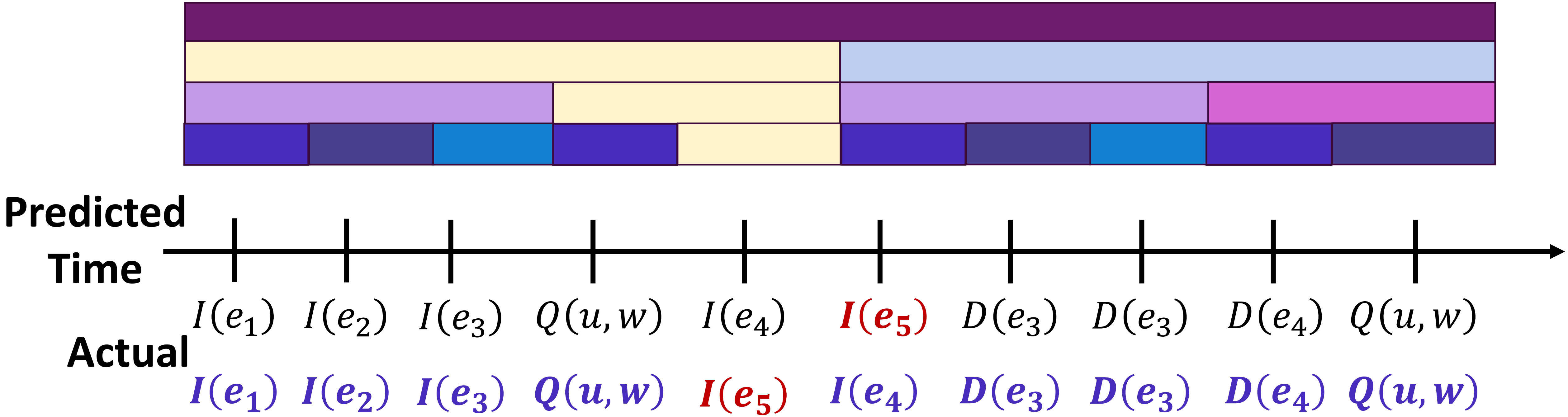
Issue: large subtree divides predicted and real timestamps



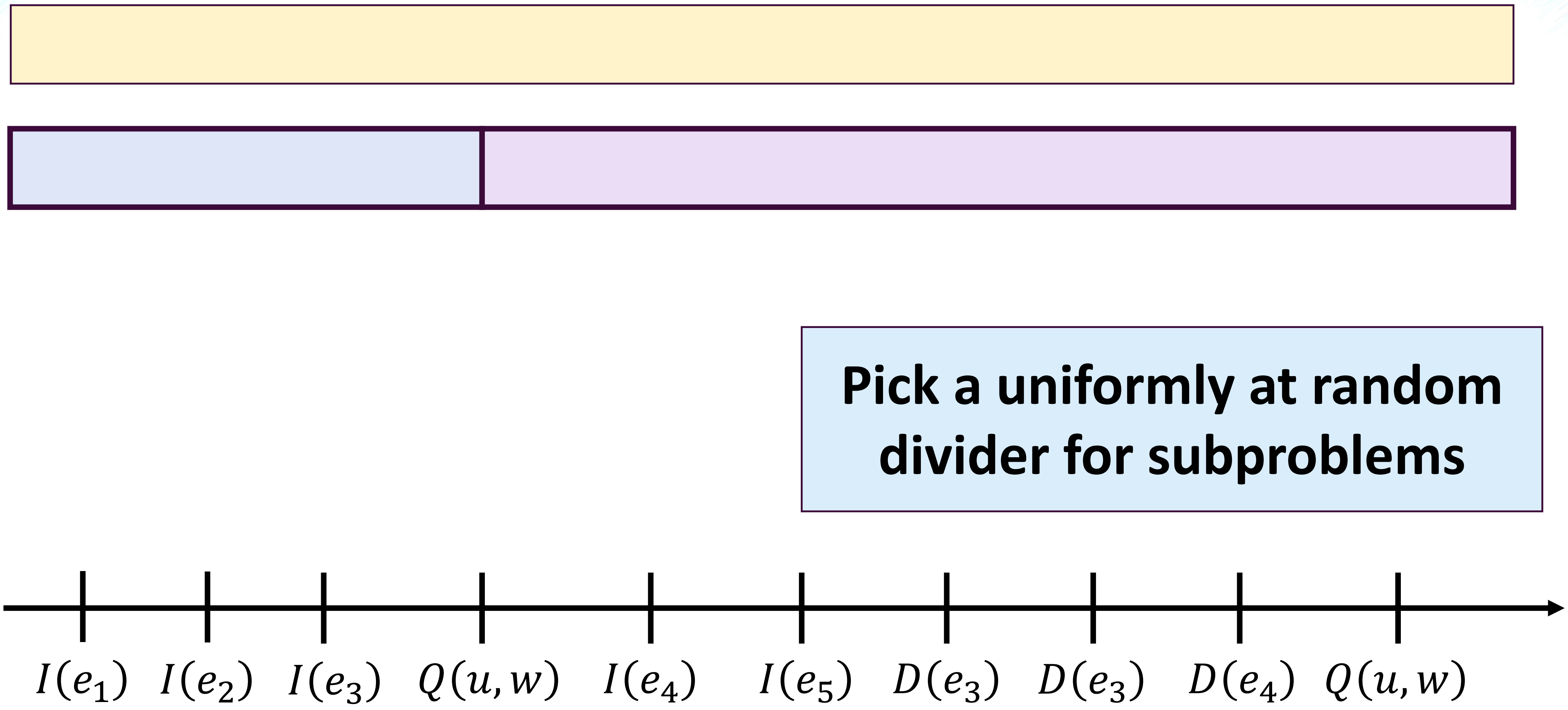
Offline to Online **First Attempt:** **Failed**

Issue: large subtree divides predicted and real timestamps

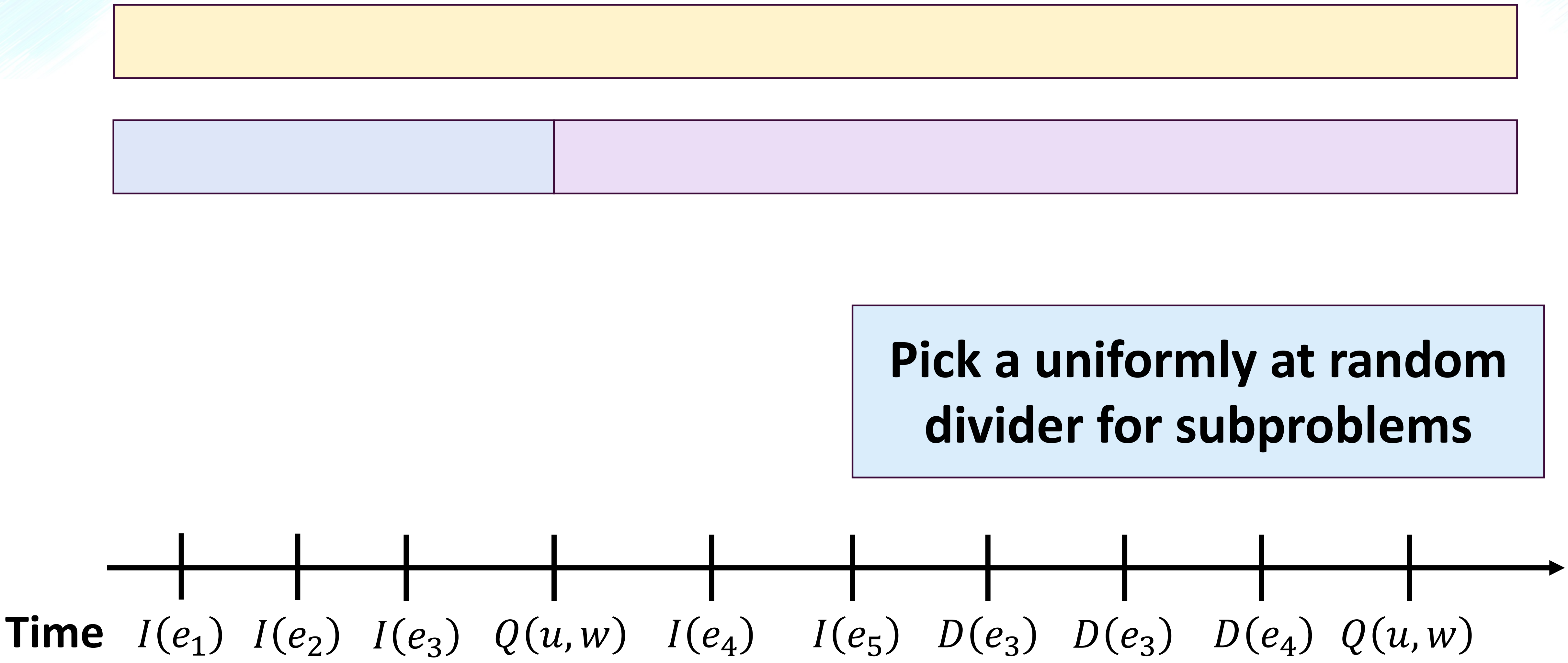
L_1 Error: **1**; Update time: $\boldsymbol{O(n)}$



Random Partition Tree Data Structure

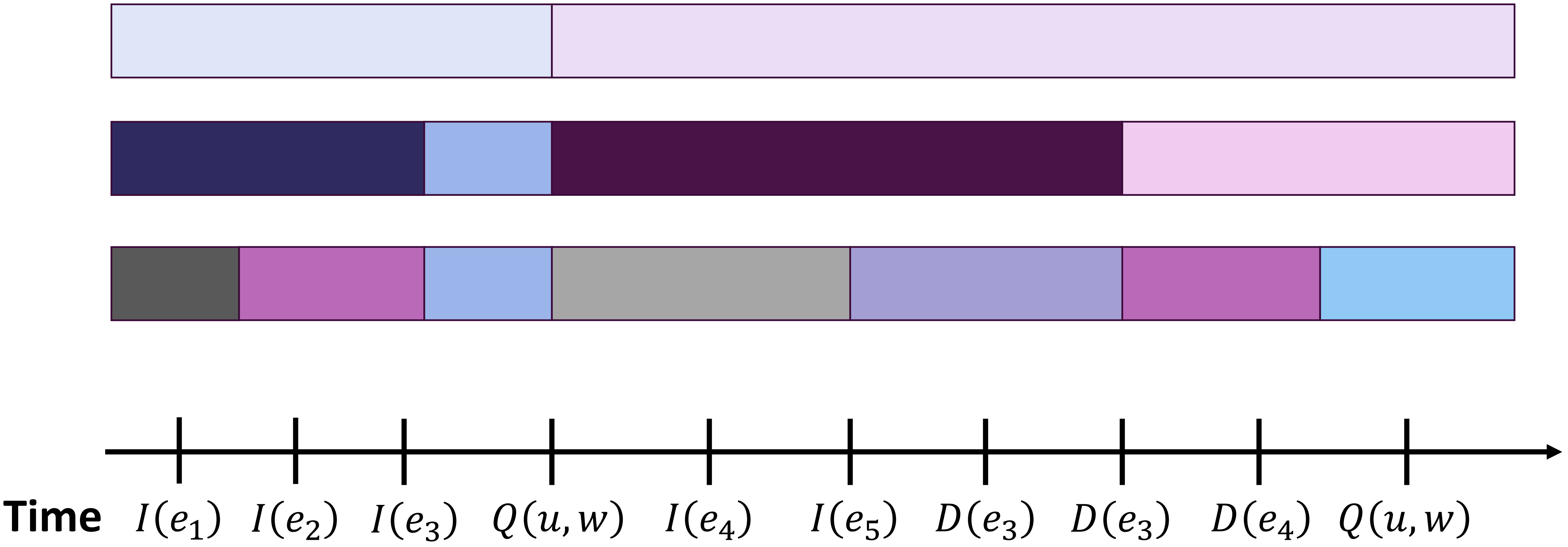


Random Partition Tree Data Structure



Random Partition Tree Data Structure

Run offline divide-and-conquer algorithm on randomly picked subproblems



Random Partition Tree Data Structure

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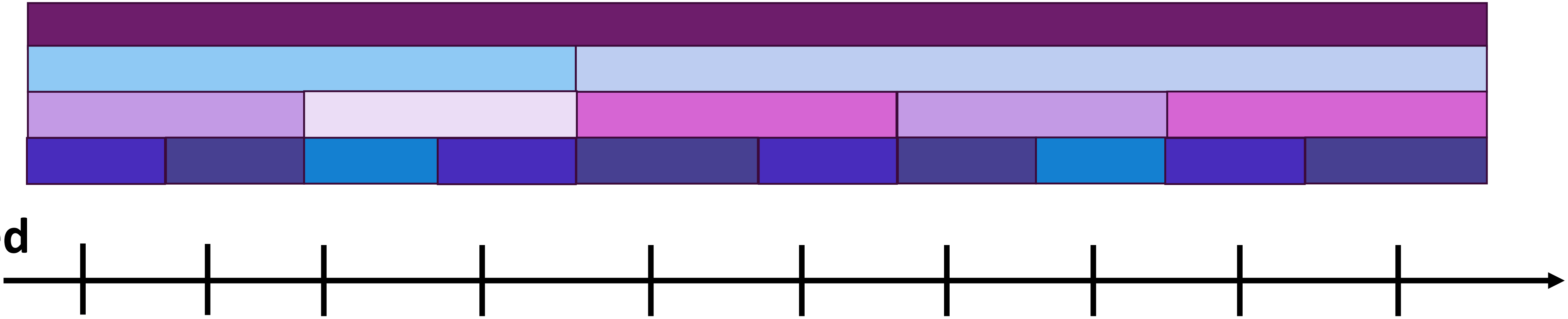

Predicted	$I(e_1)$	$I(e_2)$	$I(e_3)$	$Q(u,w)$	$\boldsymbol{I(e_4)}$	$\boldsymbol{I(e_5)}$	$D(e_3)$	$D(e_3)$	$D(e_4)$	$Q(u,w)$
Actual	$\boldsymbol{I(e_1)}$	$\boldsymbol{I(e_2)}$	$\boldsymbol{I(e_3)}$	$\boldsymbol{Q(u,w)}$	$\boldsymbol{I(e_5)}$	$\boldsymbol{I(e_4)}$	$\boldsymbol{D(e_3)}$	$\boldsymbol{D(e_3)}$	$\boldsymbol{D(e_4)}$	$\boldsymbol{Q(u,w)}$

Random Partition Tree Data Structure

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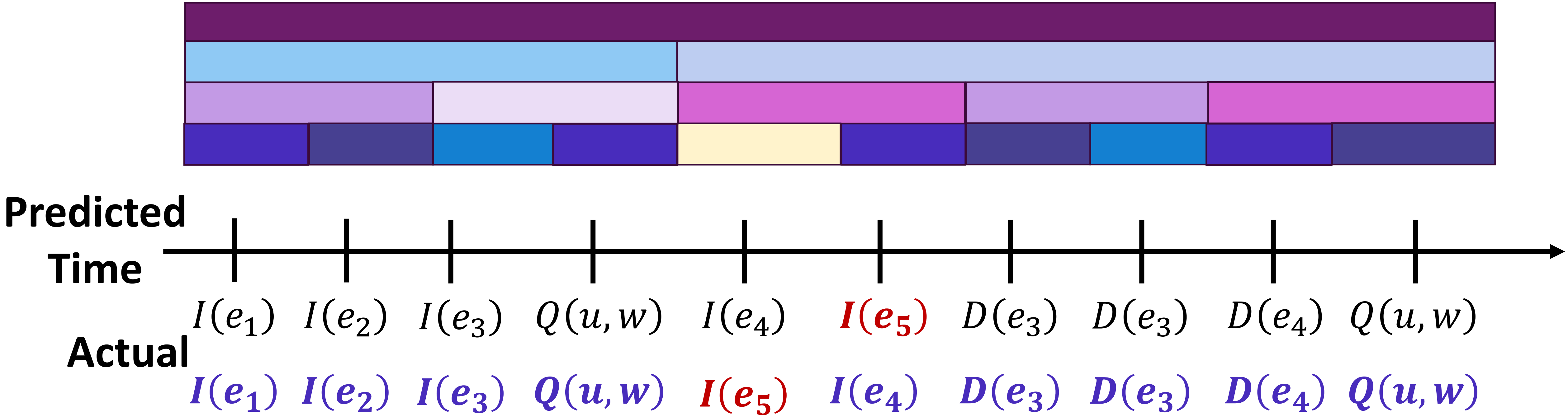
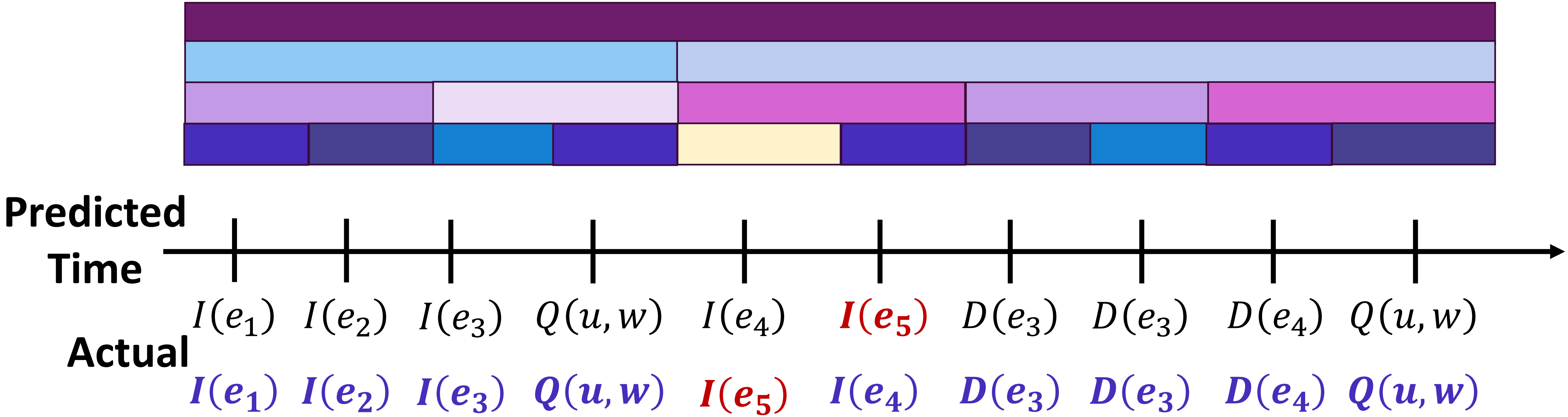


Random Partition Tree Data Structure

Purpose of the random partition tree: in expectation size of subproblem (largest subtree going in between) equal to L_1 error



Random Partition Tree Data Structure

Purpose of the random partition tree: in expectation size of subproblem (smallest subtree) equal to L_1 error

Proof: Coupling argument for splitting subproblems to drawing dividers:

1. For each divider between two timestamps in the original sequence, draw value (called rank) in $[0, 1]$

Random Partition Tree Data Structure

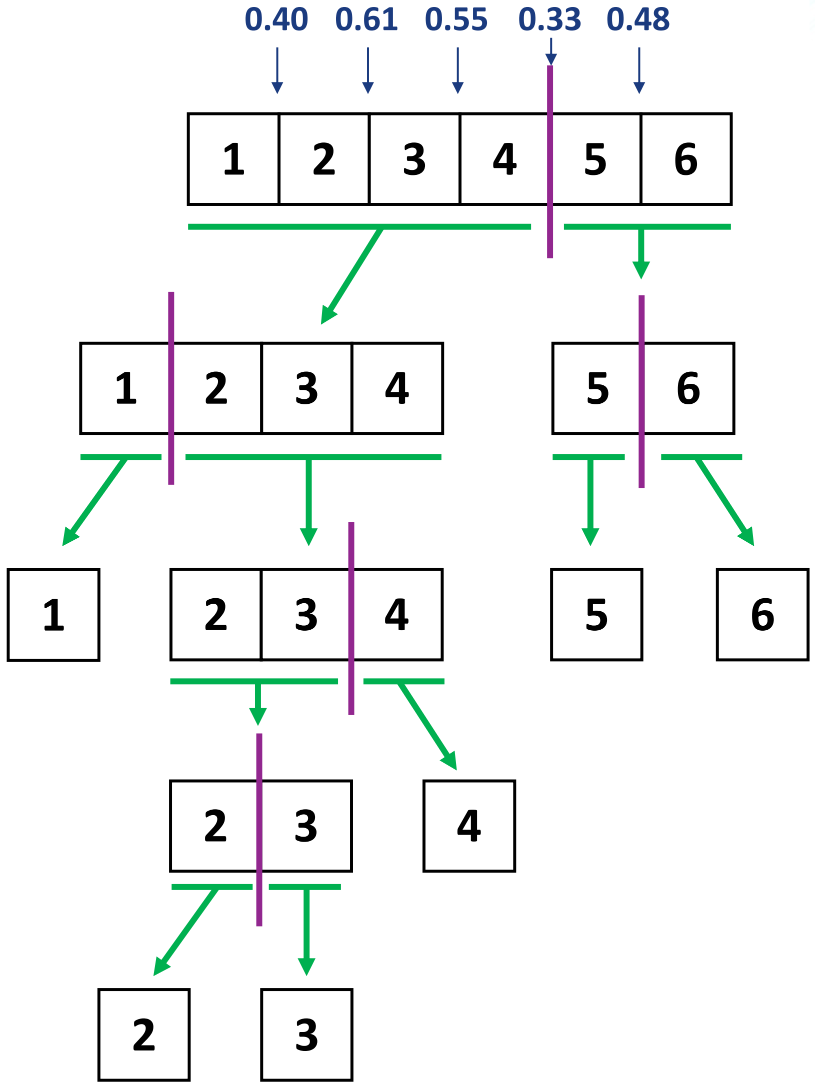
Purpose of the random partition tree: in expectation size of subproblem (smallest subtree) equal to L_1 error

Proof: Coupling argument for splitting subproblems to drawing dividers:

1. For each divider between two timestamps in the original sequence, draw value (called rank) in $[0, 1]$
2. Divide sequence of updates lowest rank first

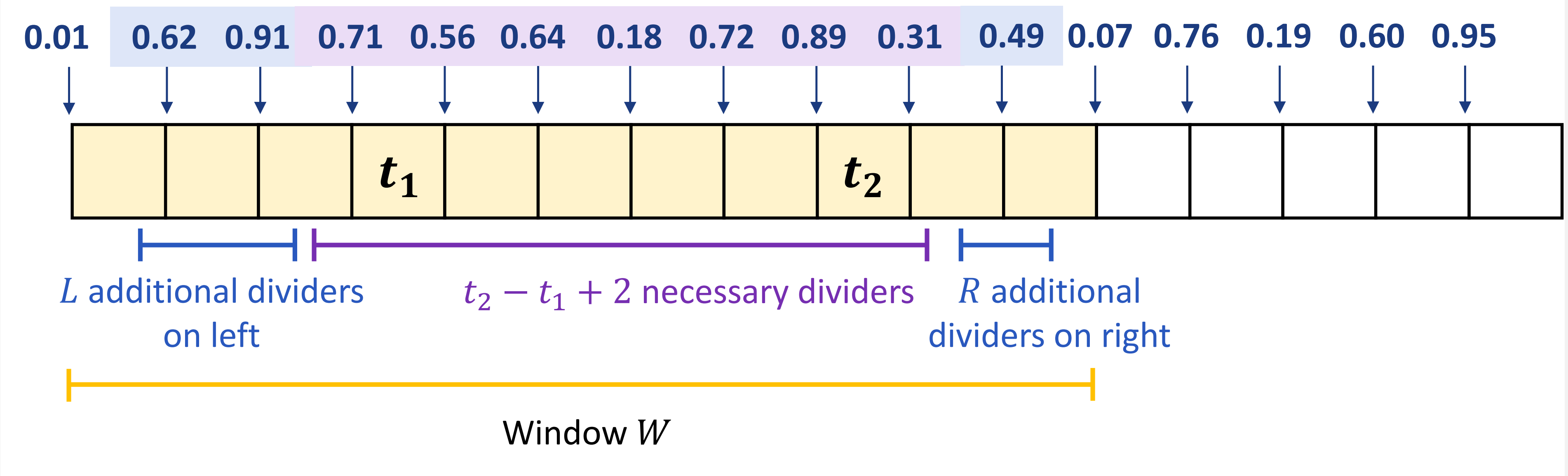
Random Partition Tree Data Structure

Example Random Tree produced via drawing dividers



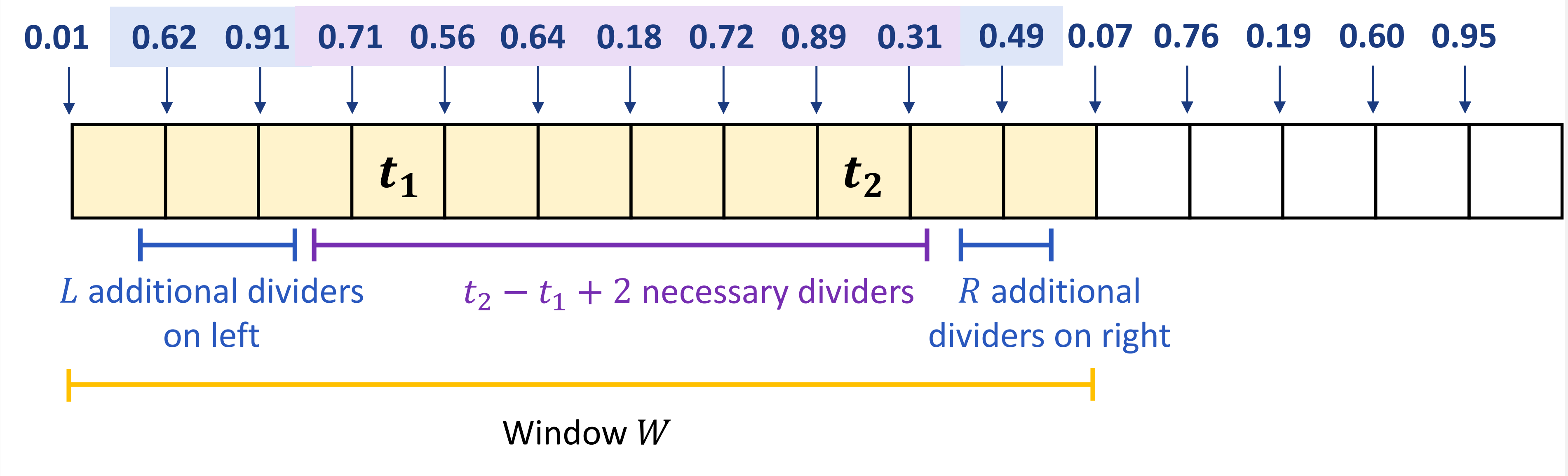
Random Partition Tree Data Structure

Consider an error that occurred at times t_1, t_2



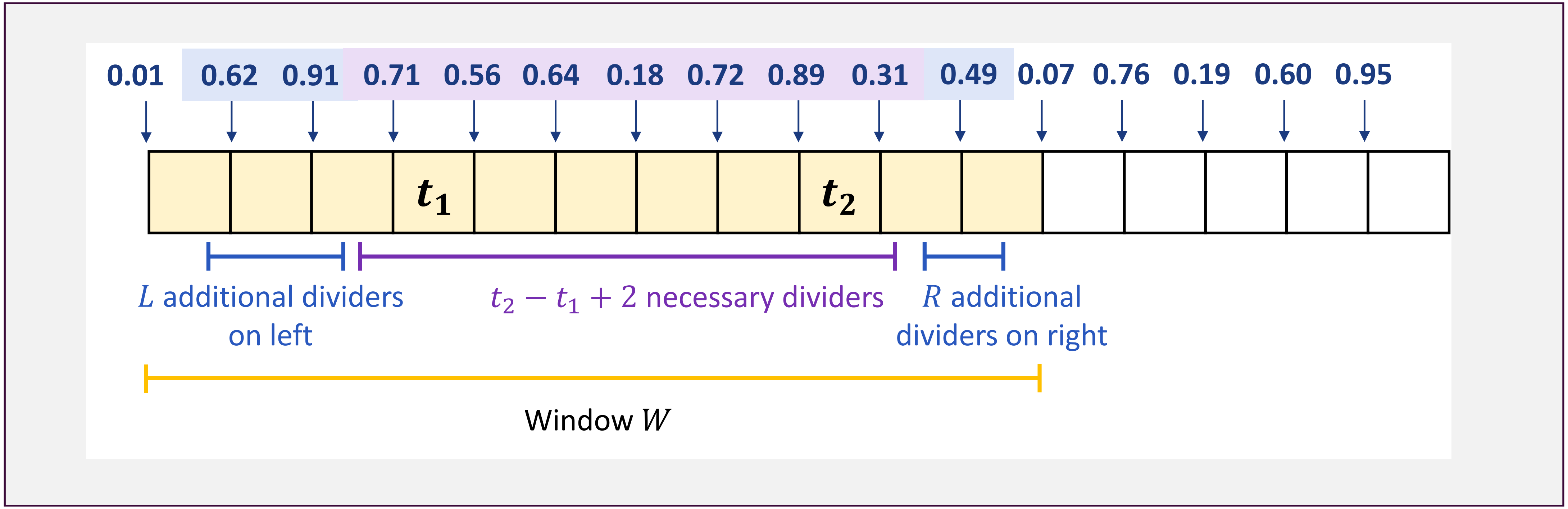
Random Partition Tree Data Structure

Expected window W size is equal to $L + R + (t_2 - t_1 + 1)$



Random Partition Tree Data Structure

$$\mathbf{E}[W] = O(|t_2 - t_1| \cdot \log(T))$$



Predicted-Updates Result

$$\mathbf{E}[W] = O\left(|t_2 - t_1| \cdot \log(T)\right)$$

Expected Work per error t_1, t_2: $\tilde{O}(|t_2 - t_1|)$

for each recomputation over window $[t_1, t_2]$ (leaf dominated divide-and-conquer)

Predicted-Updates Result

$$\mathbf{E}[W] = O\big(|t_2 - t_1| \cdot \log(T)\big)$$

Expected Work per error t_1, t_2: $\tilde{O}(|t_2 - t_1|)$

Total expected work: $\tilde{O}(|\mathbf{p} - \mathbf{r}|_1)$

Boosting to high probability via $O(\log n)$ independent trials

Predicted-Updates Result

- Runtime in terms of L_1-error between predictions and real
  - $\mathbf{p}$ vector of **predicted** timestamps
  - $\mathbf{r}$ vector of **real** timestamps
  - L_1 error: $|\mathbf{p} - \mathbf{r}|_1$

$\boldsymbol{update}$ is **worst-case update time** of **incremental/decremental** algorithm

Runtime same as partially dynamic with total

$$\tilde{O}\left(\left(||\mathbf{p} - \mathbf{r}||_1 + T\right) \cdot \boldsymbol{update}\right)$$

time for T updates

[L-Srinivas COLT '24]

The Predicted-Updates Model

	Best Fully Dynamic		Best Predicted-Updates when $\|\|\mathbf{p}-\mathbf{r}\|\|_1 = \tilde{O}(T)$	
Planar Digraph APSP	$\tilde{O}\left(n^{2/3}\right)$	[FR06, Kle05]	$\tilde{O}(\sqrt{n})$	[DGWN22]
Triconnectivity	$O(n^{2/3})$	[GIS99]	$\tilde{O}(1)$	[HR20, PSS17]
k-Edge Connectivity	$n^{o(1)}$	[JS22]	$\tilde{O}(1)$	[CDK+21]
APSP	$\left(\frac{256}{k^2}\right)^{4/k}$-Approx $\tilde{O}\left(n^k\right)$ update $\tilde{O}(n^{k/8})$ query	[FGNS23]	$(2r-1)^k$-Approx $\tilde{O}\left(m^{1/(k+1)}n^{k/r}\right)$	[CGH+20]
AP Maxflow/Mincut	$O(\log(n)\log\log n)$-Approx $\tilde{O}\left(n^{2/3+o(1)}\right)$	[CGH+20]	$O\left(\log^{8k}(n)\right)$-Approx. $\tilde{O}\left(n^{2/(k+1)}\right)$	[Gor19, GHS19]
MCF	$(1+\varepsilon)$-Approx $\tilde{O}(1)$ update $\tilde{O}(n)$ query	[CGH+20]	$O(\log^{8k}(n))$-Approx. $\tilde{O}\left(n^{2/(k+1)}\right)$ update $\tilde{O}(P^2)$ query	[Gor19, GHS19]
Uniform Sparsest Cut	$2^{O(\log^{5/6}(n))}$-Approx $2^{O(\log^{5/6}(n))}$ update $O(\log^{1/6}(n))$ query	[GRST21]	$O\left(\log^{8k}(n)\right)$-Approx $\tilde{O}\left(n^{2/(k+1)}\right)$ $O(1)$ query	[Gor19, GHS19]
Submodular Max	$1/4$-Approx $\tilde{O}(k^2)$	[DFL+23]	0.3178-Approx $\tilde{O}(\text{poly}(k))$	[FLN+22]

Conclusion

Link Prediction – Predict edges in a network using Networkx

Last Updated : 08 May, 2020

- **Practicality of link-prediction:**
  - Lots of work on insertion link prediction

RESEARCH ARTICLE | COMPUTER SCIENCES |

Link prediction using low-dimensional node embeddings: The measurement problem

Nicolas Menand and C. Seshadhri Authors Info & Affiliations

Towards Better Evaluation for Dynamic Link Prediction

Farimah Poursafaei*, Shenyang Huang*, Kellin Pelrine, Reihaneh Rabbany
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Learning Spectral Graph Transformations for Link Prediction

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Link prediction

Article Talk

From Wikipedia, the free encyclopedia

Conclusion

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  - **Link deletions** much less well-studied

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Ongoing Sridharbaskari-Srinivas-**L** ‘24