Structural Rounding: Approximation Algorithms for Graphs Near an Algorithmically Tractable Class

STOC 2018

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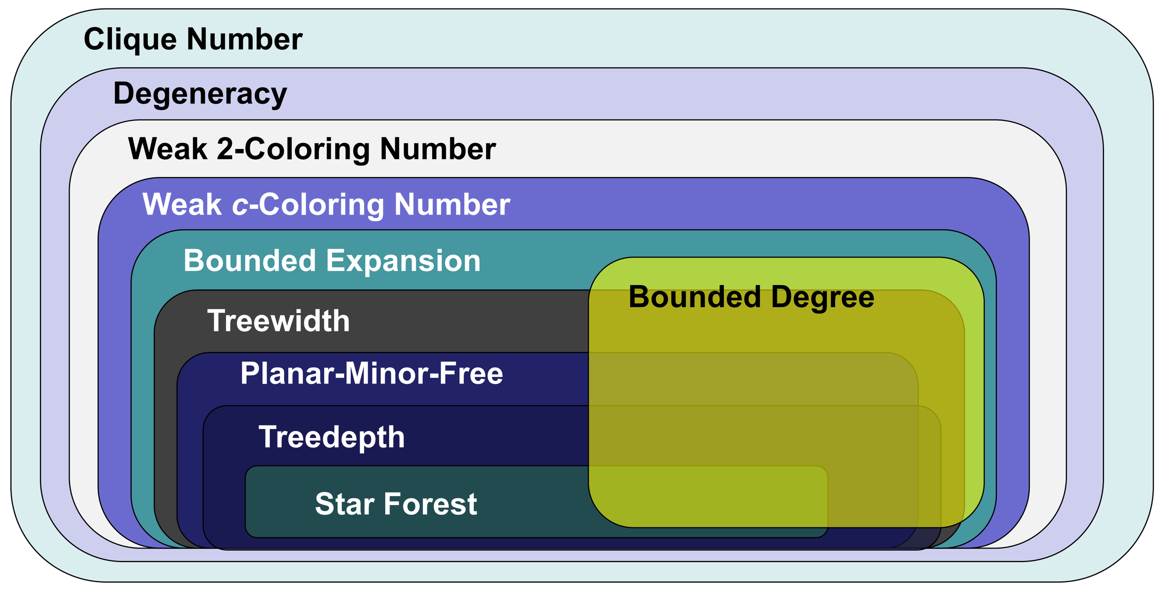
Motivation

Some real-world networks are small perturbations of graphs from a structural class due to natural variations or noise caused by measurement error.

Develop algorithms for such γ-close to a structural class $\mathcal{C}$ graphs for some γ edits (vertex deletions, edge deletions, edge contractions).



Structural Rounding Framework

- **Stability:** A graph minimization (resp. maximization) problem Π is stable under edit operation ψ with constant c' if

$$OPT_\Pi(G') \le OPT_\Pi(G) + c'\gamma$$
$$\text{(resp. } OPT_\Pi(G') \ge OPT_\Pi(G) - c'\gamma)$$

given G' γ-editable from G under ψ. Π is closed under ψ when $c' = 0$.

- **Structural Lifting:** Π can be structurally lifted w.r.t. ψ with constant c if, given edit sequence $\psi_1, \psi_2, \ldots, \psi_k$ where $k \le \gamma$ that edits G into G', a solution S' for G' can be converted in polytime to S for G such that

$$Cost_\Pi(S) \le Cost_\Pi(S') + c \cdot k$$
$$\text{(resp. } Cost_\Pi(S) \ge Cost_\Pi(S') - c \cdot k).$$

- **(α, β)-approx:** An alg for $(\mathcal{C}_\lambda, \psi)$-EDIT is a bicriteria (α, β)-approx if number of edits is at most α times the optimal number of edits into $\mathcal{C}_\lambda$, and $\lambda \le \beta \cdot \lambda^*$.

$(\mathcal{C}_\lambda, \psi)$-EDIT

Input: Graph $G = (V, E)$, paremterized family $\mathcal{C}_\lambda$ of graphs, target parameter value λ^*, edit operation ψ

Problem: Find k edits $\psi_1, \psi_2, \ldots, \psi_k$ such that
$$\psi_k\left(\psi_{k-1}\left(\cdots \psi_2(\psi_1(G))\right)\right) \in \mathcal{C}_\lambda.$$

Objective: Minimize k and λ where $\lambda \ge \lambda^*$

Structural Rounding Approximation Theorem: Suppose Π is stable under ψ with constant c' and can be structurally lifted with constant c. If Π has a $\rho(\lambda)$-approx in $\mathcal{C}_\lambda$ and $(\mathcal{C}_\lambda, \psi)$-EDIT has a polytime (α, β)-approx, then there exists a polytime

$$((1 + c'\alpha\delta) \cdot \rho(\beta\lambda) + c\alpha\delta)\text{-approx}$$
$$\text{(resp. } ((1 - c'\alpha\delta) \cdot \rho(\beta\lambda) - c\alpha\delta)\text{-approx)}$$

for Π on any $(\delta \cdot OPT_\Pi(G))$-close graph to $\mathcal{C}_\lambda$.

Editing Results: Upper and Lower Bounds

Editing to Bounded Degeneracy:
- **Approximation algorithm:**
  - ✓ Local Ratio Technique by Bar-Yehuda et. al.
  - ✓ LP-based: (3, 3)-approx for edge deletion and (4, 4)-approx for vertex deletion
  - ✓ Integrality gap is $\Omega(n)$ so cannot hope for non-bicriteria approx using this approach
- **Lower bound:**
  - ✓ $o(\log n)$-inapprox vertex deletions distinguishing *r* and *r+1*
  - ✓ $o(\log r)$-inapprox edge deletions distinguishing *r* and *r+1*
  - ✓ Reduction from SET COVER

Editing to Bounded Treewidth:
- **Approximation algorithm:**
  - ✓ Relationship between vertex separators and treewidth
  - ✓ Combine Bodleaender's $O(\log n)$-approx alg for treewidth and Fiege et al.'s $O(\sqrt{\log w})$-approx alg for vertex separators
  - ✓ $\left(O(\log^{1.5} n), O(\sqrt{\log w})\right)$-approx. vertex deletions
  - ✓ Generated tree decompositions have $O(\log n)$ height
  - ✓ Pathwidth is at most width times the height of a tree decomp
  - ✓ $\left(O(\log^{1.5} n), O(\sqrt{\log w} \cdot \log n)\right)$-approx. pathwidth vertex deletions

Editing to Bounded Weak c-Coloring Number:
- Used to define bounded expansion
- Characterizes bounded degeneracy $c = 1$
- **Lower bound:**
  - ✓ $o(\log n)$-inapprox vertex deletions, edge deletions, edge contractions
  - ✓ Reduction from SET COVER, similar to reductions for degeneracy

Editing to Treedepth 2 (Star Forests):
- Preliminary results for treedepth
- **Approximation algorithm:**
  - ✓ Reduction to HITTING SET
  - ✓ 4-approx for vertex deletion
  - ✓ 3-approx for edge deletion
- **Lower bound:**
  - ✓ Reduction from VERTEX COVER for vertex deletions
  - ✓ $(2 - \epsilon)$-inapprox (assuming UGC) vertex deletions
  - ✓ Reduction from MINIMUM DOMINATING SET-*B* for edge deletions
  - ✓ APX-complete for edge deletions

Table of Editing Results

Graph Family $\mathcal{C}_\lambda$	Edit Operation ψ		
	Vertex Deletion	**Edge Deletion**	**Edge Contraction**
Bounded Degree (d) [d-BDD-V, d-BDD-E]	$O(\log d)$-approx. [5] $(\ln d - C \cdot \ln\ln d)$-inapprox.	Polynomial time [6]	—
Bounded Clique Number (b) [b-CN-V]	— $o(\log n)$-inapprox. when $b = \Omega\left(n^{\frac{1}{2}}\right)$	—	—
Bounded Degeneracy (r) [r-DE-V, r-DE-E]	$\left(\frac{4m-\beta rn}{m-rn}, \beta\right)$-approx. $\left(\frac{1}{\epsilon}, \frac{2}{1-\epsilon}\right)$-approx. $(\epsilon < 1)$ $o(\log n)$-inapprox.	$\left(\frac{1}{\epsilon}, \frac{2}{1-2\epsilon}\right)$-approx. $\left(\epsilon < \frac{1}{2}\right)$ $(\ln r - C \cdot \ln\ln r)$-inapprox.	—
Bounded Weak c-Coloring Number (t) [t-BWE-V-c, t-BWE-E-c, t-BWE-C-c]	— $o(t)$-inapprox. when $t = o(\log n)$	— $o(t)$-inapprox. when $t = o(\log n)$	— $o(t)$-inapprox. when $t = o(\log n)$
Bounded Treewidth (w) [w-TW-V, w-TW-E]	$\left(O(\log^{1.5} n), O(\sqrt{\log w})\right)$-approx. $o(\log n)$-inapprox. when $w = \Omega\left(n^{\frac{1}{2}}\right)$	$\left(O(\log n \log\log n), O(\log w)\right)$-approx. [2] —	—
Bounded Pathwidth (w) [w-PW-V, w-PW-E]	$\left(O(\log^{1.5} n), O(\sqrt{\log w} \cdot \log n)\right)$-approx. —	$\begin{pmatrix} O(\log n \log\log n), \\ O(\log w \cdot \log n) \end{pmatrix}$-approx. [2] —	—
Star Forest [SF-V, SF-E]	4-approx. $(2-\epsilon)$-inapprox. (UGC)	3-approx. APX-complete	—

Structural Rounding Applicable Problems

Problem	Edit Type ψ	c'	c	Graph Class $\mathcal{C}_\lambda$	$\rho(\lambda)$
Independent Set (IS)	vertex deletion	1	1	degeneracy r	$1/(r+1)$
Independent Set (IS)	vertex deletion	1	0	treewidth w	1 [3]
Vertex Cover (VC)	vertex deletion	0	1	treewidth w	1 [3]
Feedback Vertex Set (FVS)	vertex deletion	0	1	treewidth w	1 [3]
Minimum Maximal Matching (MMM)	vertex deletion	0	1	treewidth w	1 [3]
Chromatic Number (CRN)	vertex deletion	0	1	treewidth w	1 [3]
Independent Set (IS)	edge deletion	0	1	degeneracy r	$1/(r+1)$
Dominating Set (DS)	edge deletion	1	0	degeneracy r	$O(r^2)$ [7]
$(\ell -)$ Dominating Set (DS)	edge deletion	1	0	treewidth w	1 [1, 4]
Edge $(\ell -)$ Dominating Set (EDS)	edge deletion	1	1	treewidth w	1 [4]
$(\ell -)$ Dominating Set (DS)	edge contraction	0	1	treewidth w	1
Edge $(\ell -)$ Dominating Set (EDS)	edge contraction	0	1	treewidth w	1
Connected $(\ell -)$ Dominating Set	edge contraction	0	1	treewidth w	1
Connected Edge $(\ell -)$ Dominating Set	edge contraction	0	1	treewidth w	1
(Weighted) TSP Tour	edge contraction	0	2	treewidth w	1

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