Scheduling with Communication Delay in Near-Linear Time

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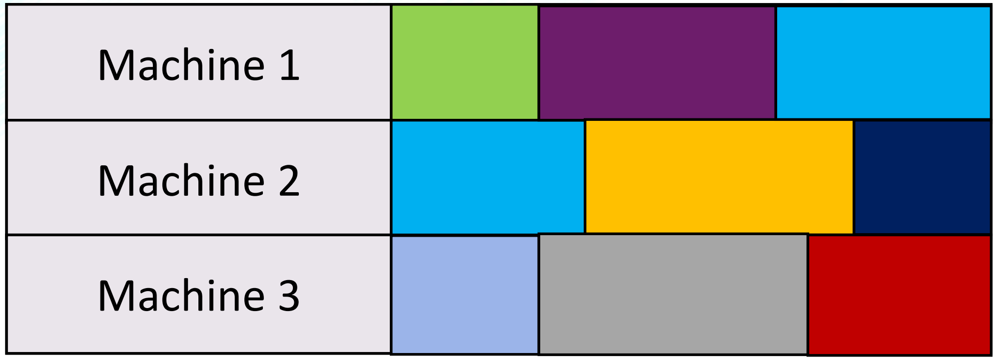
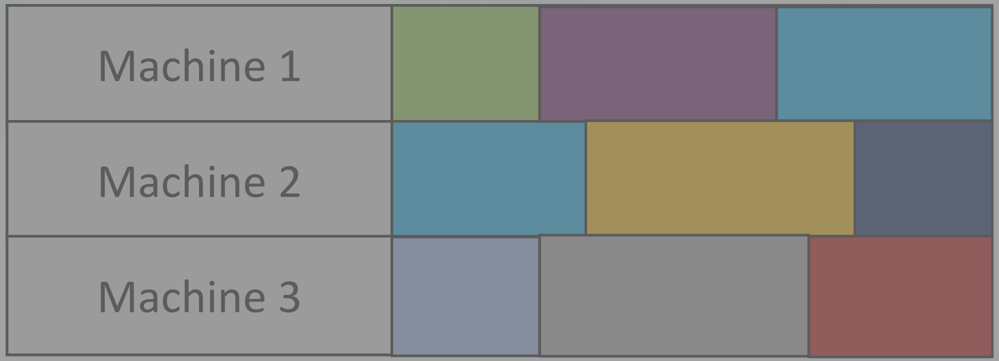


Scheduling is a classical problem in theory and in practice

- Cluster data processing management (Google Cloud Dataflow, Spark, Hadoop, Mesos...etc.)
- Machine learning (scheduling training, e.g., Tensorflow…etc.)



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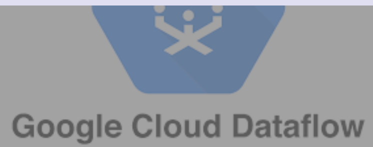
Efficient Scheduling is Important in Large Data Centers



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Research Question and Goal

How computationally expensive is it to perform approximately-optimal scheduling?

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Even simple formulations of exact scheduling are **NP Hard**

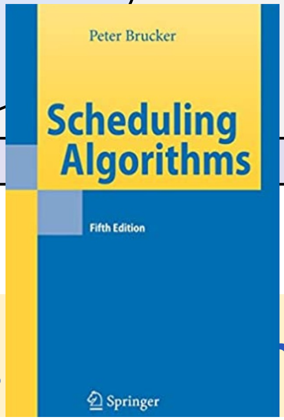
Communication Delay

Rich body of literature for designing good **approximation algorithms** for multiprocessor scheduling

Job

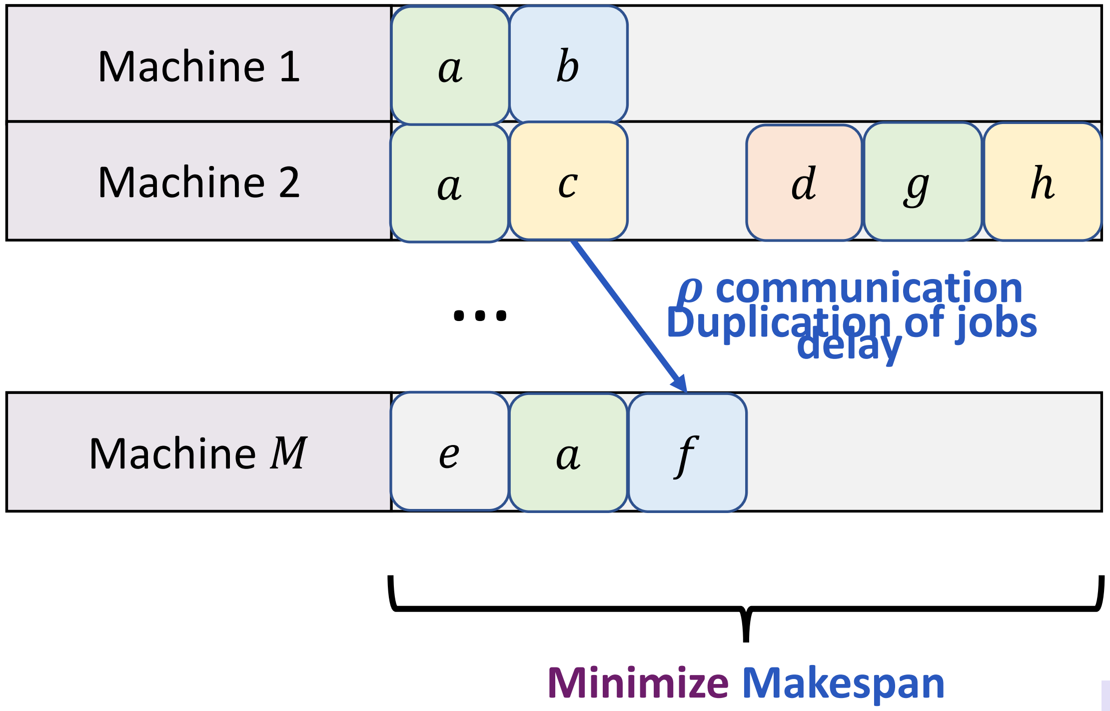
Open: efficient algorithms with good approximations

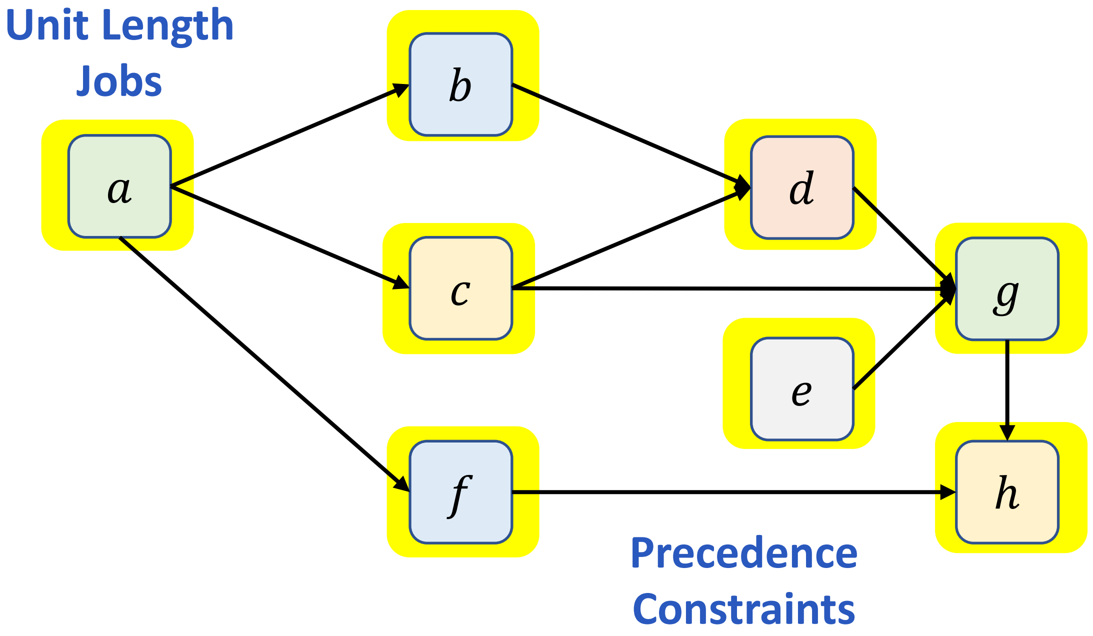
Job Duplication



Goal: Minimiz ne while computing good approx. schedules

Scheduling with Communication Delay





Runtime as Close to Linear as Possible

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- **With duplication:**
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Our Result: $O(\log \rho / \log \log \rho)$-approximation, $O(n \ln M + m \ln^3 n \ln \rho / \ln \ln \rho)$ runtime, whp, assuming schedule has length at least ρ

Lepere-Rapine Algorithm

Lepere-Rapine Algorithm (+Modifications)

- Small subgraph: A maximal subgraph of the input where each vertex has at most 2ρ ancestors

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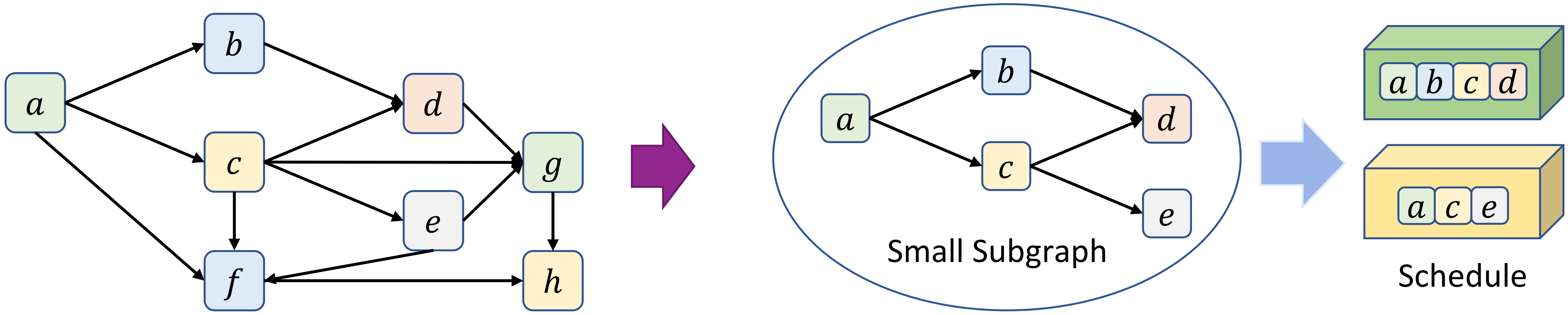

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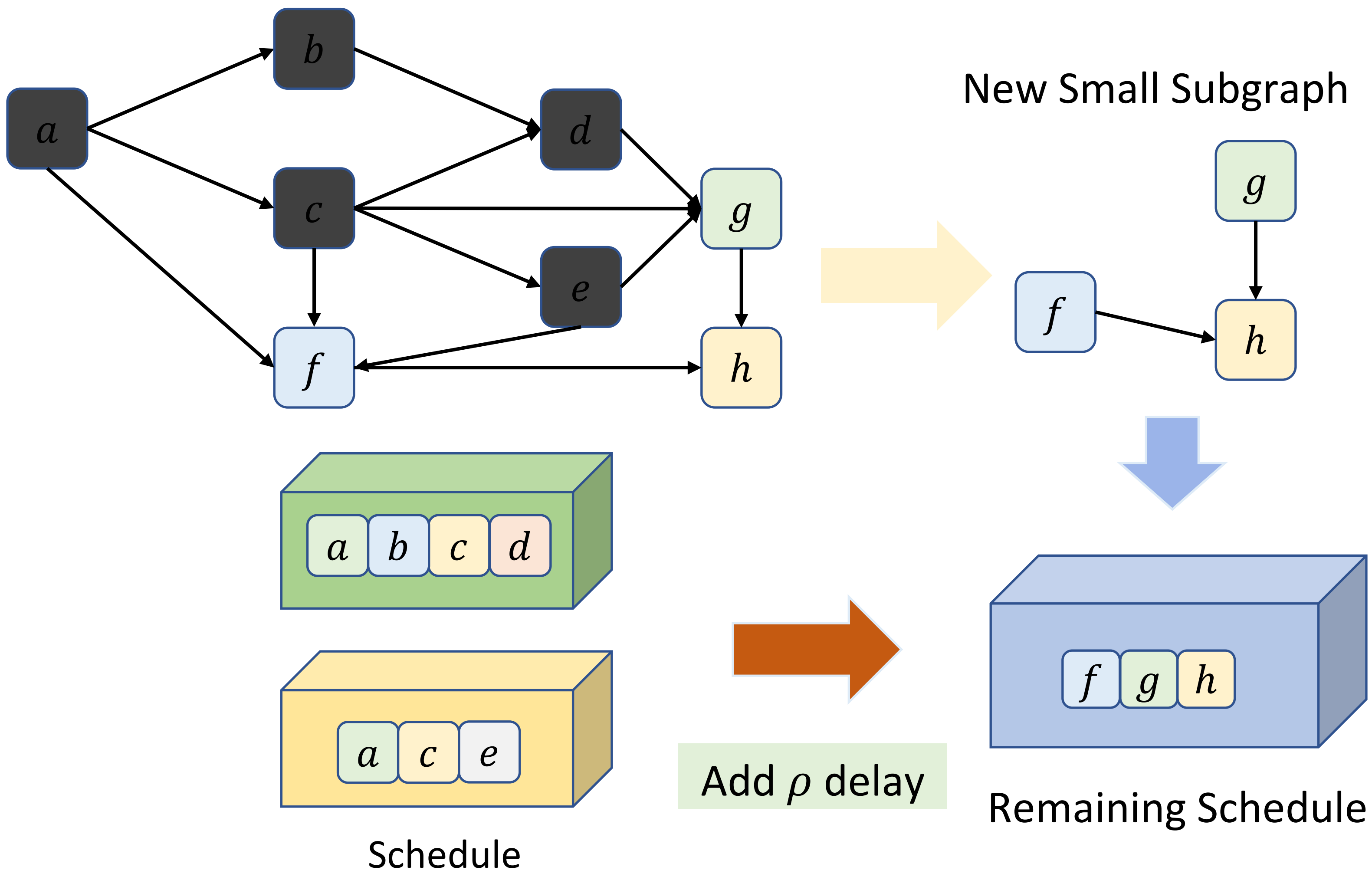

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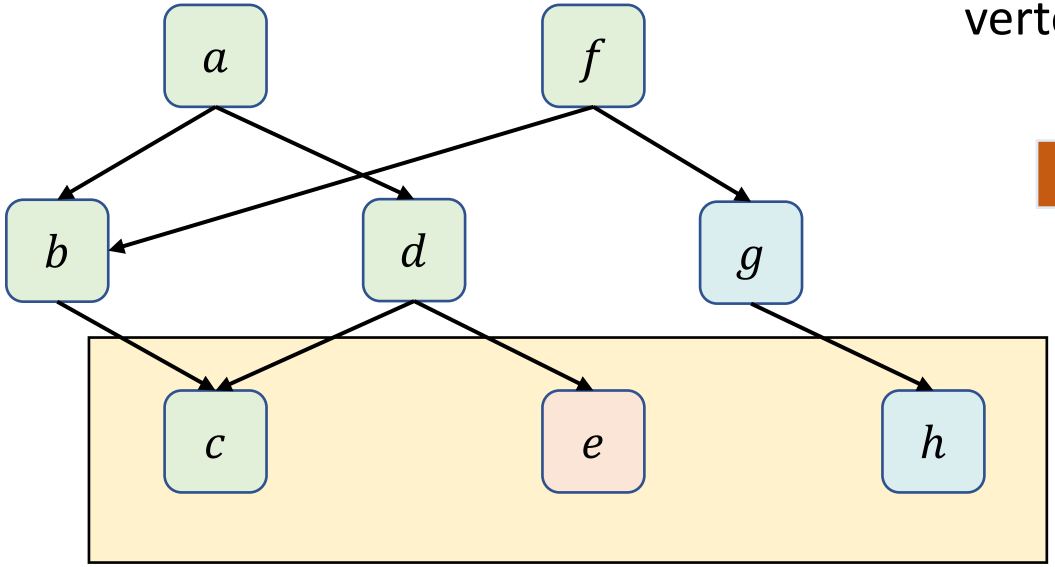

> ½ ancestors + vertex are **not** in batch

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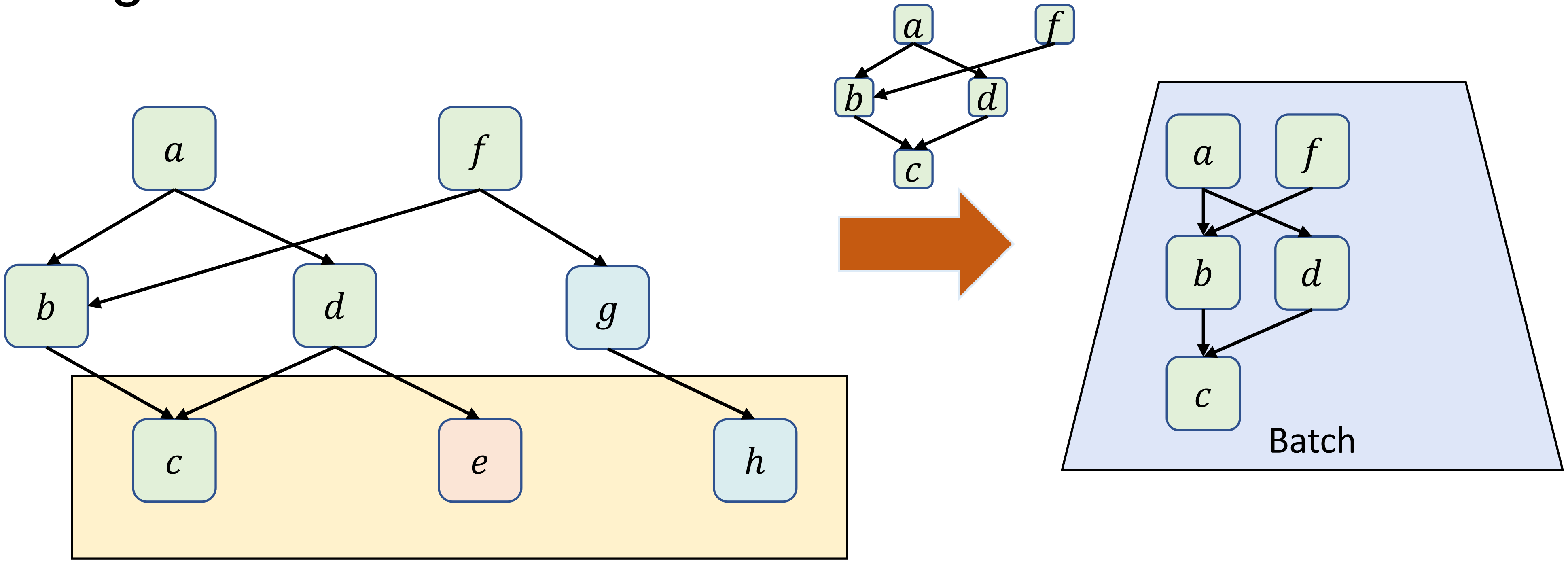

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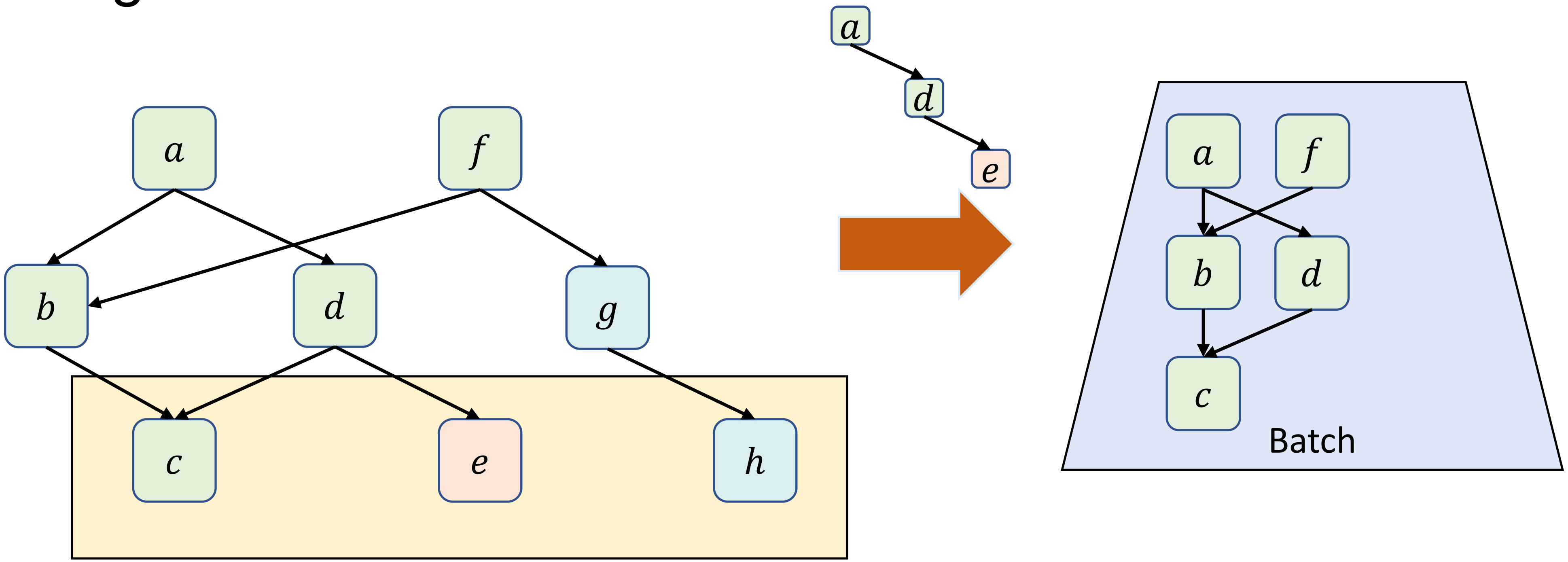

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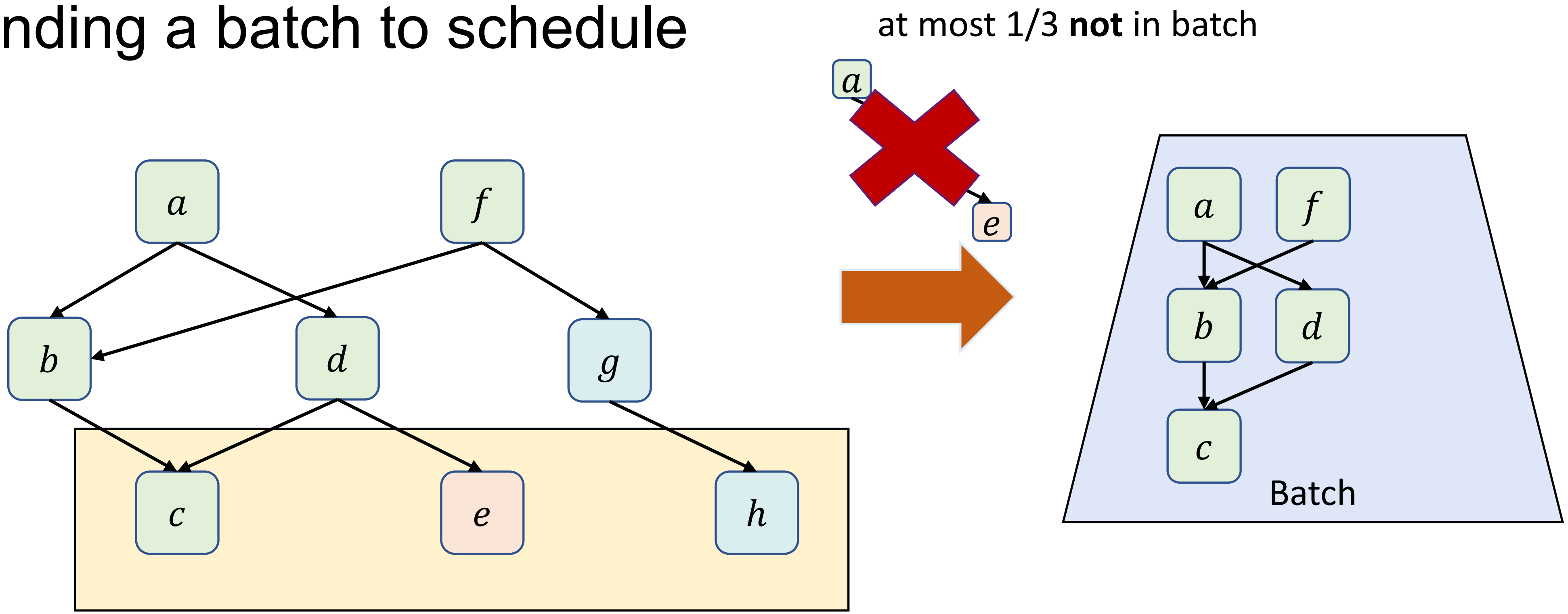

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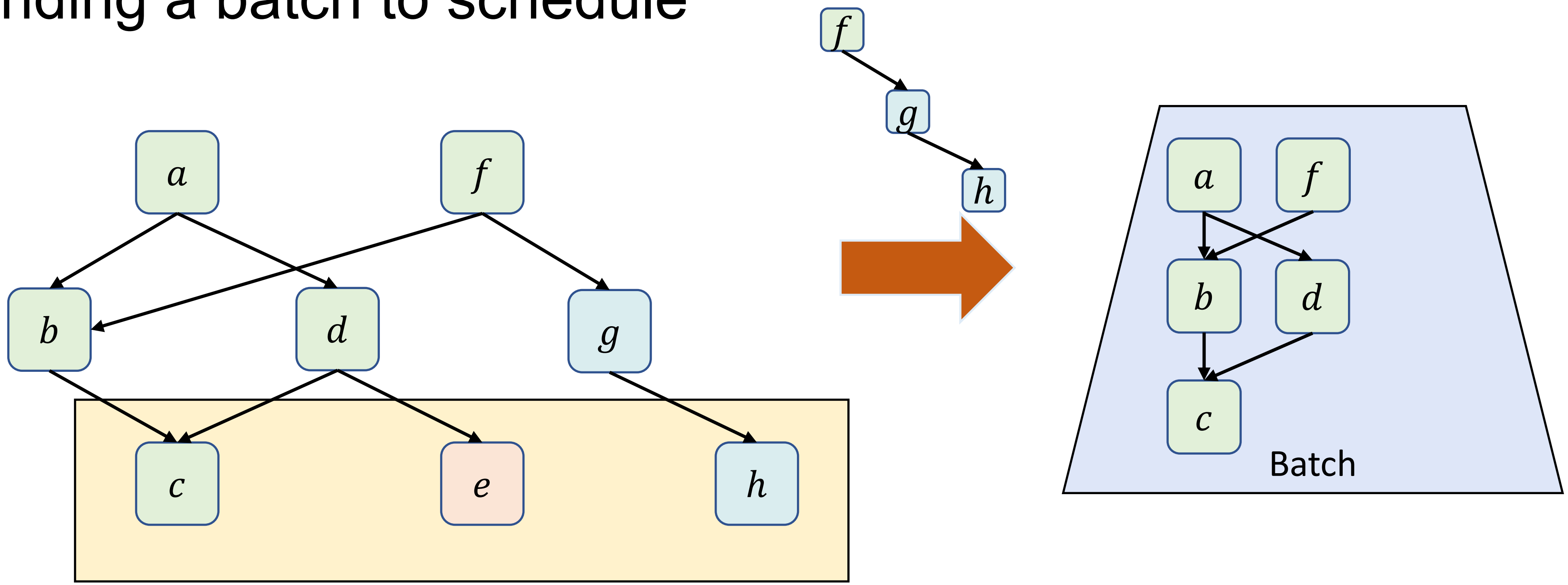

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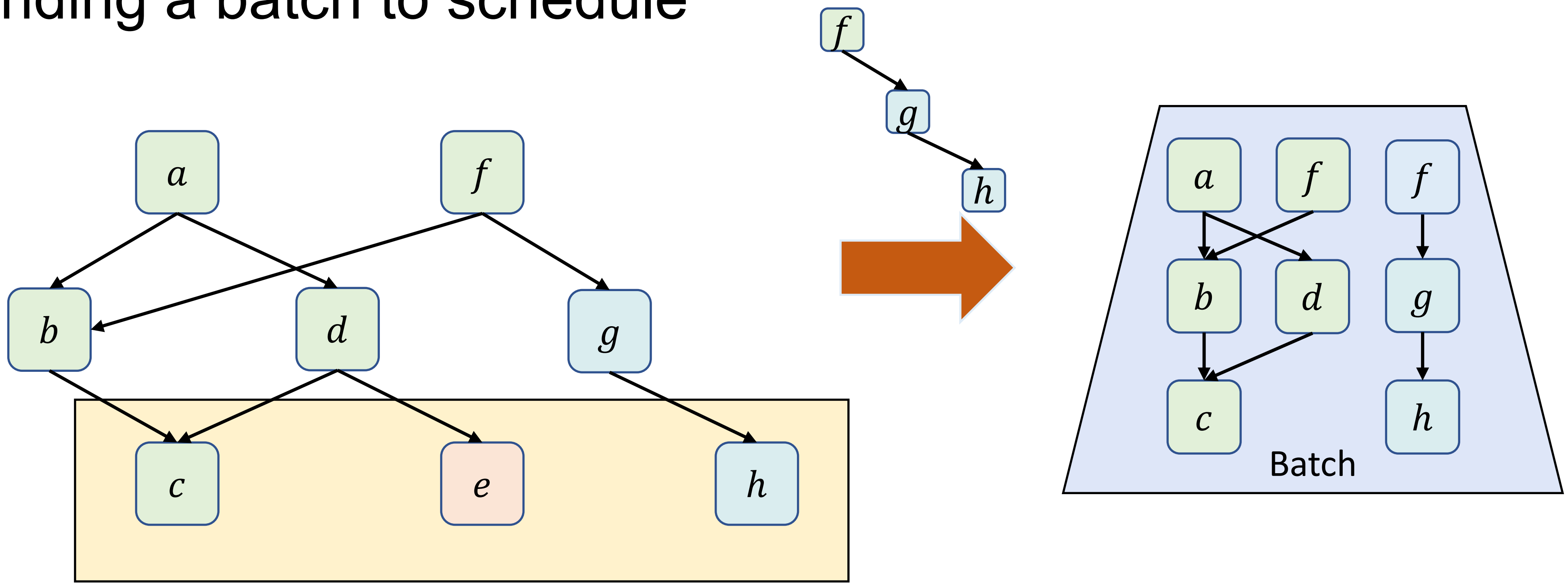

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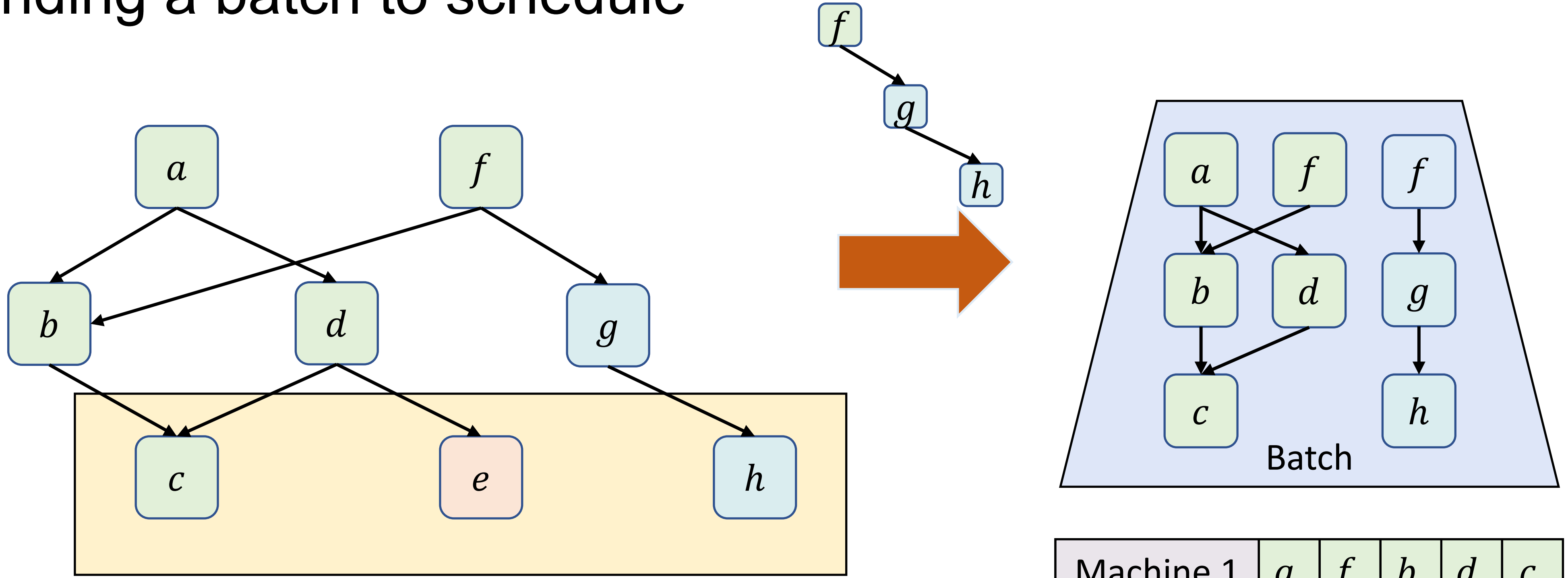

Machine 1	a	f	b	d	c
Machine 2	f	g	h		

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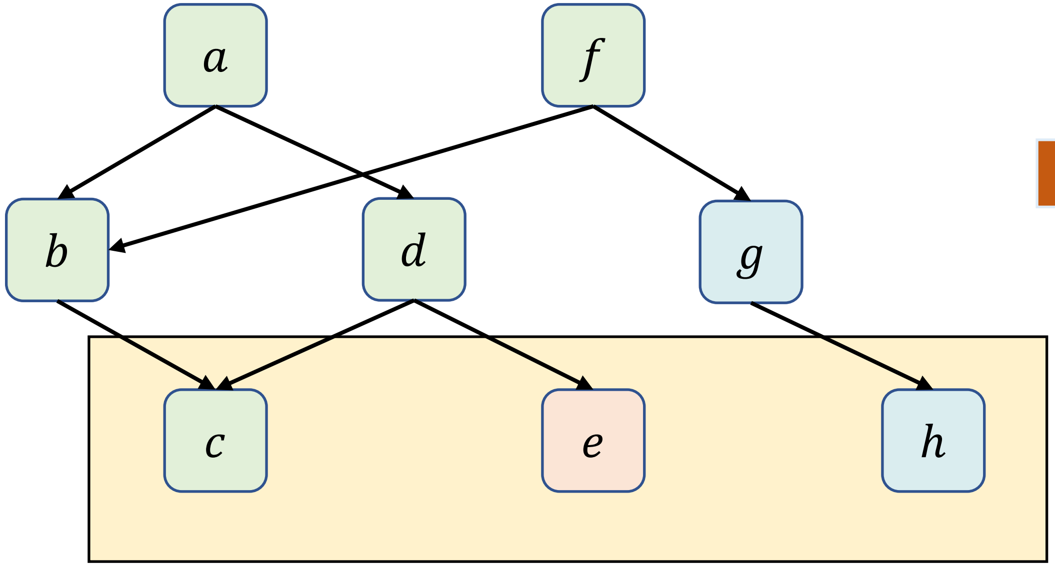

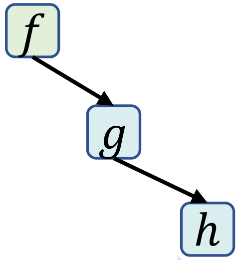

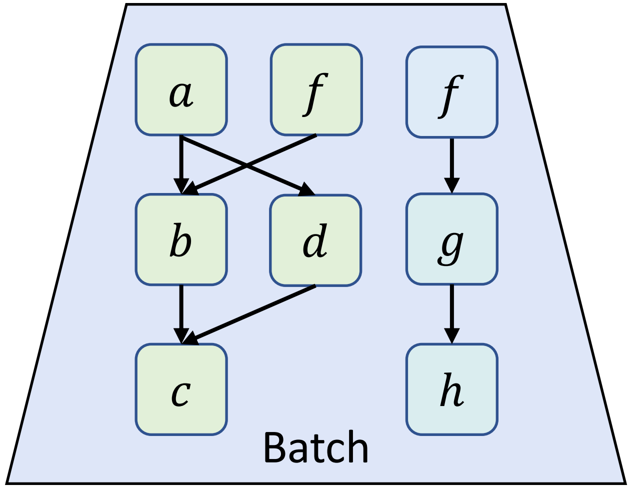

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Graham's list scheduling, 1971

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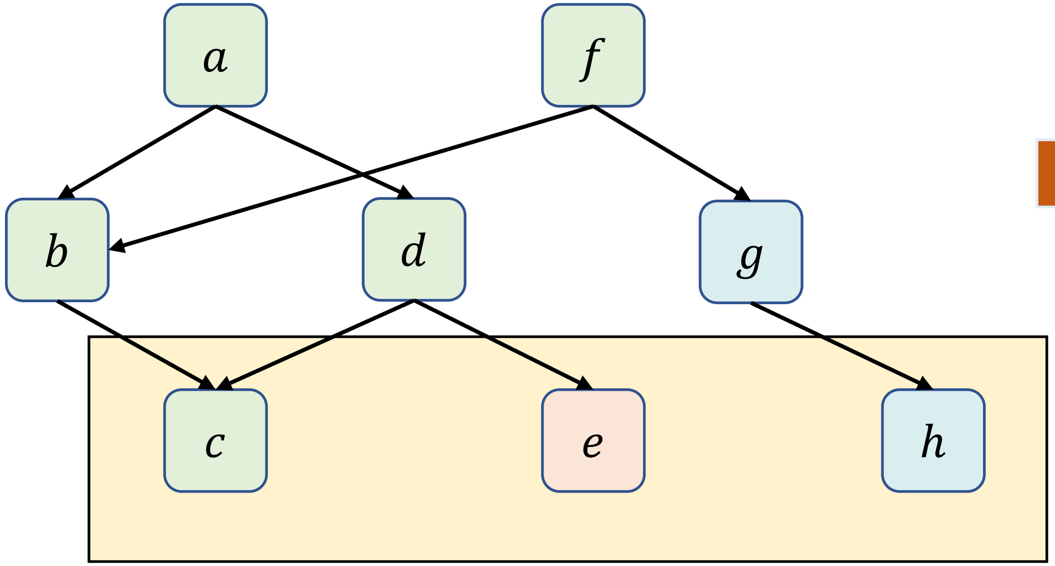
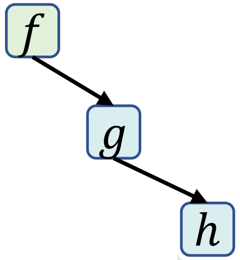
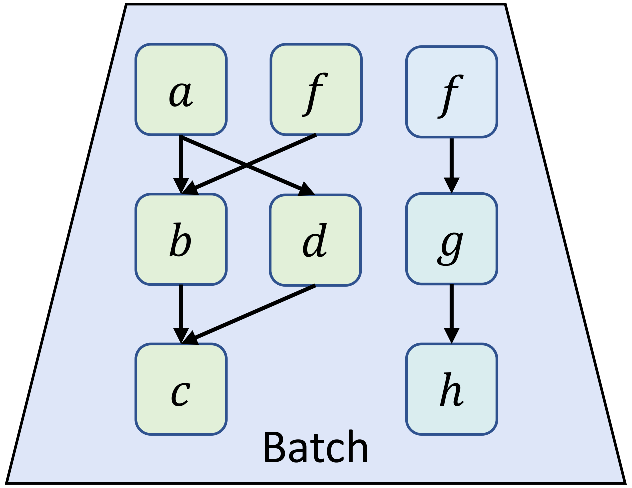
$$\Omega(n\rho^2 + m\rho)$$







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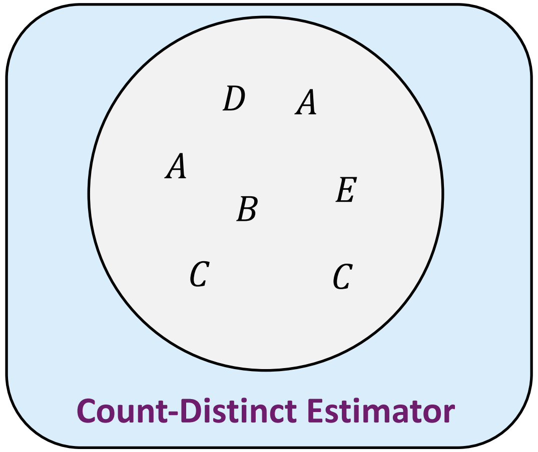

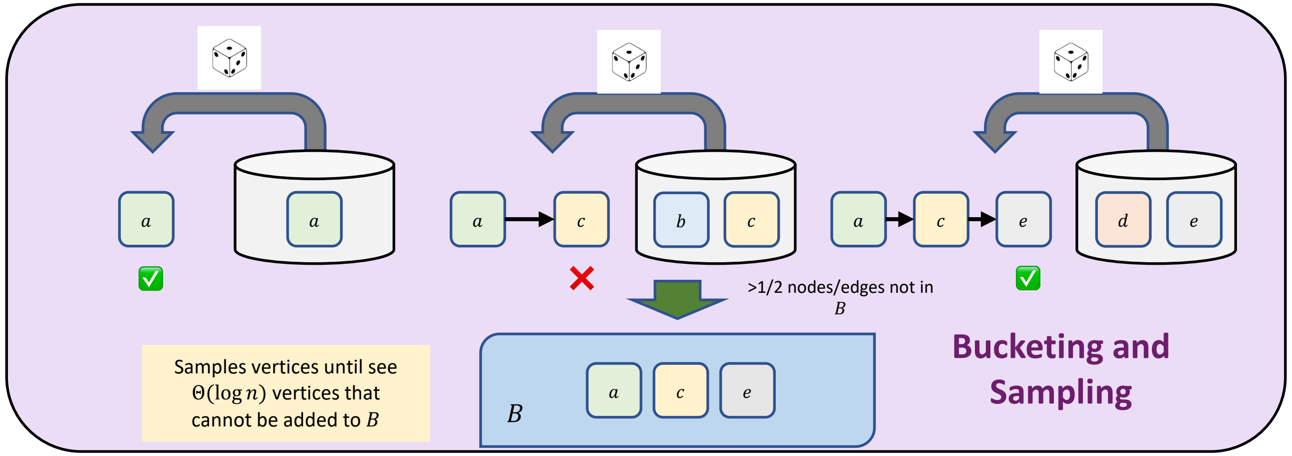

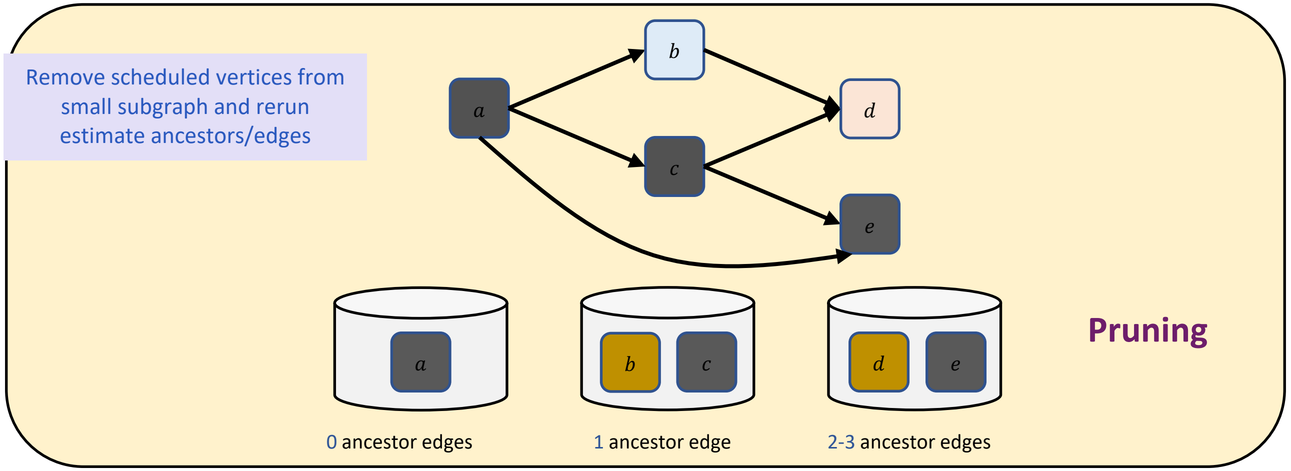

Near-Linear Time Scheduling

- Estimating the Number of Ancestors

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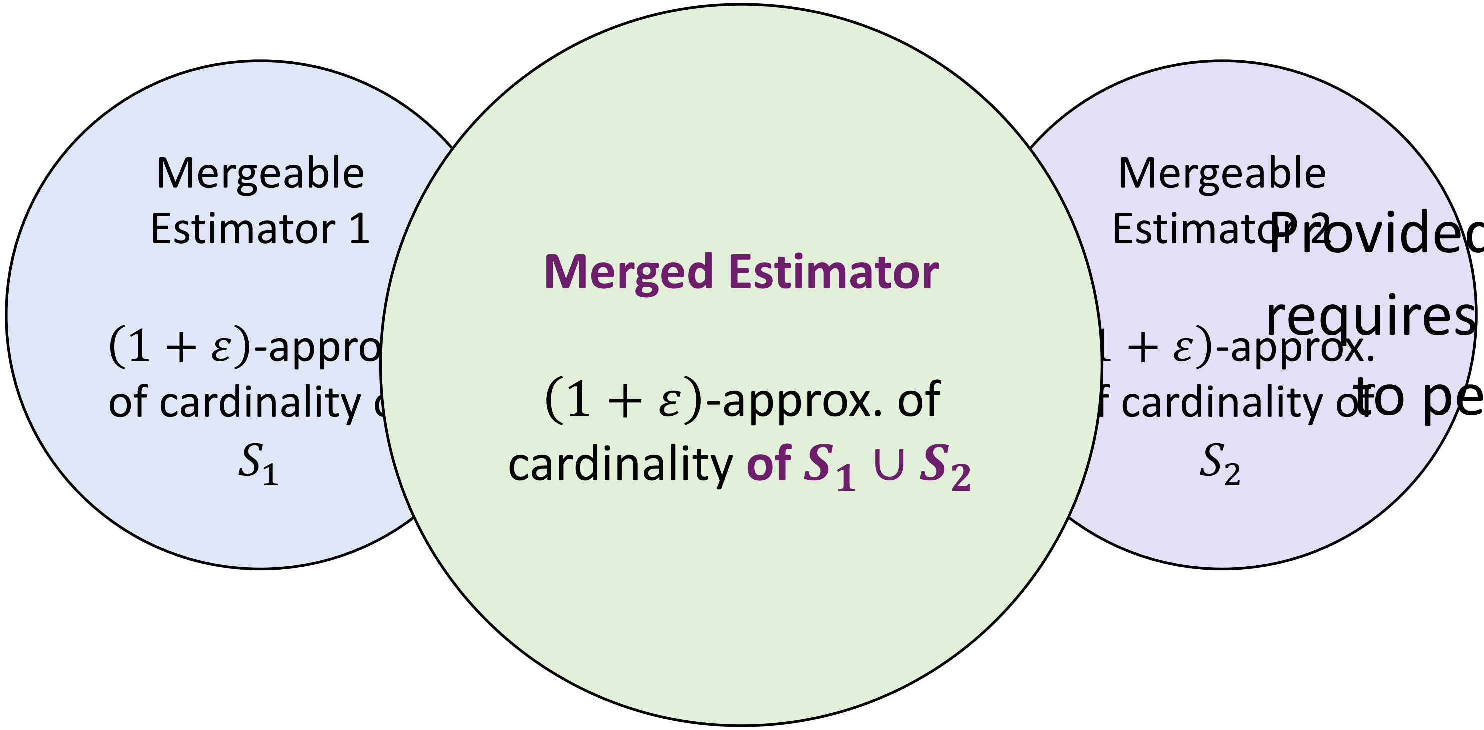

Provided $|S_1 \cup S_2| = n$, requires $O(\log^2 n)$ time to perform merge

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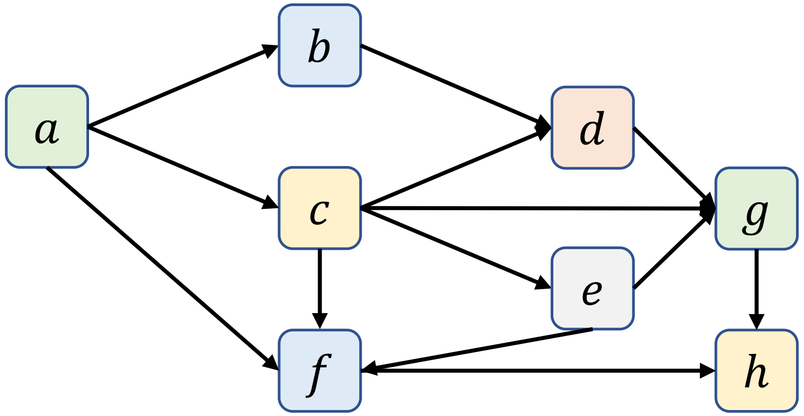

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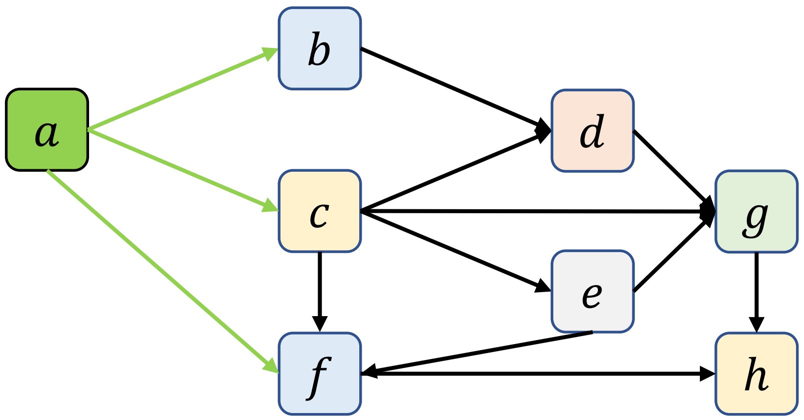

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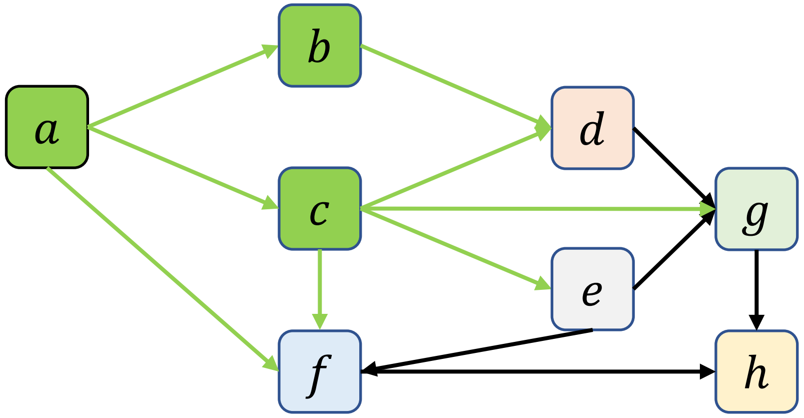

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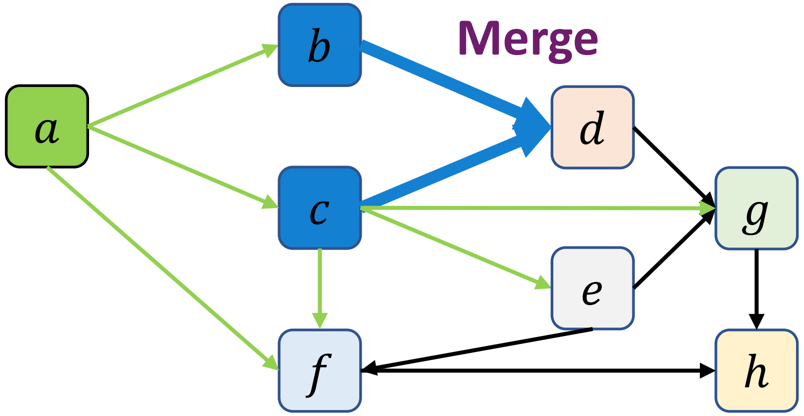

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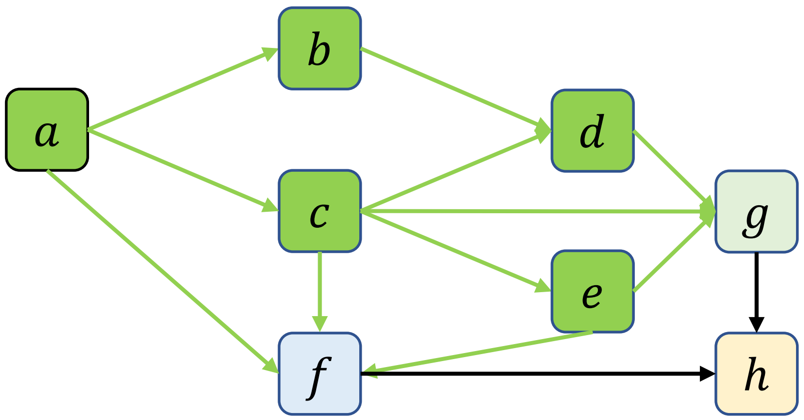

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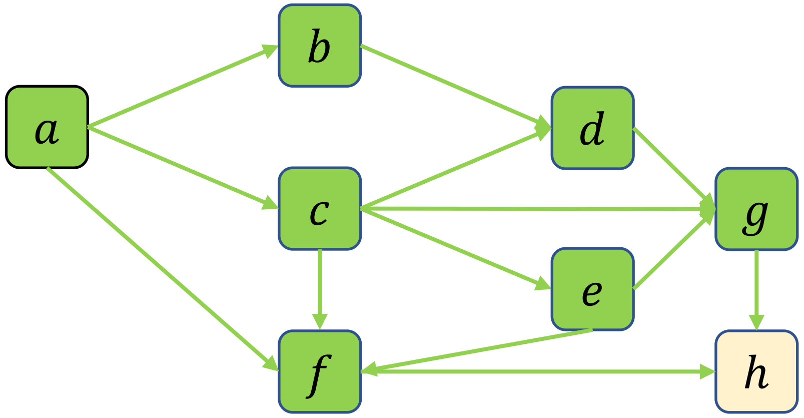

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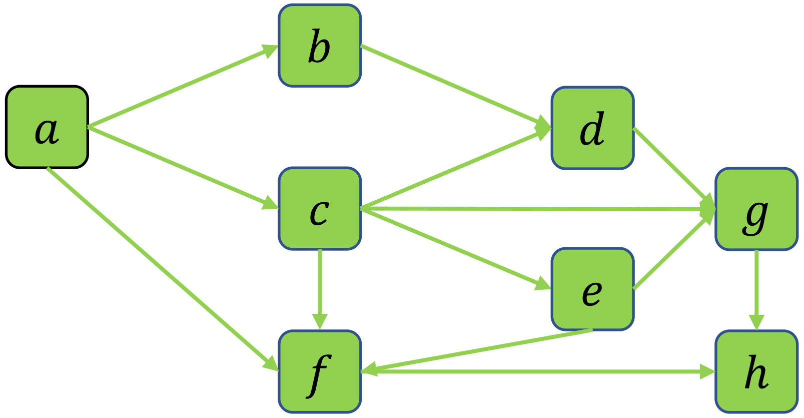

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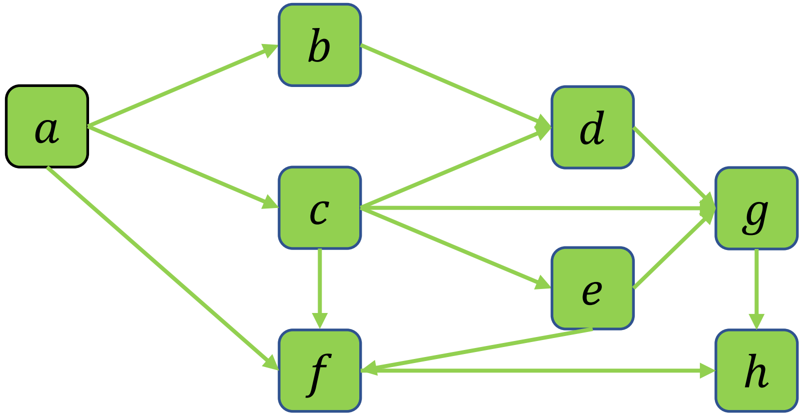
$(1 \pm \epsilon)$-approx. on number of **ancestors and edges**



$O\left((n+m)\log^2 n\right)$

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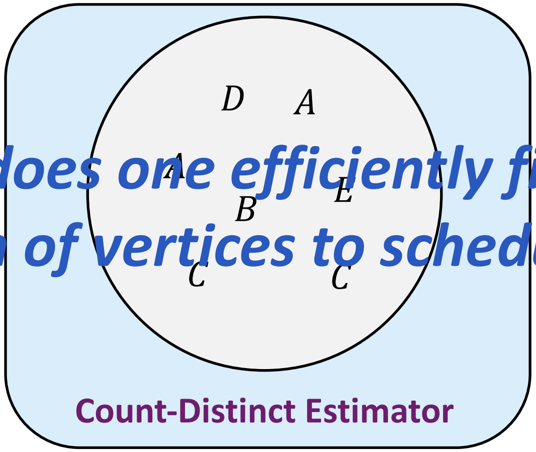
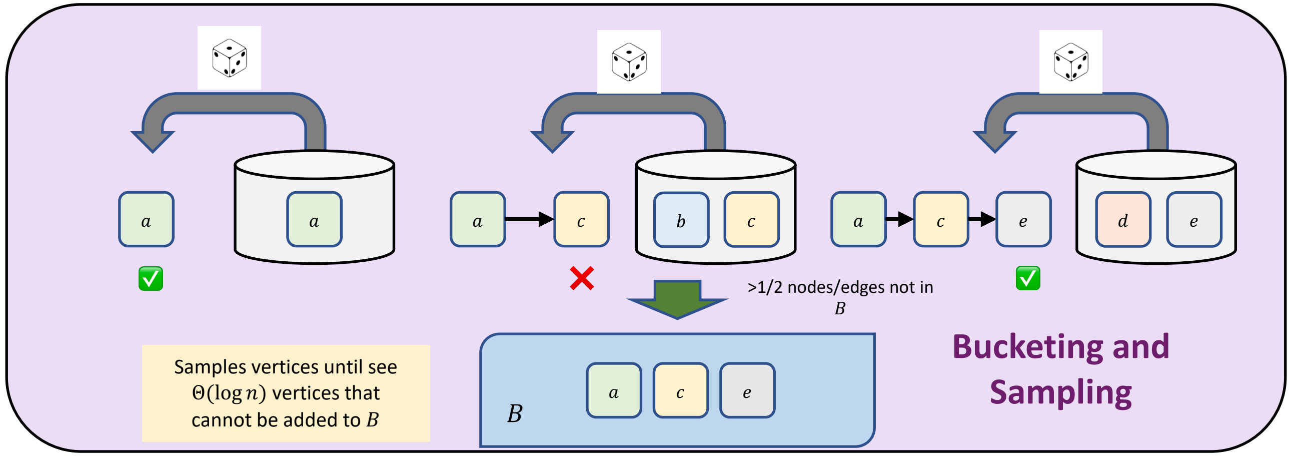
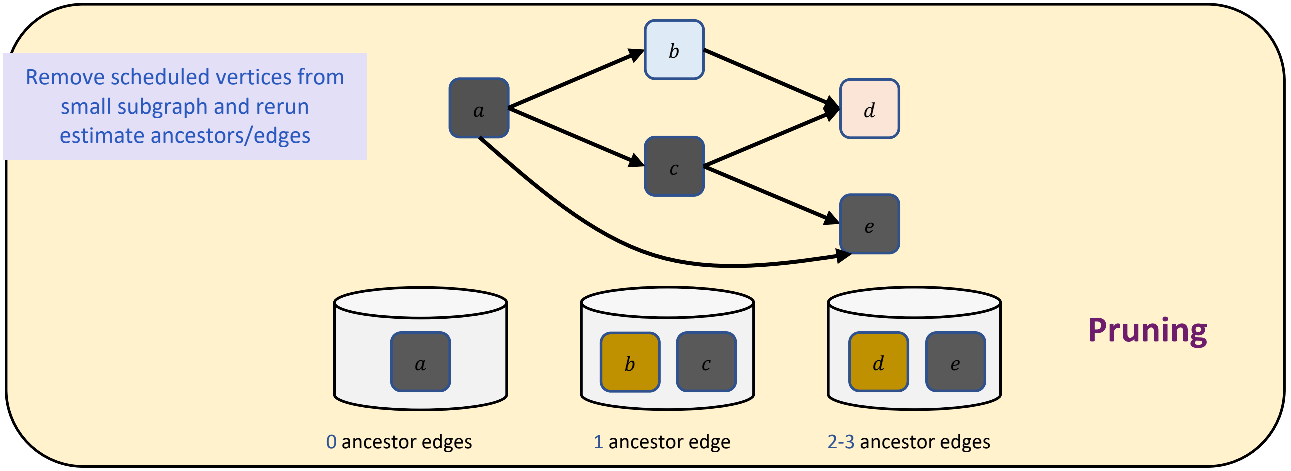
How does one efficiently find a batch of vertices to schedule?

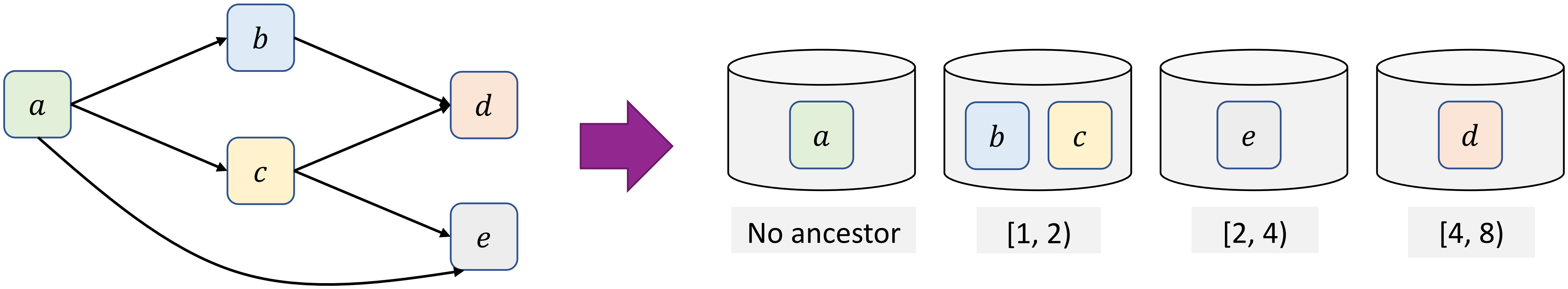






Scheduling Small Subgraph

- Partition vertices into buckets where v is in bucket i if the number of ancestor edges is in $[2^i, 2^{i+1})$ (starting with $i = 0$, first bucket for nodes with no ancestors)



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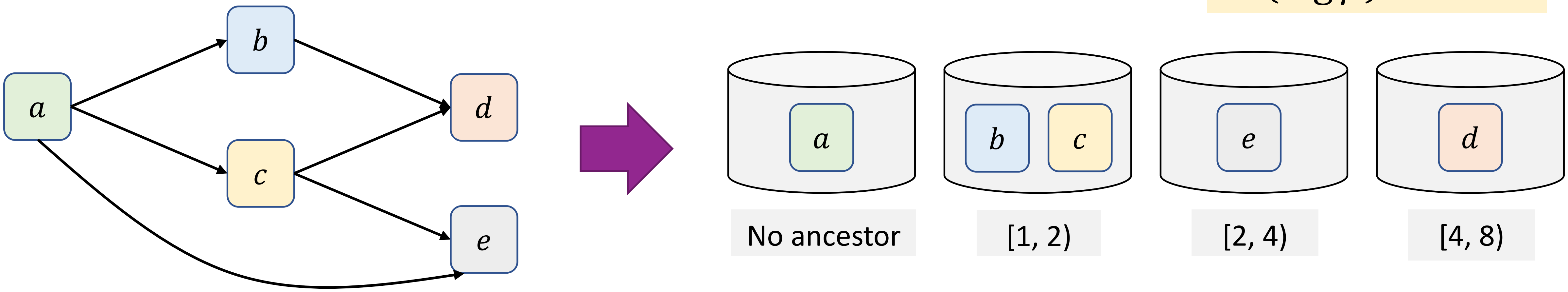
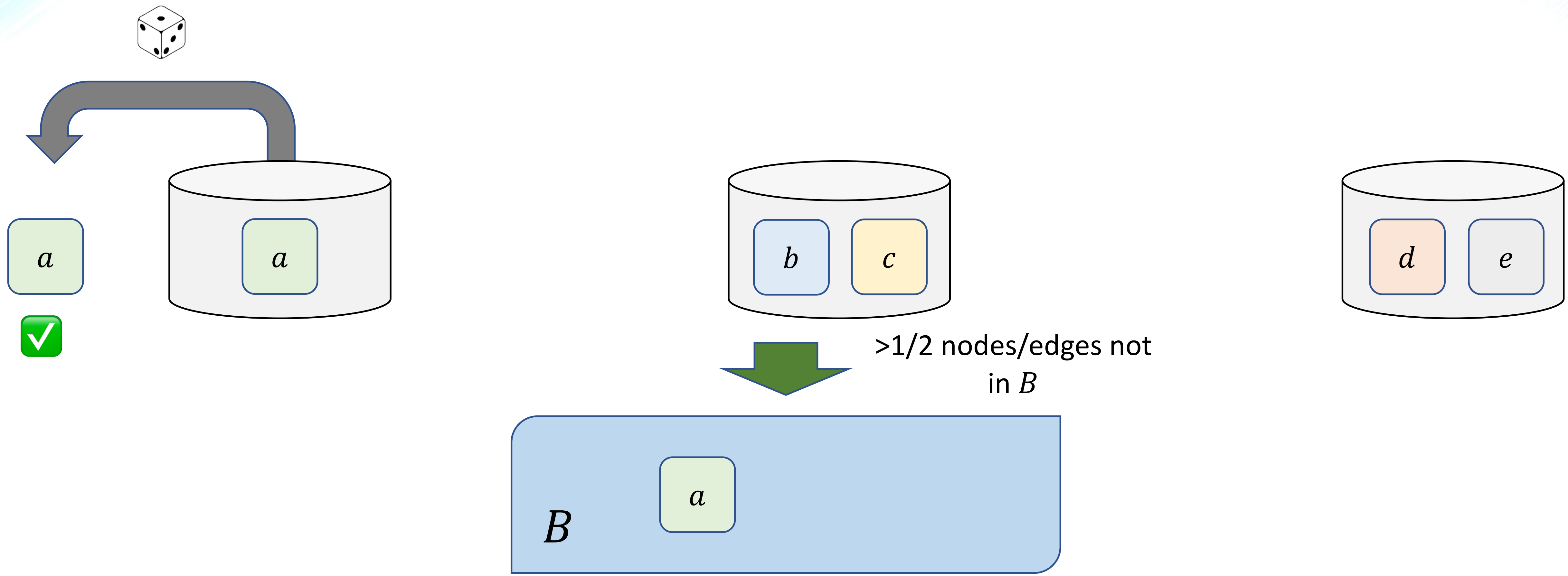
$O(\log \rho)$ buckets



Sample Vertices from Buckets



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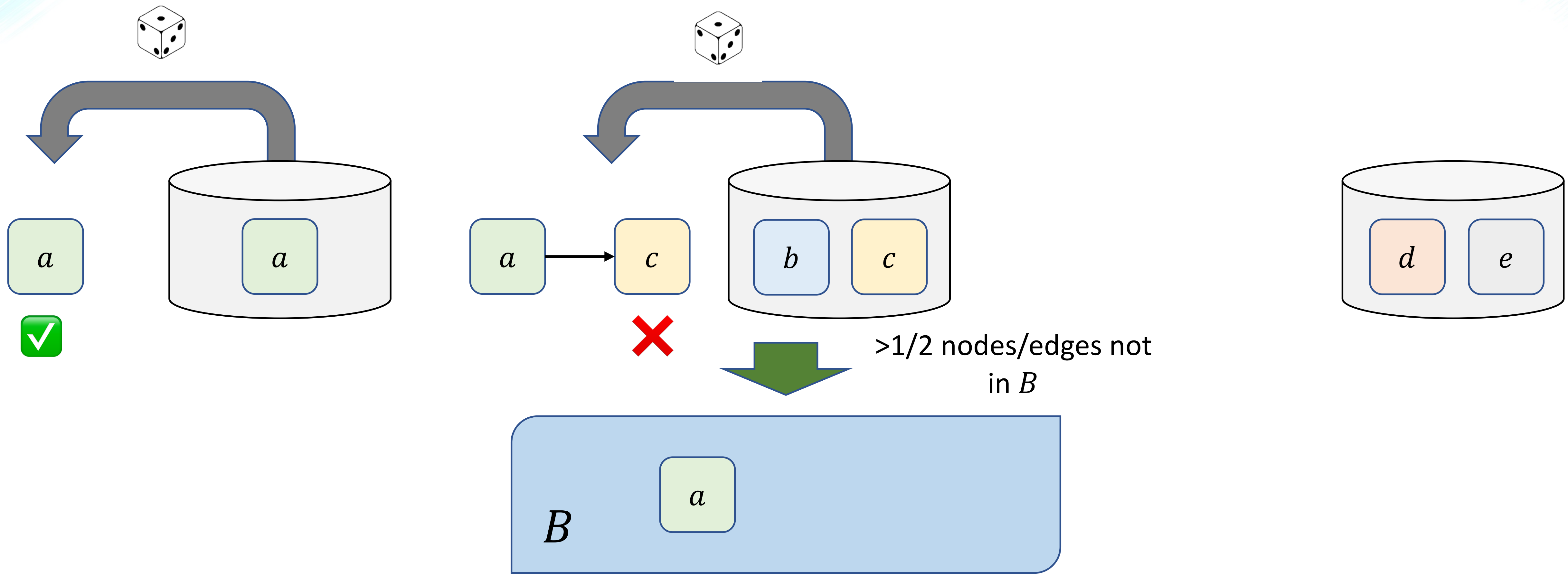

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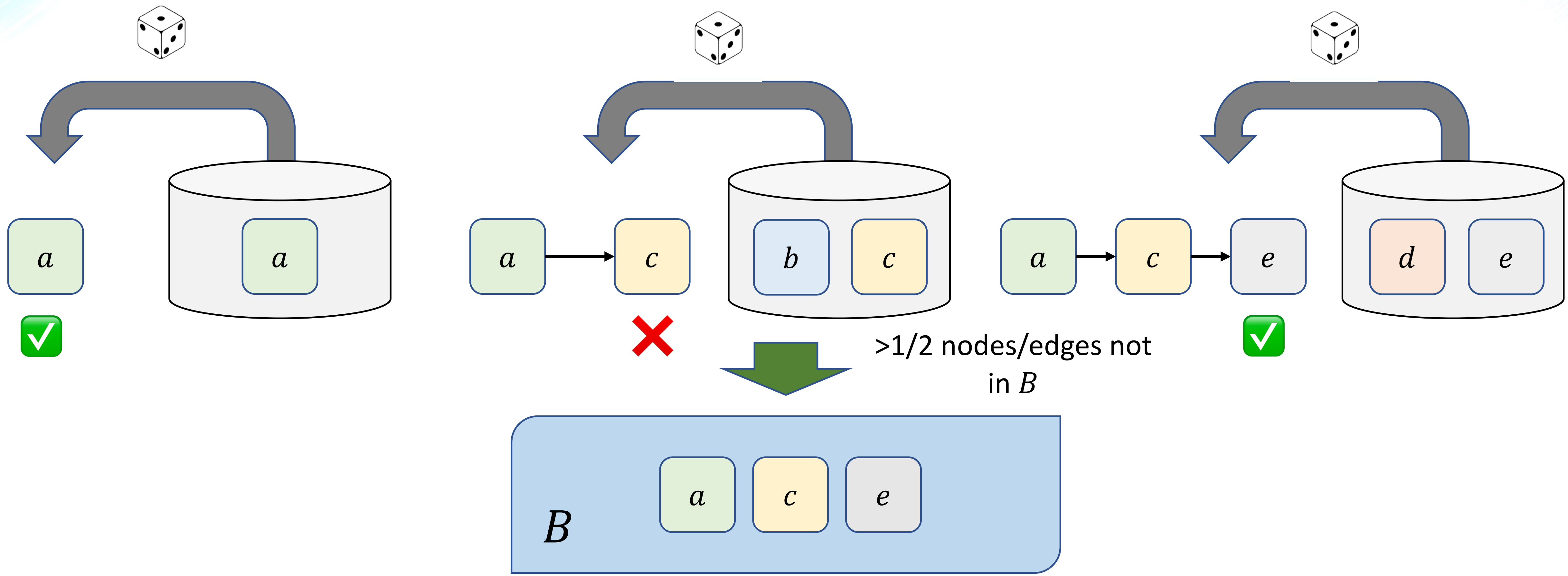

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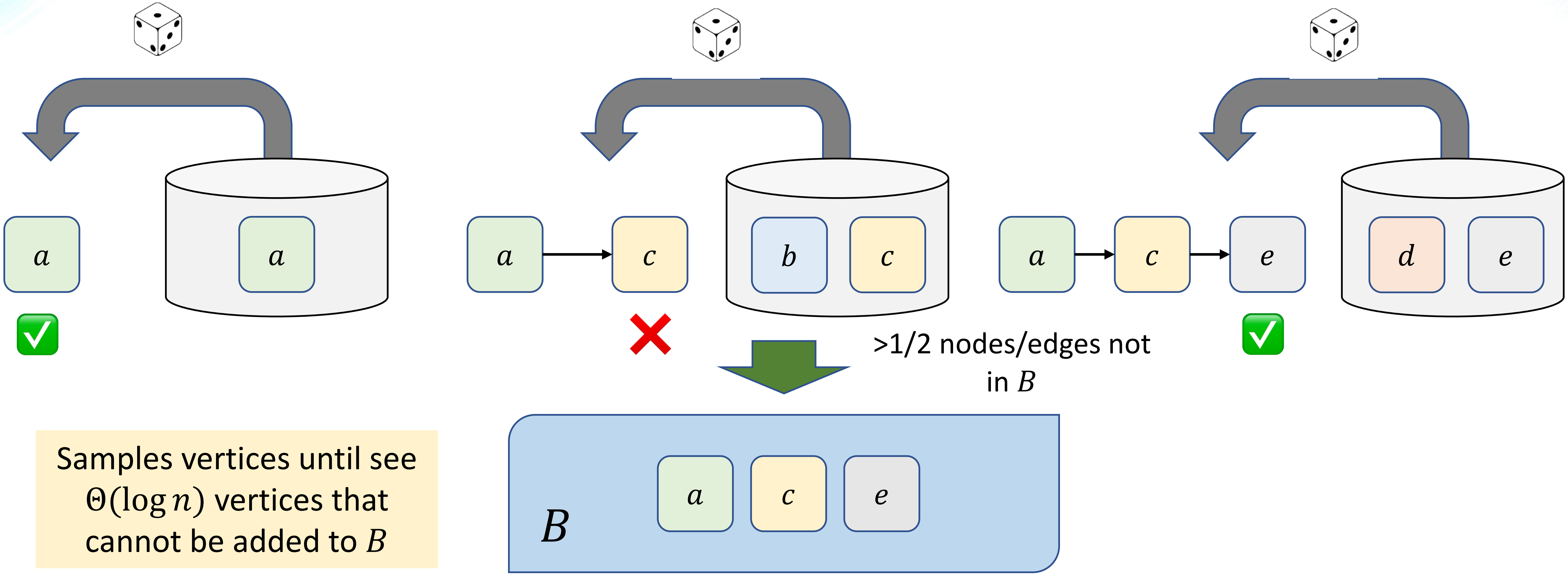

Samples vertices until see $\Theta(\log n)$ vertices that cannot be added to B

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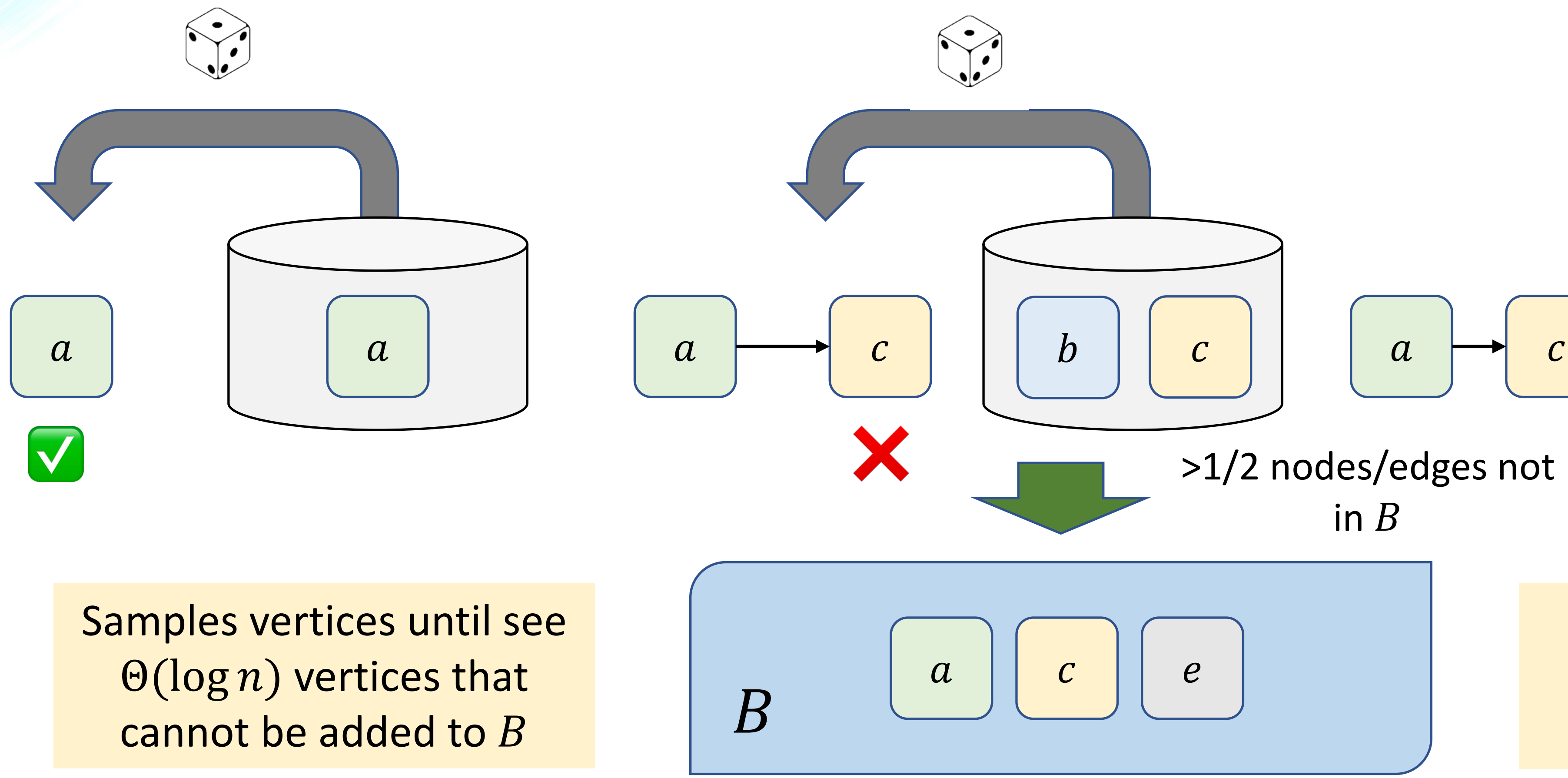
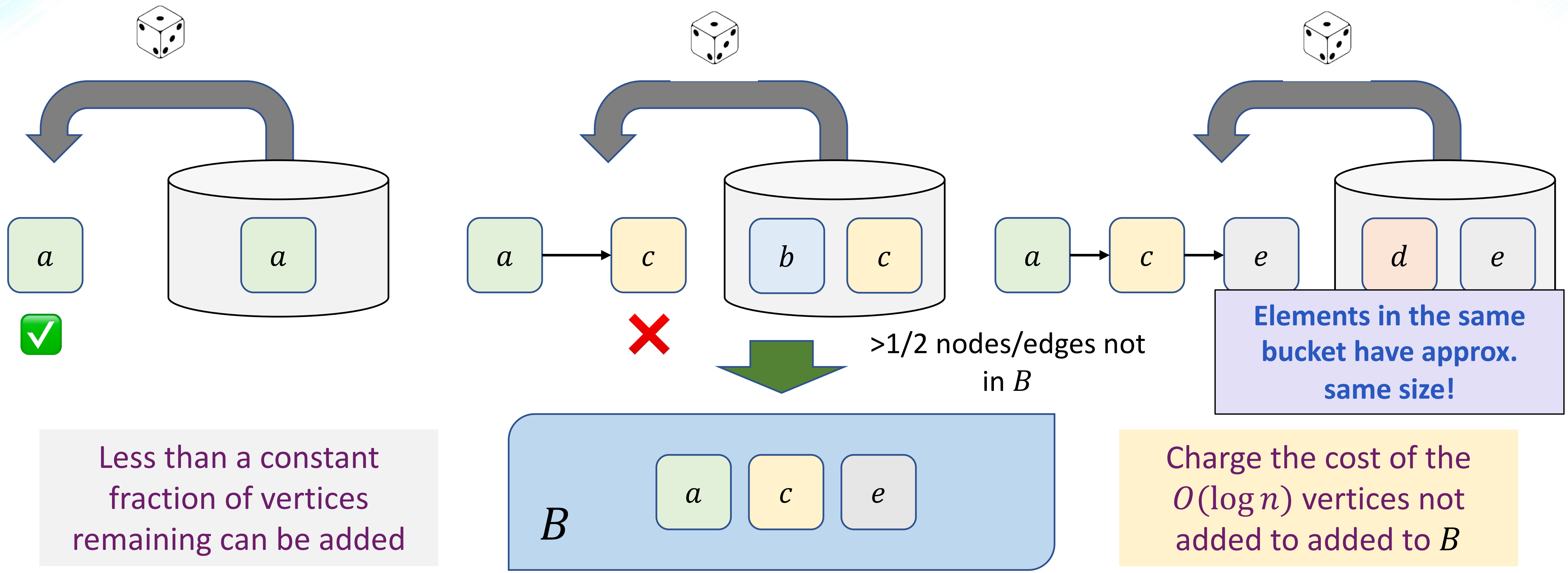


Elements in the same bucket have approx. same size!

Samples vertices until see $\Theta(\log n)$ vertices that cannot be added to B

Charge the cost of the $O(\log n)$ vertices not added to added to B

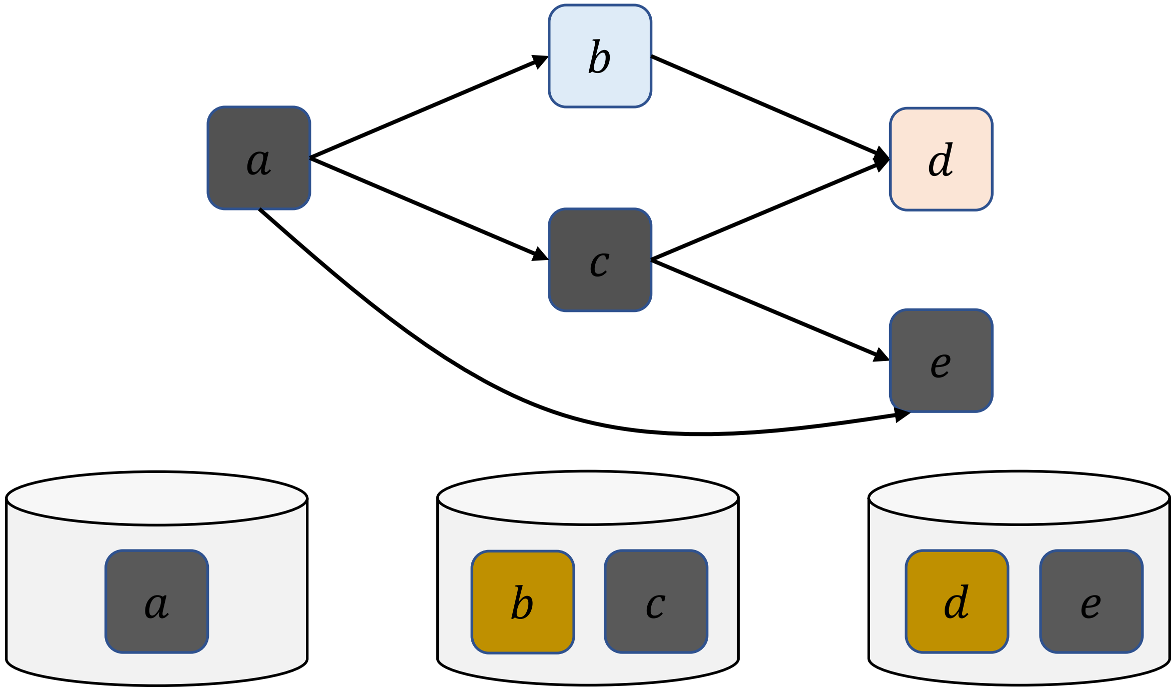
Sample Vertices from Buckets



Less than a constant fraction of vertices remaining can be added

Charge the cost of the $O(\log n)$ vertices not added to added to B

Pruning Vertices



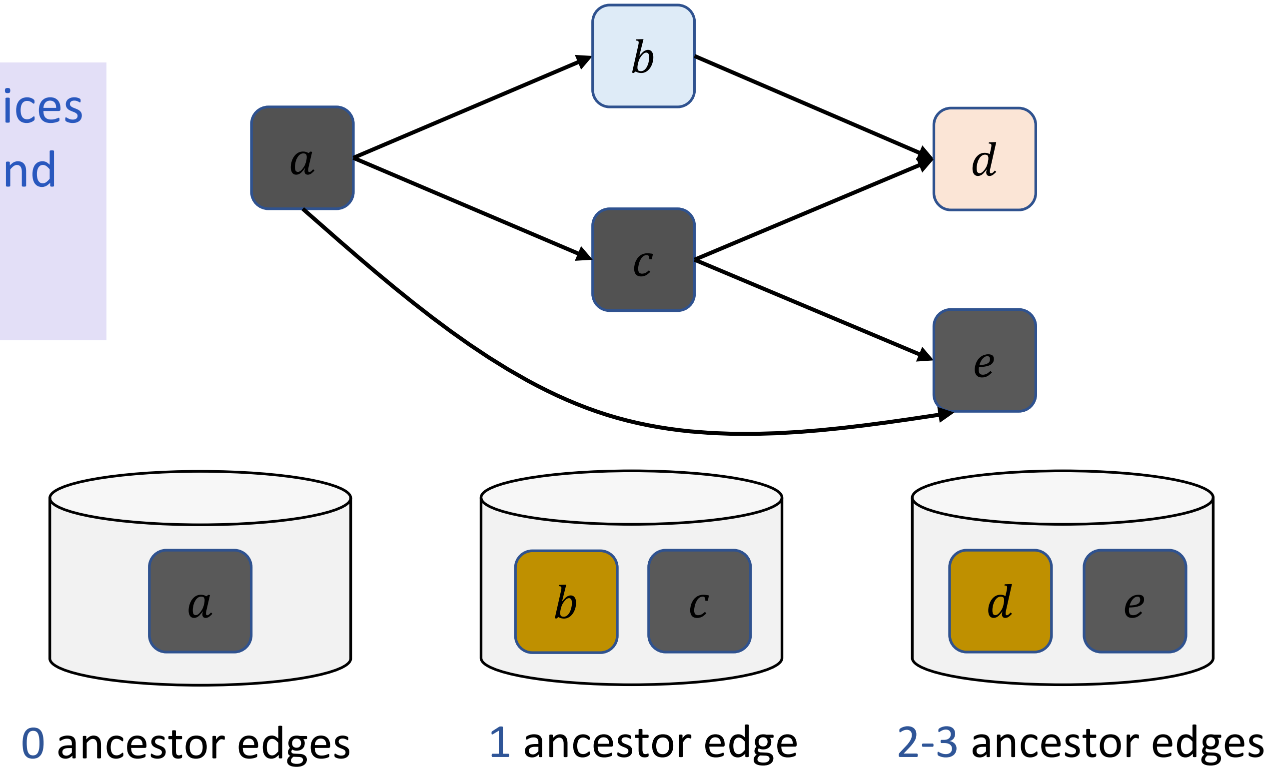
0 ancestor edges

1 ancestor edge

2-3 ancestor edges

Pruning Vertices

Remove scheduled vertices from small subgraph and rerun estimate ancestors/edges



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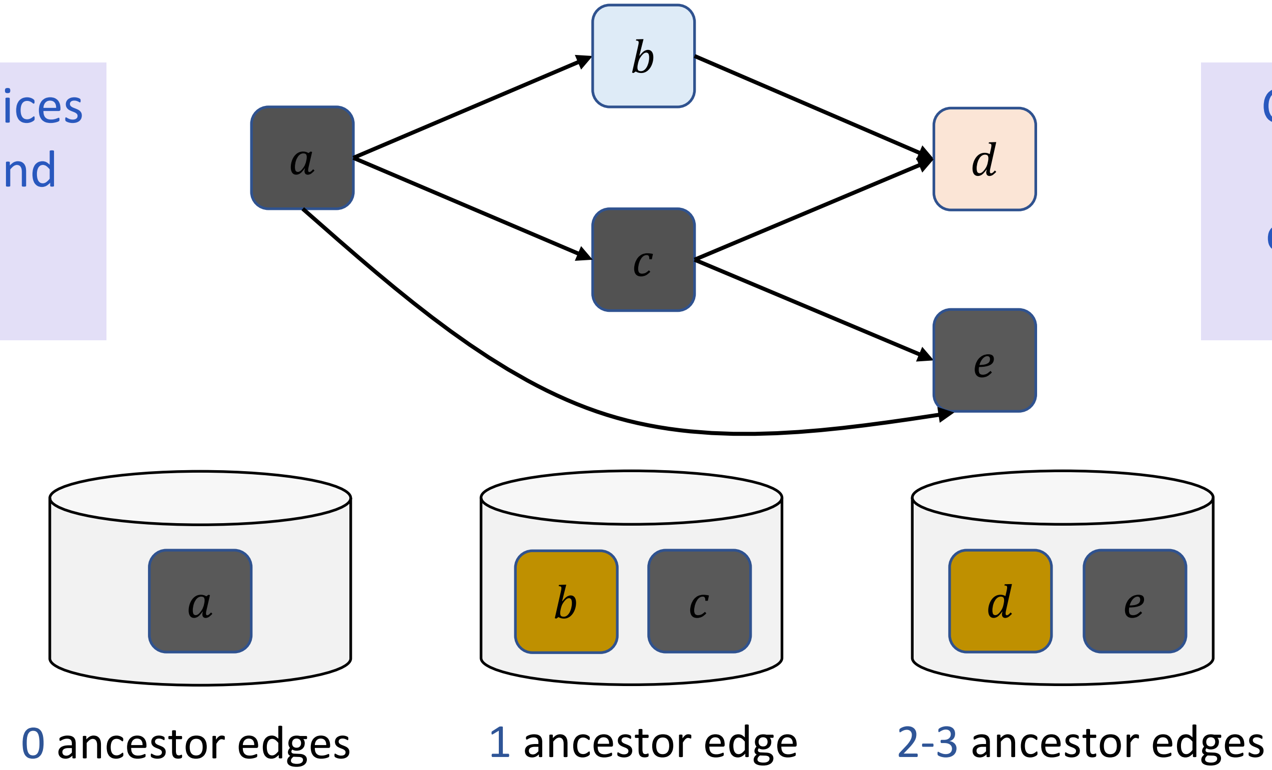

Calculate ratio between new estimate and old estimate—**prune if less than 1/2**

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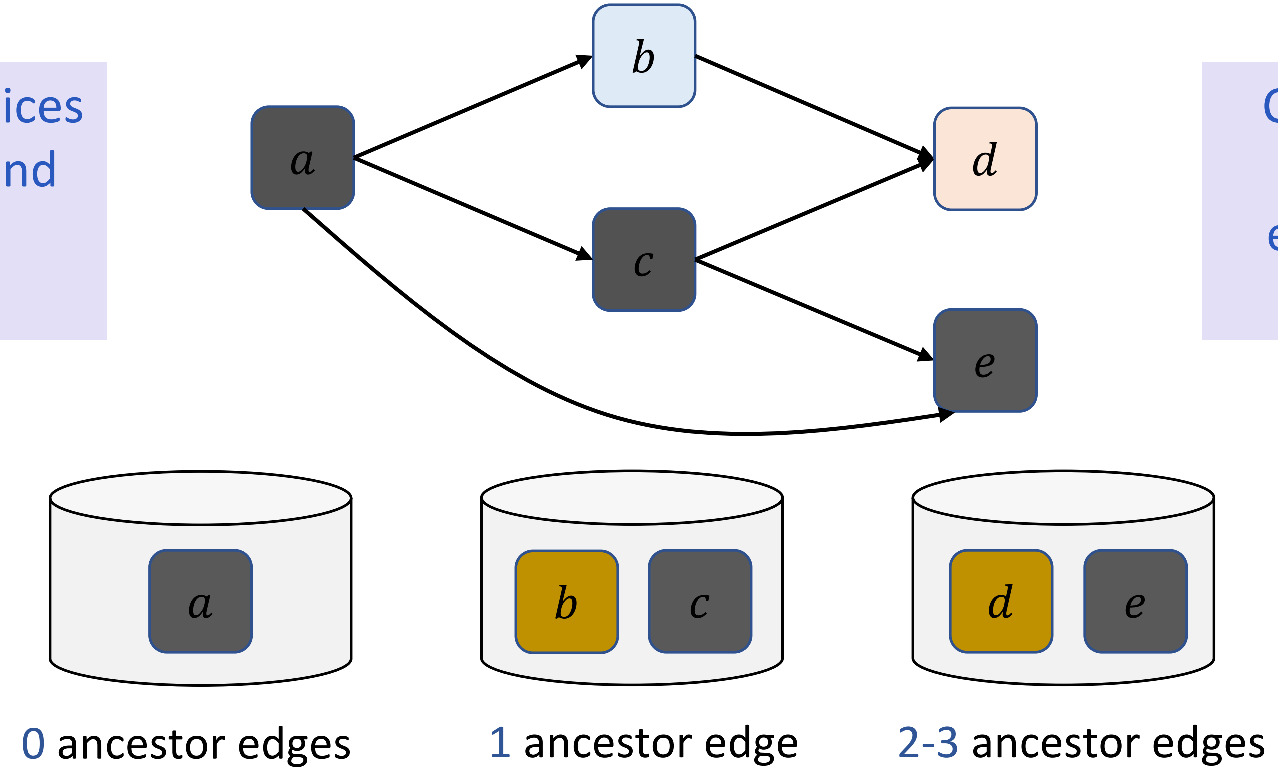
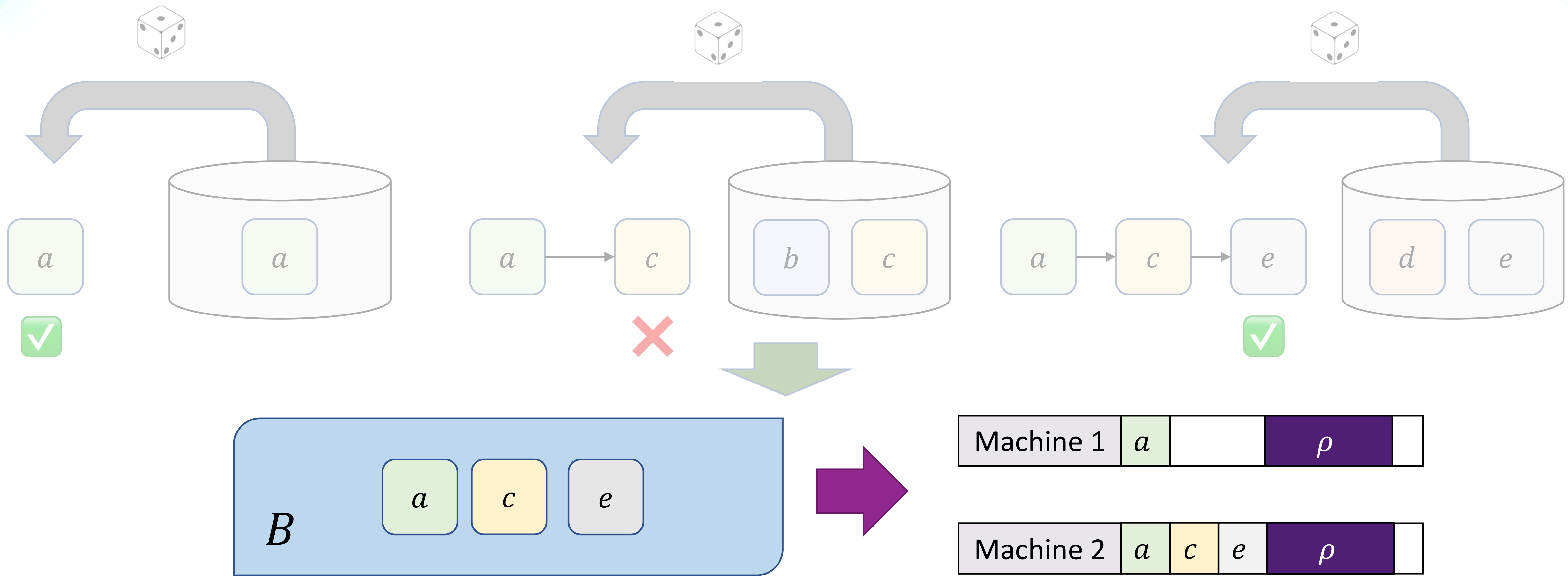


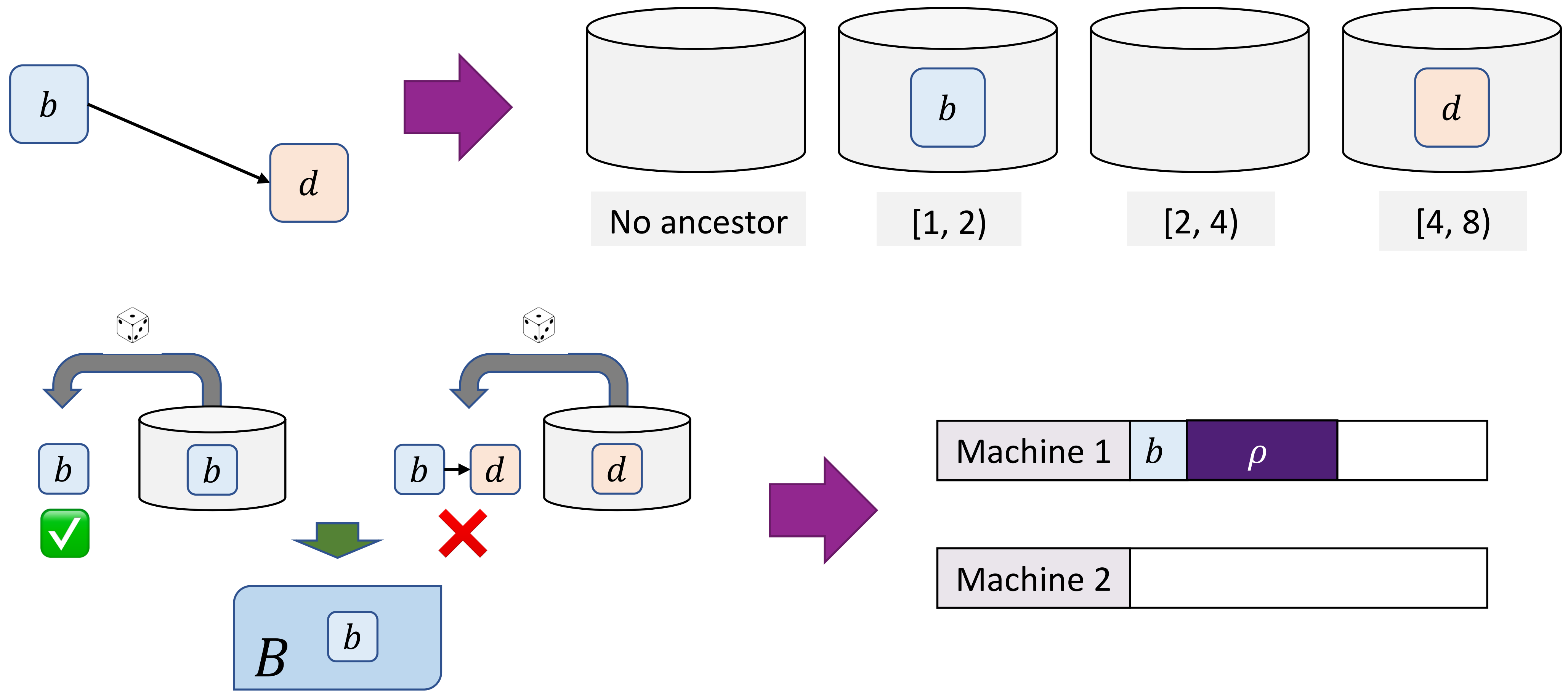
Calculate ratio between new estimate and old estimate—**prune if less than 1/2**

$O((n_S + m_S)\log^2 n)$

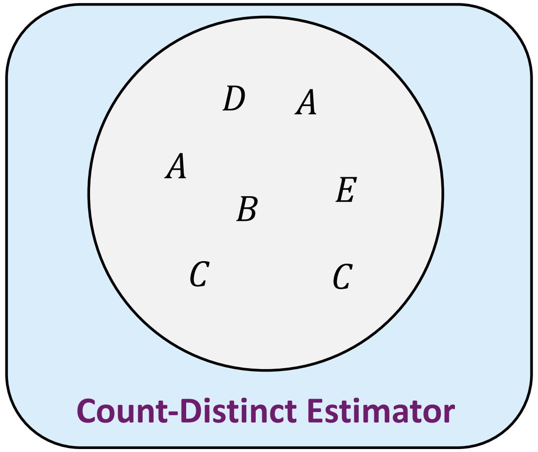
Schedule Bucket

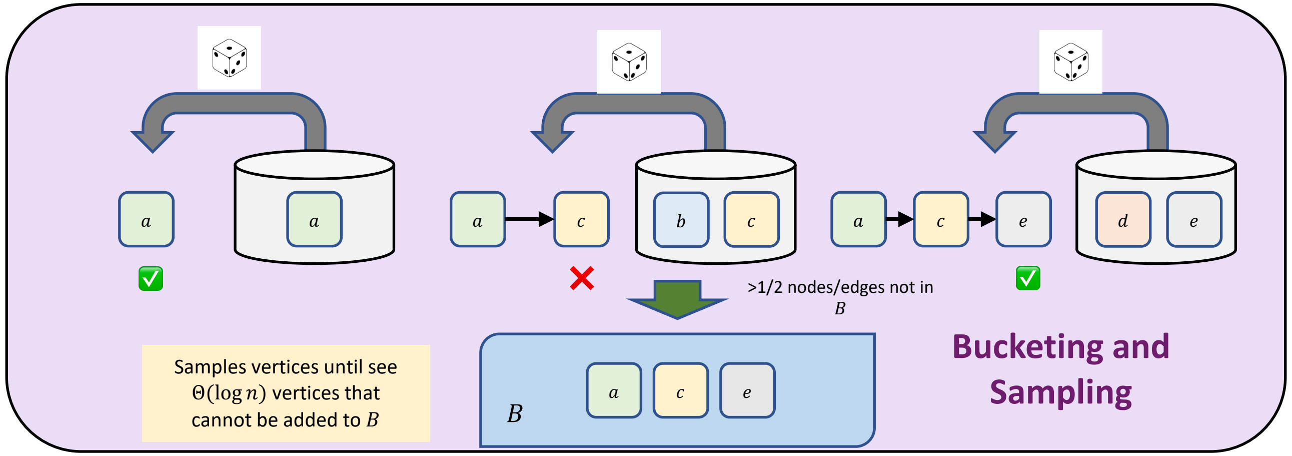


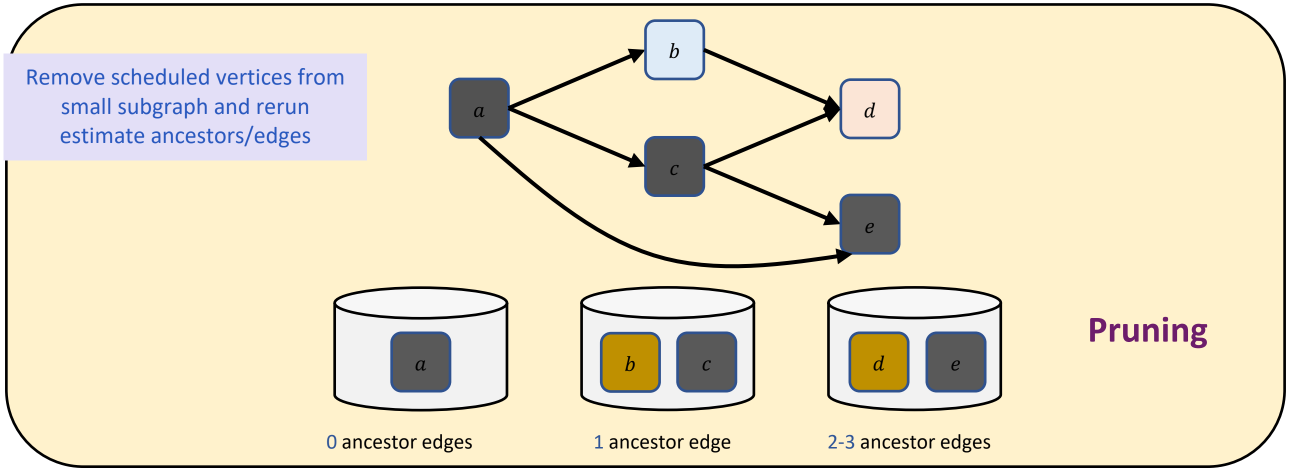
Schedule Remaining Vertices



Our Result: $O(\log \rho \,/\, \log\log \rho)$-approximation, $\tilde{O}(n+m)$ runtime, whp, assuming schedule has length at least ρ







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  - $O(\ln \rho$ …
  - $O(|E_S|$ … total cost over all iterations
- Pruning Runtime: $O(|E_S| \cdot \ln^3 n \cdot \ln \rho)$

Total: $O(m \ln^3 n \ln \rho + n \ln M)$

Conclusion

Main challenge: efficiently determining which jobs to schedule in a batch of jobs

Solution: size-estimation via sketching, sampling and pruning, and work charging argument

Open Questions:

1. Can we get a **linear time** algorithm?
2. Near-linear time algorithm for **non-uniform machines and non-unit jobs**.
3. Can we obtain a **linear-time transformation** for a result *without* duplication?