

CPSC 768: Scalable and Private Graph Algorithms

Lecture 3: Approximate Connected Components and Average Degree in the Sublinear Model

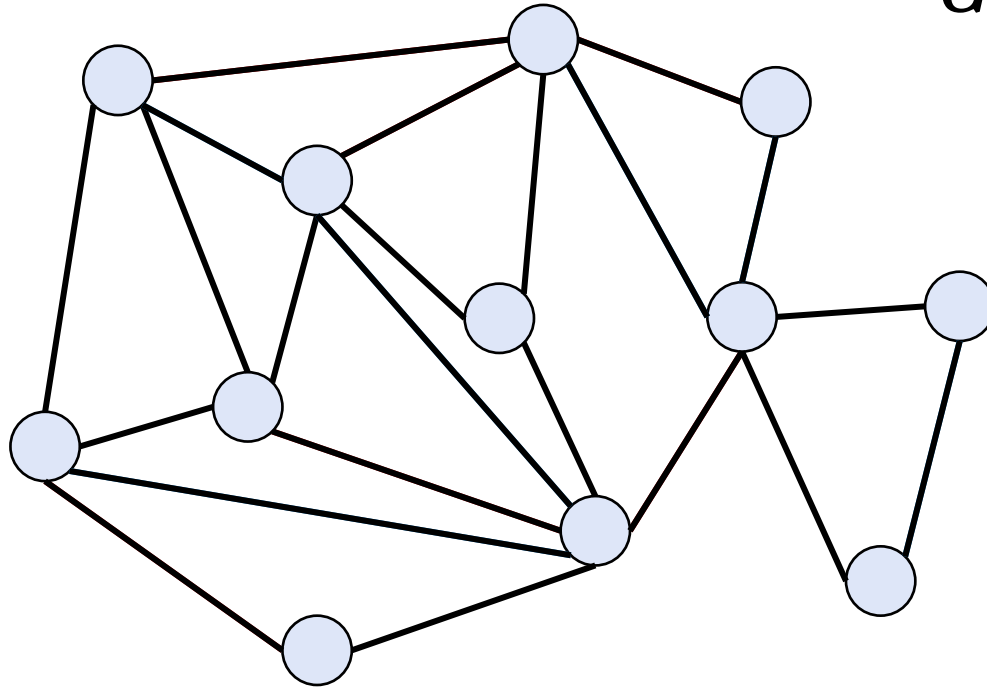
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Open Problem Session Survey

- <https://quanquanliu.com/cpsc768.html>

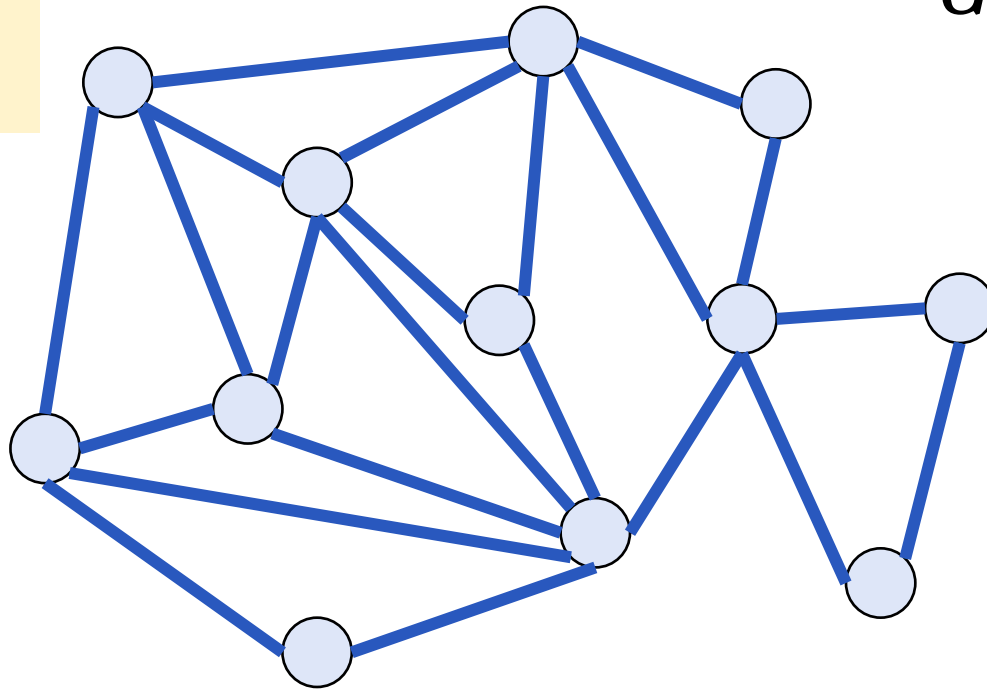
Sublinear Graph Algorithms

$$G = (V, E)$$



Sublinear Graph Algorithms

Number of
Vertices: $n = |V|$



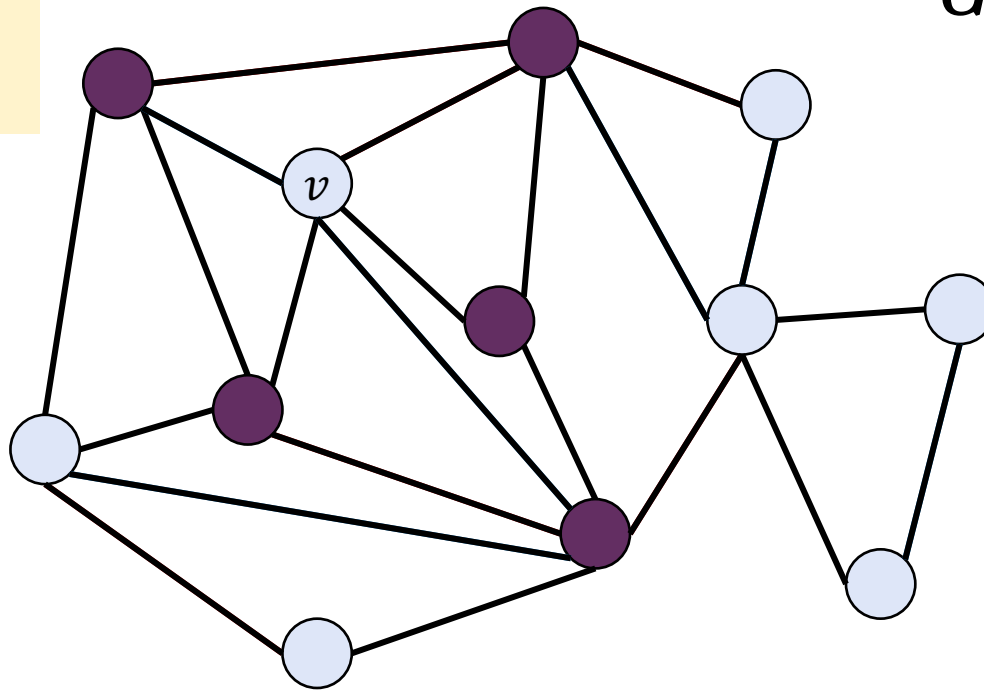
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Number of
Edges: $m = |E|$

Sublinear Graph Algorithms

Number of
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Neighbors of
Vertex v :
 $N(v)$



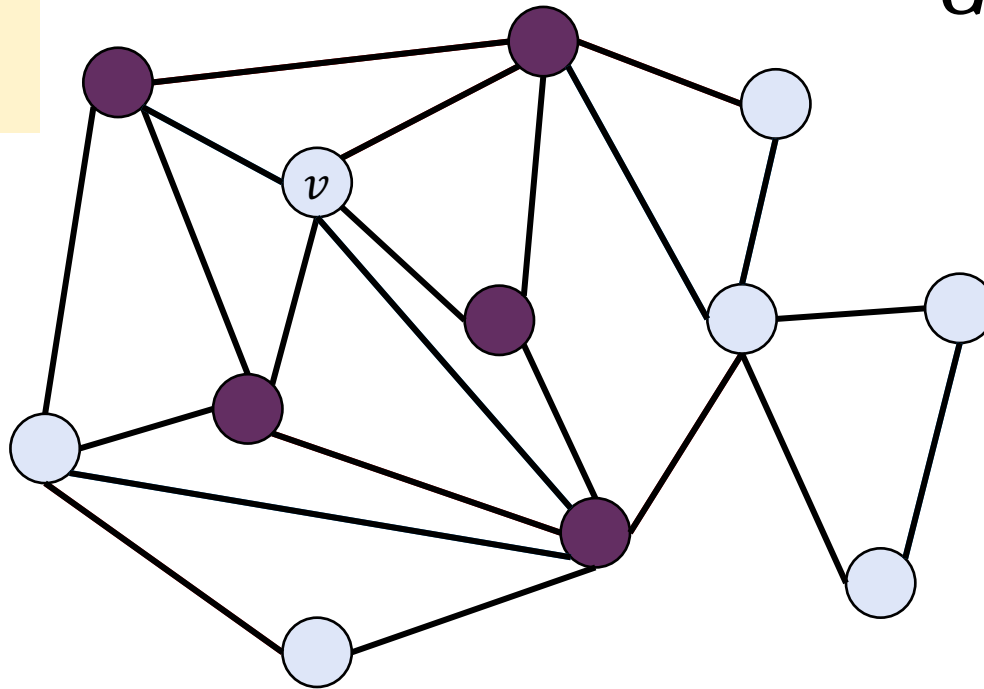
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Sublinear Graph Algorithms

Number of
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$$G = (V, E)$$

Number of
Edges: $m = |E|$

Degree of vertex
 v : $\deg(v) =$
 $|N(v)|$

Sublinear Graph Model: Query Models

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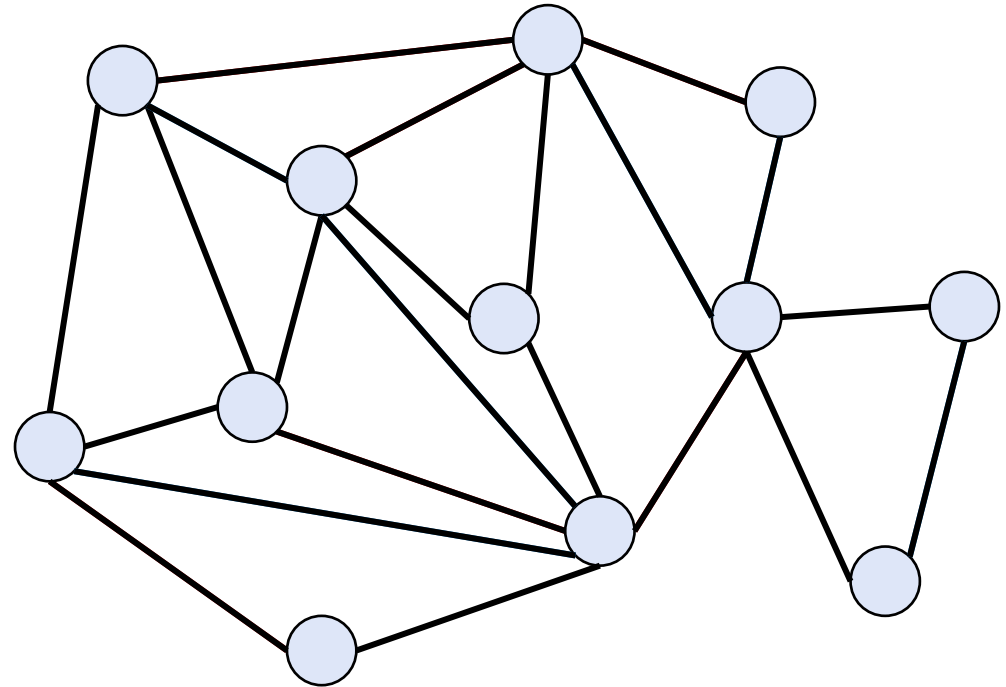
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 - **Known vertices** and **unknown edges**

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 - Cannot read the entire input
 - What types of queries can be made to the input?
 - Often queries are expected to take $O(1)$ time
- Generally assume:
 - **Known vertices** and **unknown edges**
 - Model specifies how to access edges

Sublinear Graph Model: Query Models

- **Adjacency list query model:**
 $O(1)$ time per query



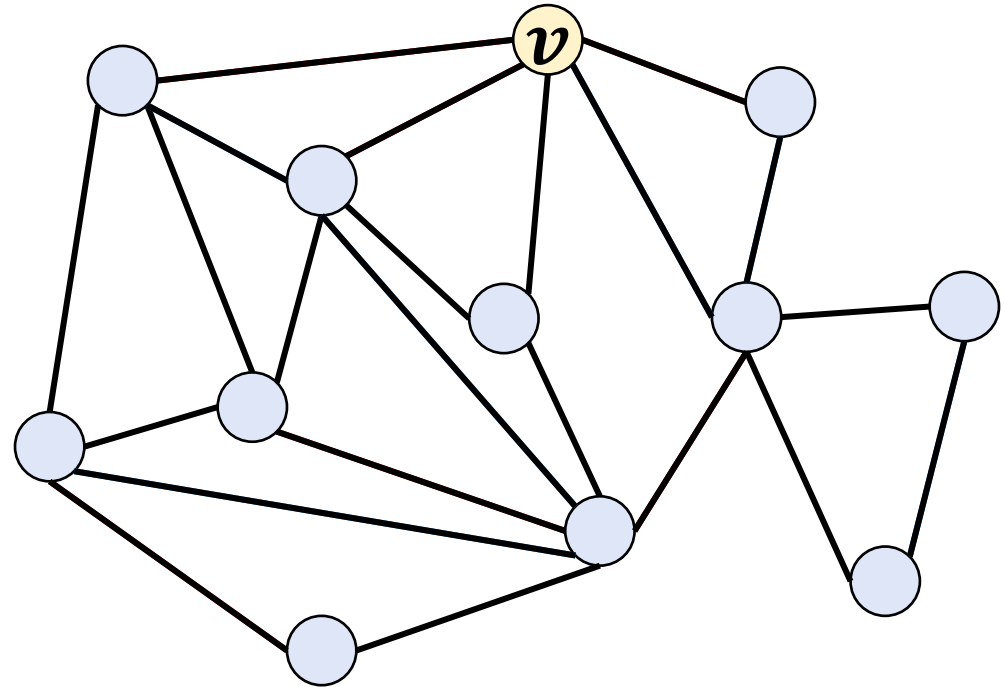
Sublinear Graph Model: Query Models

- **Adjacency list query model:**

$O(1)$ time per query

- **Degree queries:** given a vertex $v \in V$, **output** $\text{deg}(v)$

$$\text{deg}(v) = 5$$



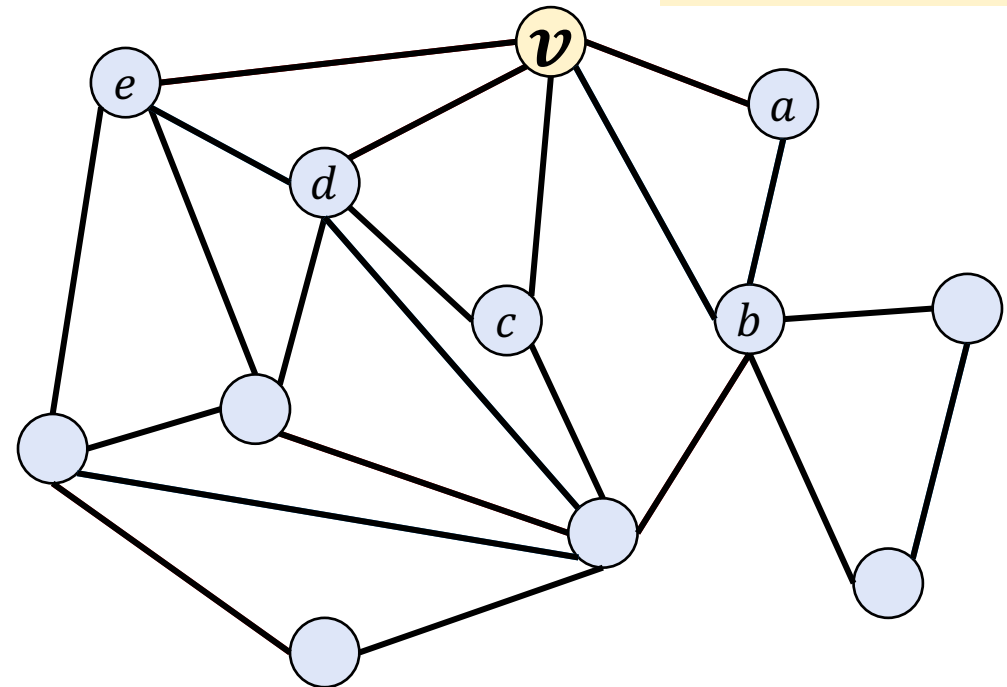
Sublinear Graph Model: Query Models

- **Adjacency list query model:**

$O(1)$ time per query

- **Degree queries:** given a vertex $v \in V$, **output $\text{deg}(v)$**
- **Neighbor queries:** given a vertex $v \in V$ and $i \in [n]$, **output the i -th neighbor of v** or \perp if $i > \text{deg}(v)$

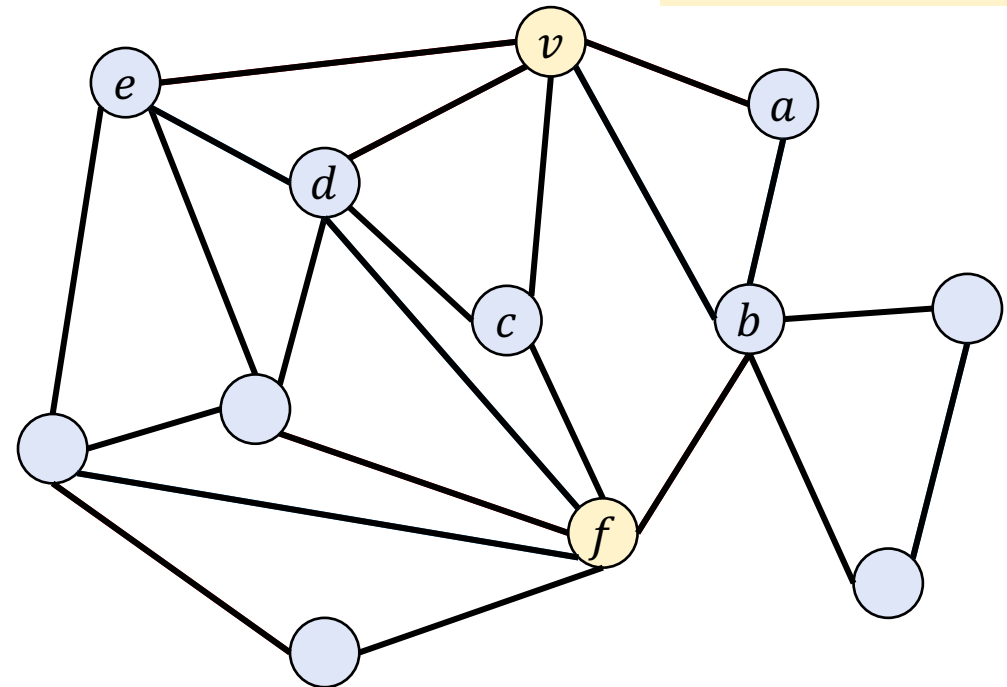
Third neighbor
of v is c



Sublinear Graph Model: Query Models

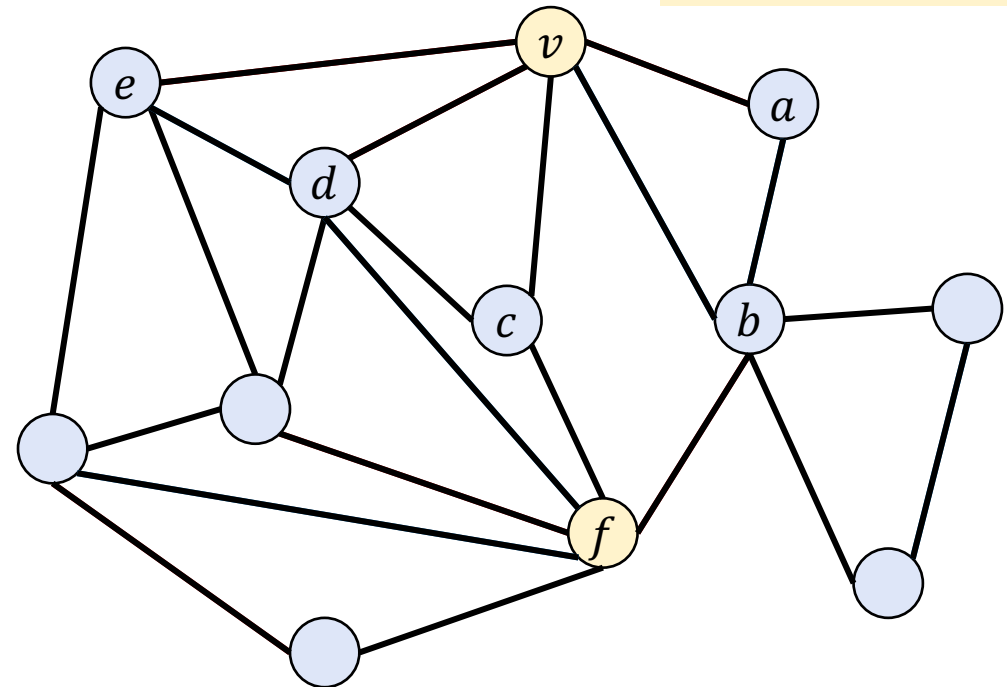
- **Adjacency matrix query model:** $O(1)$ time per query
 - **Pair queries:** given two vertices $u, v \in V$, output whether $(u, v) \in E$ is an edge or not

Is $\{v, f\}$ an edge? **NO**



Sublinear Graph Model: Query Models

- **Adjacency matrix query model:** $O(1)$ time per query
 - **Pair queries:** given two vertices $u, v \in V$, output whether $(u, v) \in E$ is an edge or not
- **General query model:** allows for all three of the above queries



Is $\{v, f\}$ an edge? **NO**