Scalable Auction Algorithms for Bipartite Maximum Matching Problems

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Joint work with

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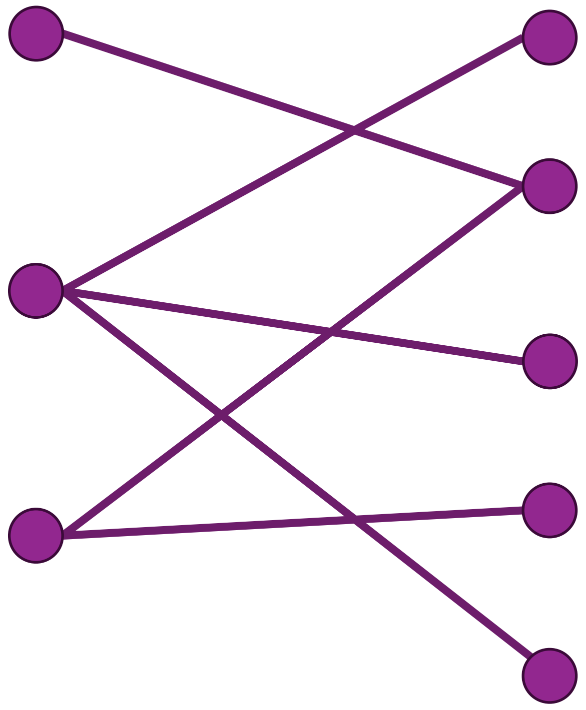
Bipartite Matching Problems

- **Maximum Matching**: return matching of maximum size



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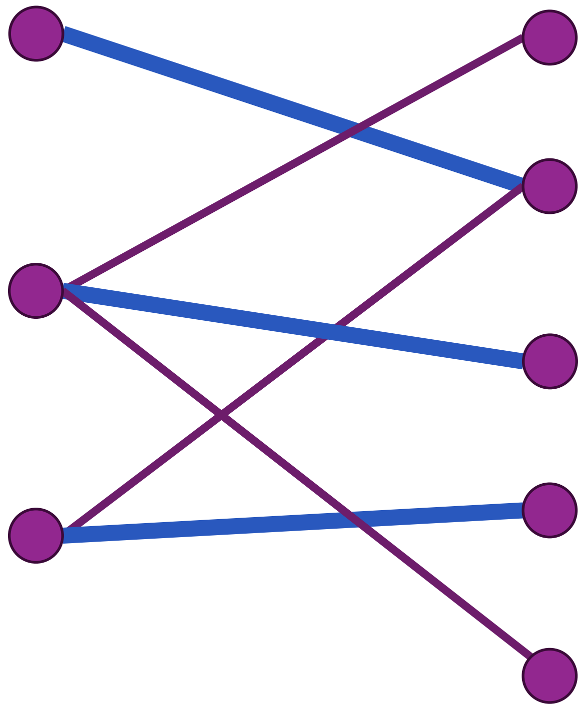
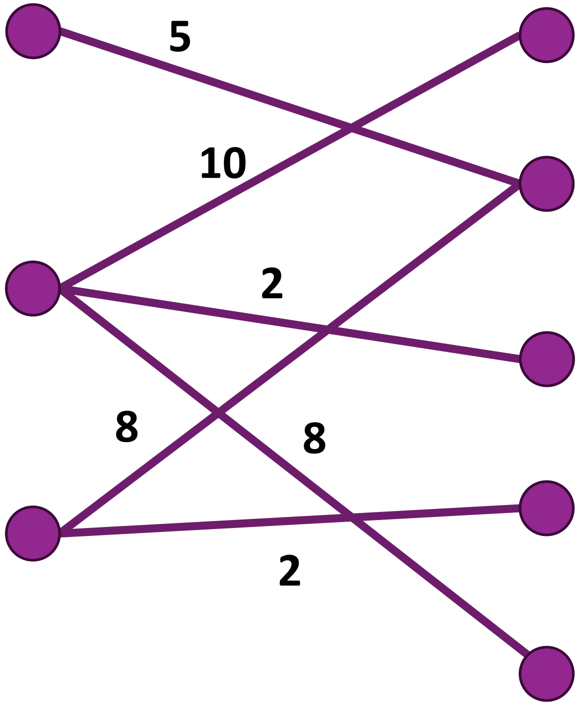


Bipartite Matching Problems

- **Maximum** **Weighted** **Matching**: return matching of maximum weight



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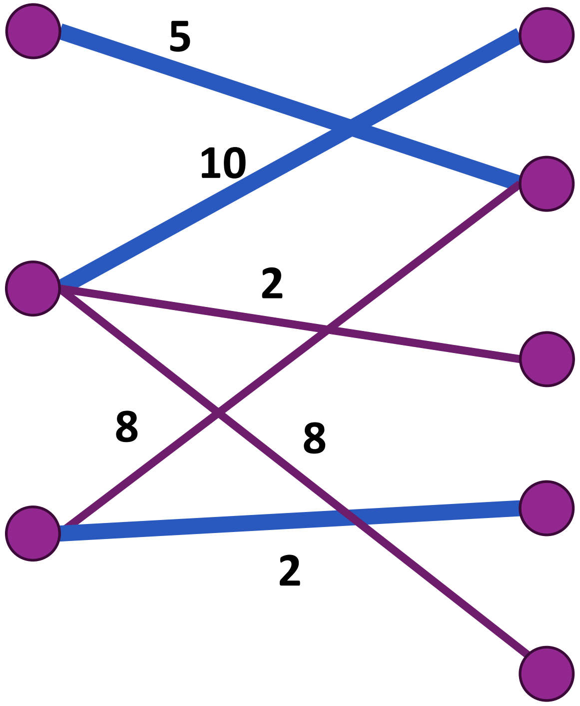
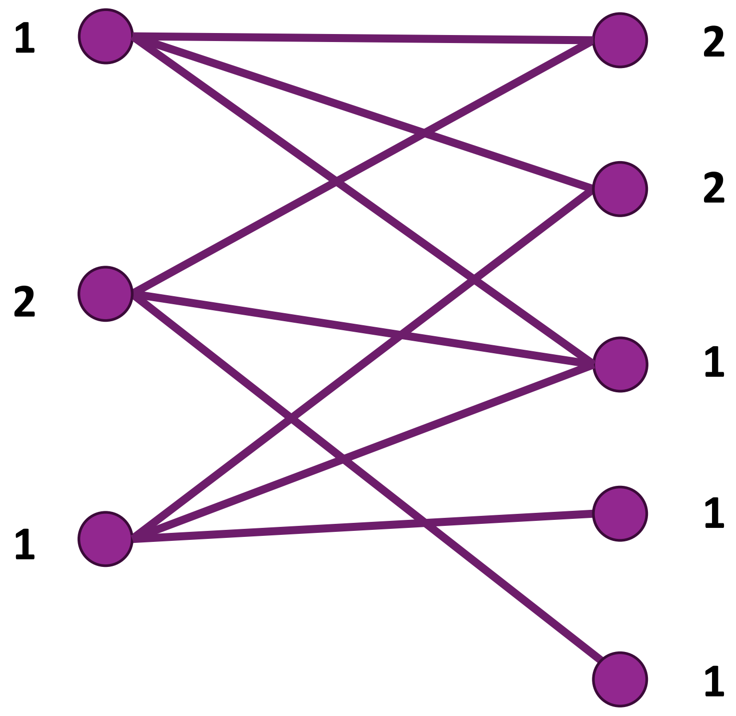


Bipartite Matching Problems

- **Maximum** ***b*-Matching**: return matching of maximum size when each node v can be matched to at most b_v nodes



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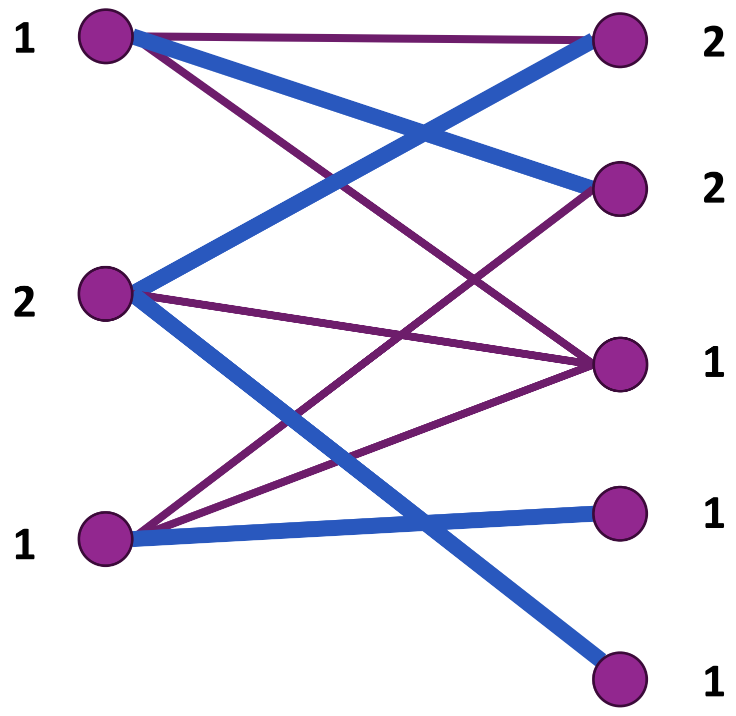

Approximate Bipartite Matching Problems

- Let M^* be the optimum matching solution
- A **$(1-\varepsilon)$-approximate** solution $\widehat{M}$ has value at least:

$$\widehat{M} \geq (1-\varepsilon) \cdot M^*$$

Auction-Based Maximum Matching

- Introduced by Demange, Gale, and Sotomayor '86 and Bertsekas '81 for **exact** (weighted) matching
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Auction-Based Maximum

Zheng and Henzinger '23 extends MWM to **sequential and dynamic models**

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Our Results

MWM = Maximum Weighted Matching
MCBM = Maximum Cardinality *b*-Matching

Model		Previous Results		Our Results	
Blackboard Distributed	MWM	$\Omega(n \log n)$ (trivial)	[DNO14]	$O\left(\frac{n \log^3(n)}{\varepsilon^8}\right)$	Theorem 3.9
	MCBM	$\Omega(nb \log n)$	trivial	$O\left(\frac{nb \log^2 n}{\varepsilon^2}\right)$	Theorem 4.8
Streaming	MWM	$O\left(\frac{\log(1/\varepsilon)}{\varepsilon^2}\right)$ pass $O\left(\frac{n \log n}{\varepsilon^2}\right)$ space	[AG11]	$O\left(\frac{1}{\varepsilon^8}\right)$ pass $O\left(n \cdot \log n \cdot \log(1/\varepsilon)\right)$ space	Theorem 3.11
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MPC	MWM	$O_\varepsilon(\log \log n)$ rounds $O_\varepsilon(n \operatorname{poly}(\log n))$ space p.m.	[GKMS19] (general)	$O\left(\frac{\log \log n}{\varepsilon^7}\right)$ rounds $O(n \cdot \log_{(1/\varepsilon)}(n))$ space p.m.	Theorem 3.15
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"Universal" solution across many different scalable models!

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First results in blackboard distributed

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Eliminate polynomial dependence in $\left(\frac{1}{\varepsilon}\right)$ in space

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Eliminate exponential dependence on $\left(\frac{1}{\varepsilon}\right)$

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Eliminate large dependence on $\left(\frac{1}{\varepsilon}\right)$ and $\log n$

Outline

- Auction Algorithm of [ALT21] for maximum cardinality matching
- Our auction algorithm for maximum weighted matching
  - Algorithm description
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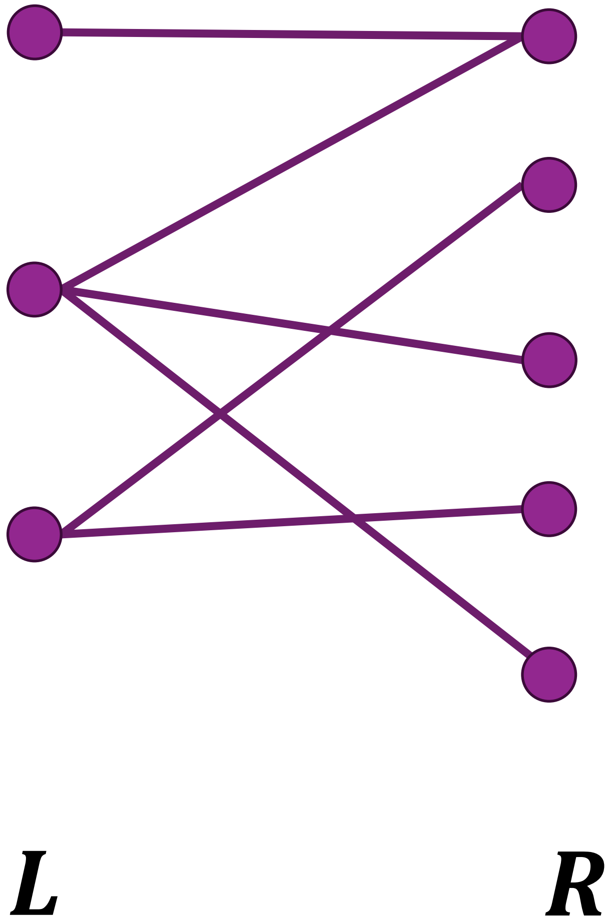
Auction Algorithm of [ALT21]

Left and Right Side of Bipartite Graph



Auction Algorithm of [ALT21]

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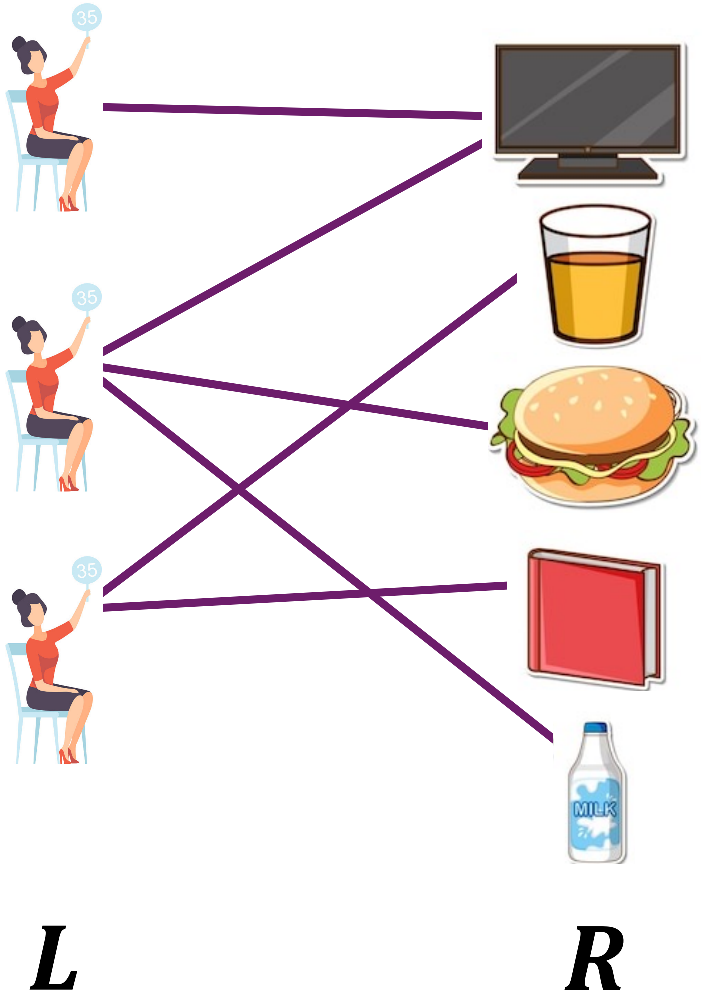
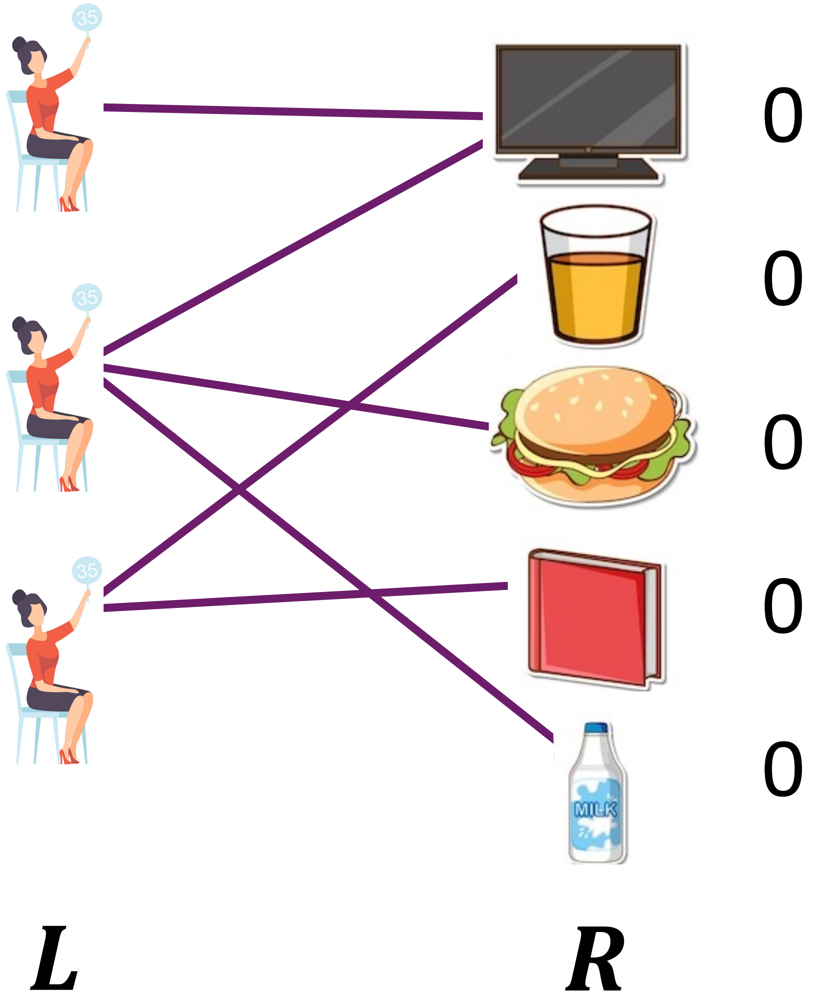


Left Side has **Bidders** and **Right Side** has **Items**

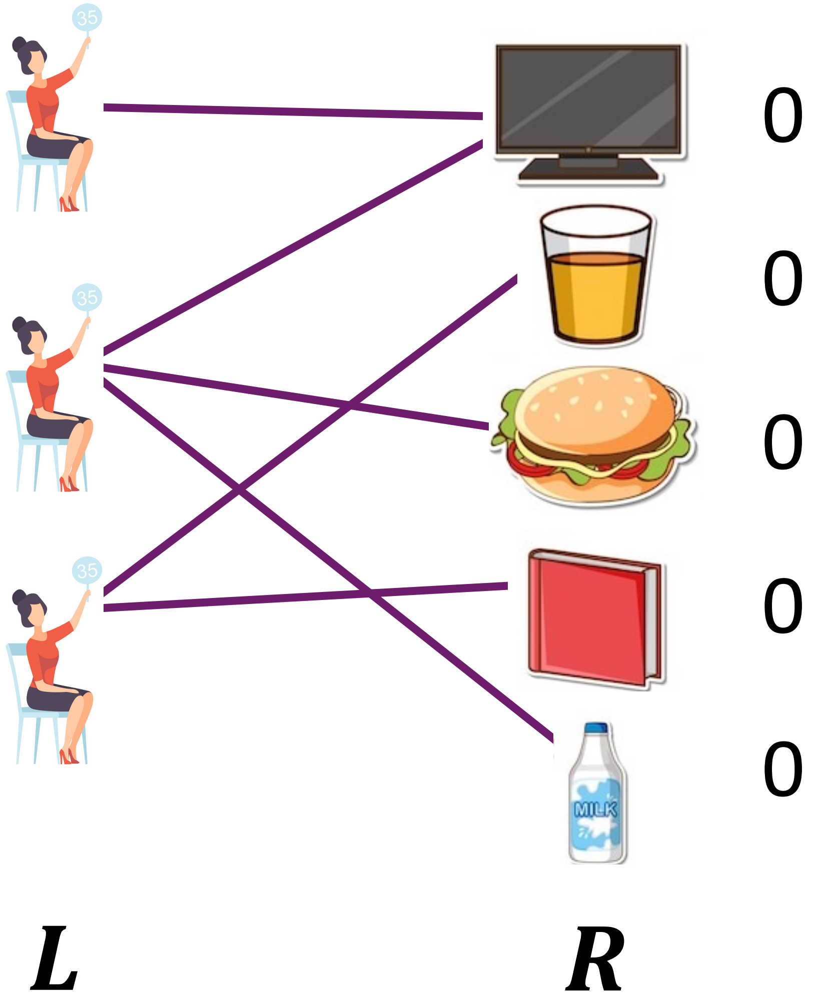
Auction Algorithm of [ALT21]



All items **start with price 0**

Auction Algorithm of [ALT21]

Iteratively, bidders **bid** on all **lowest price adjacent items**



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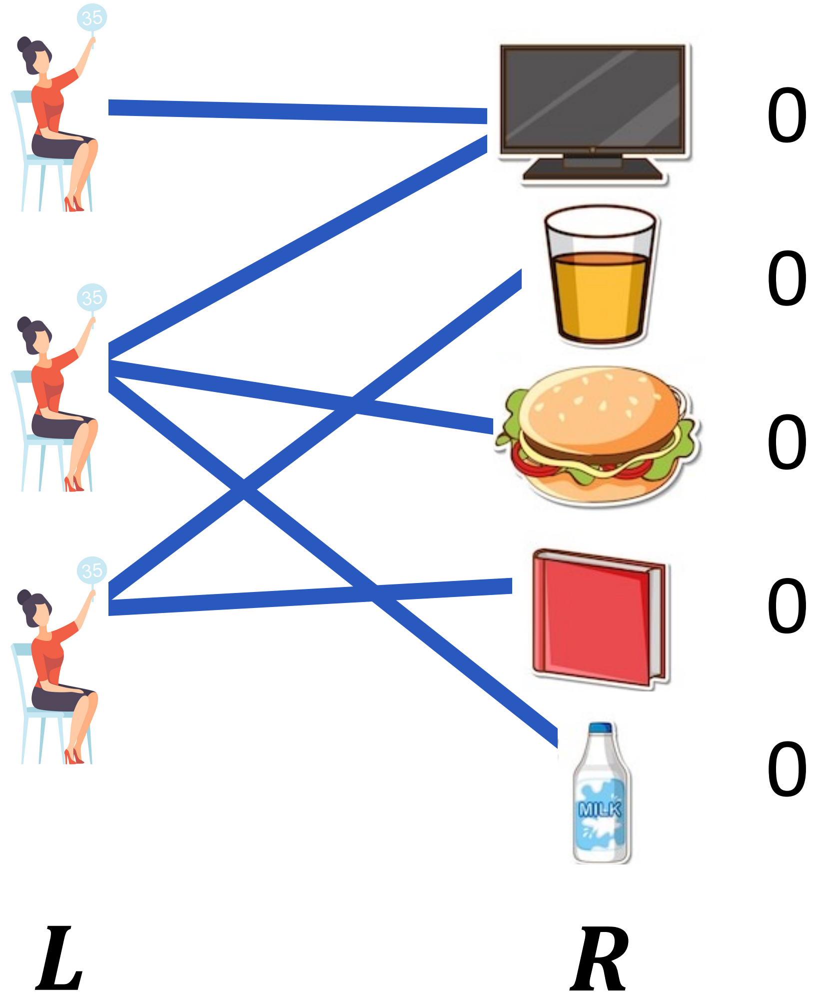

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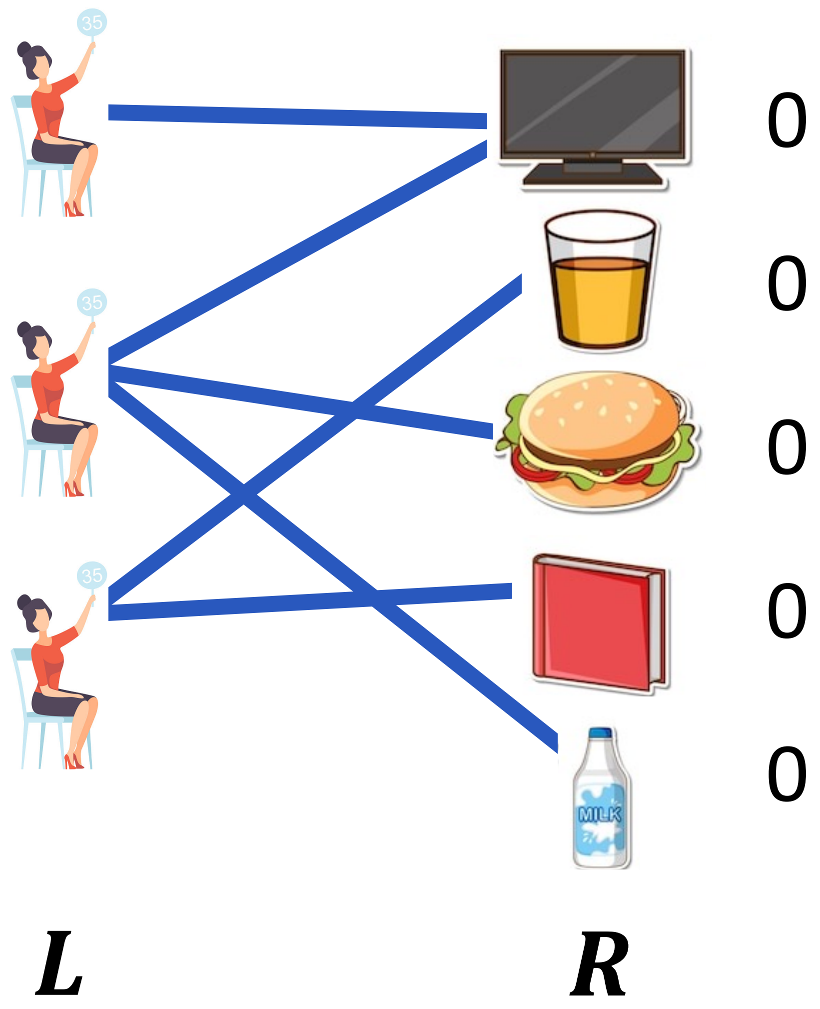
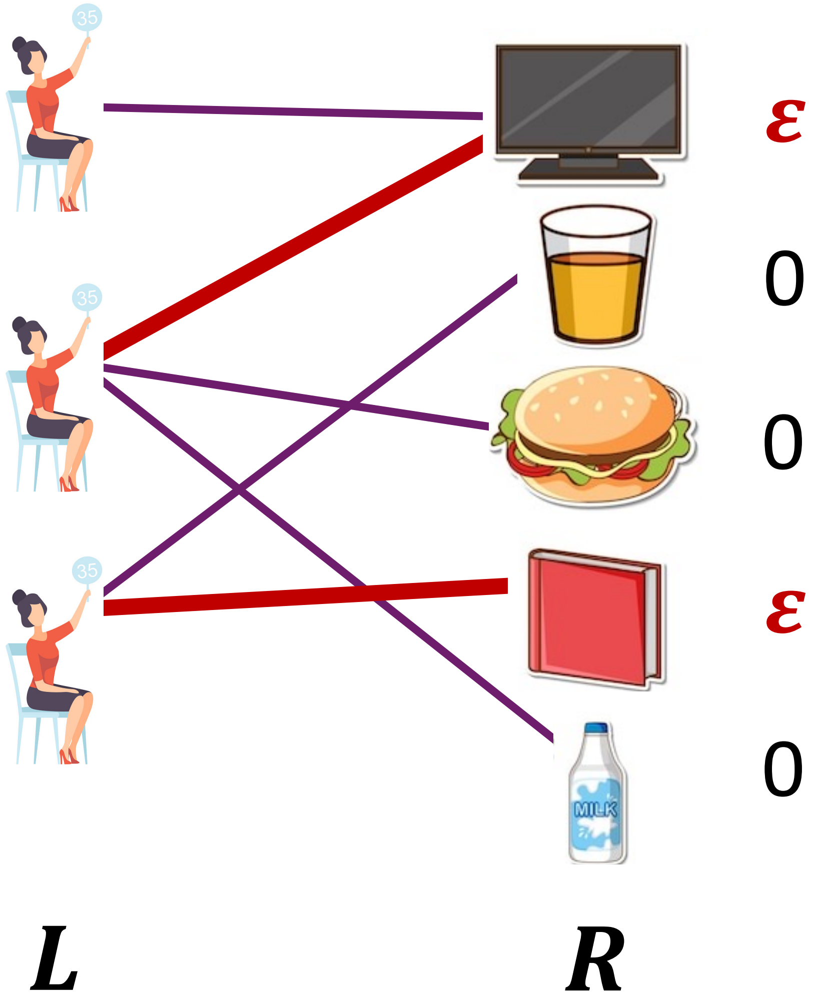


Find **maximal matching** among **induced subgraph of bid items**

Auction Algorithm of [ALT21]

Increase price of items in matching **by ε** and **maintain current matching**



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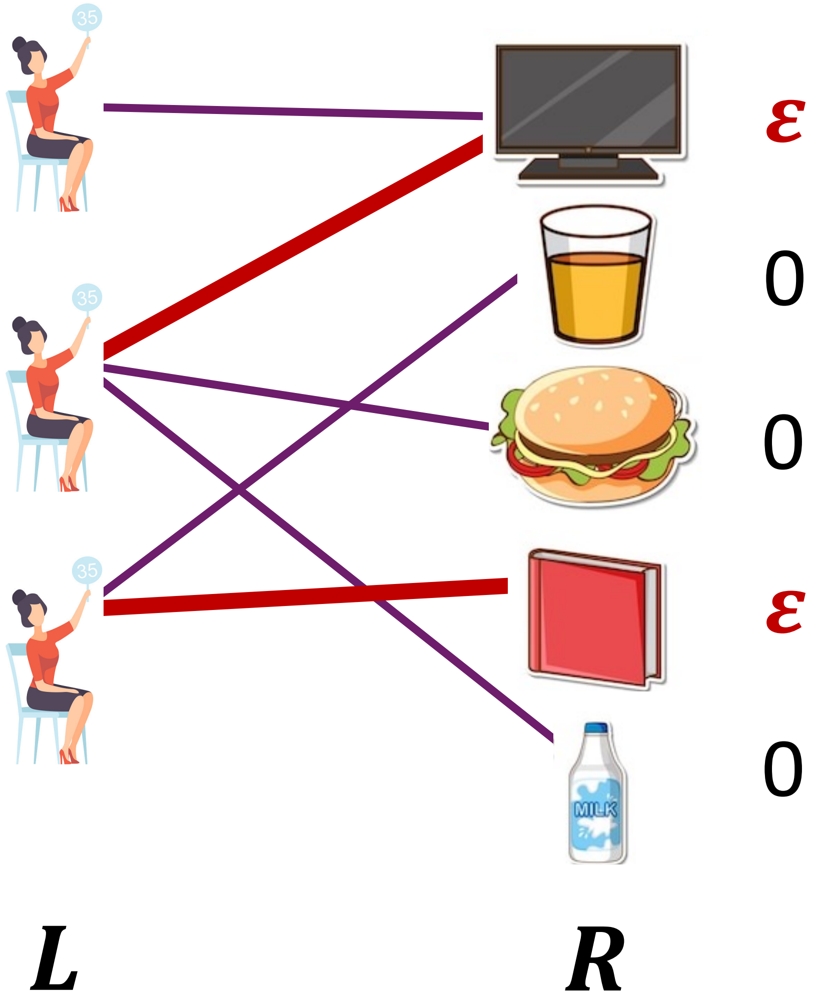
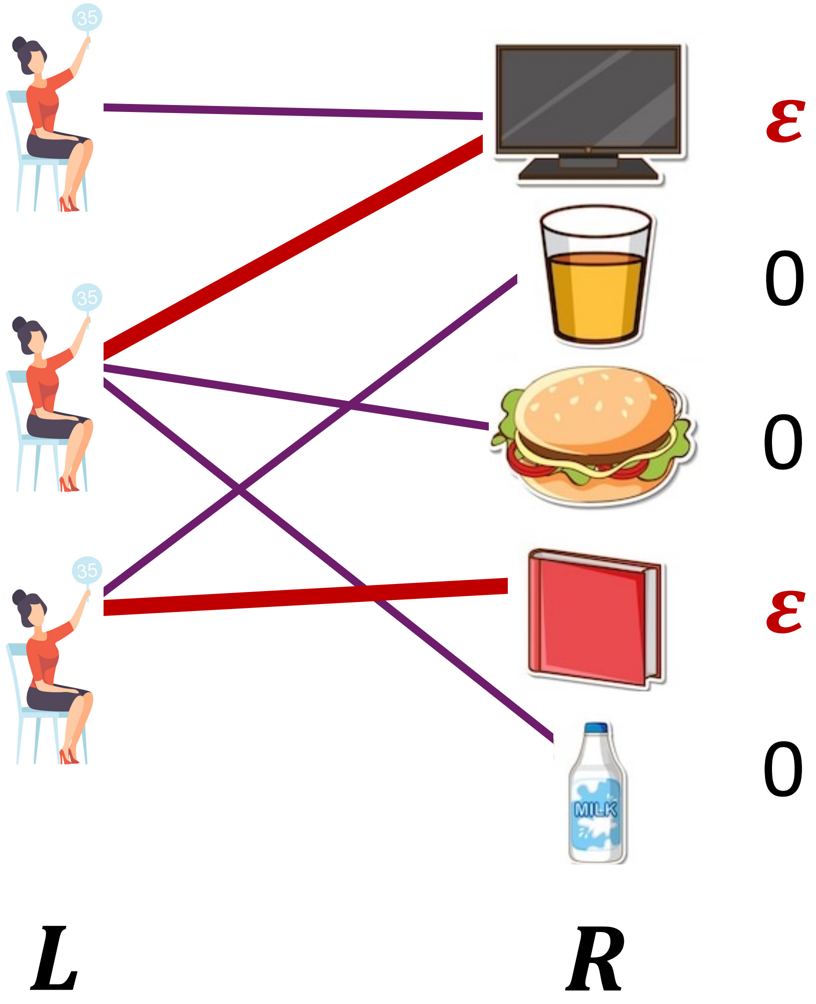


Can bid on item as long as **price < 1**

Auction Algorithm of [ALT21]

Iterate for

$\left\lceil \frac{2}{\varepsilon^2} \right\rceil$ **iterations**



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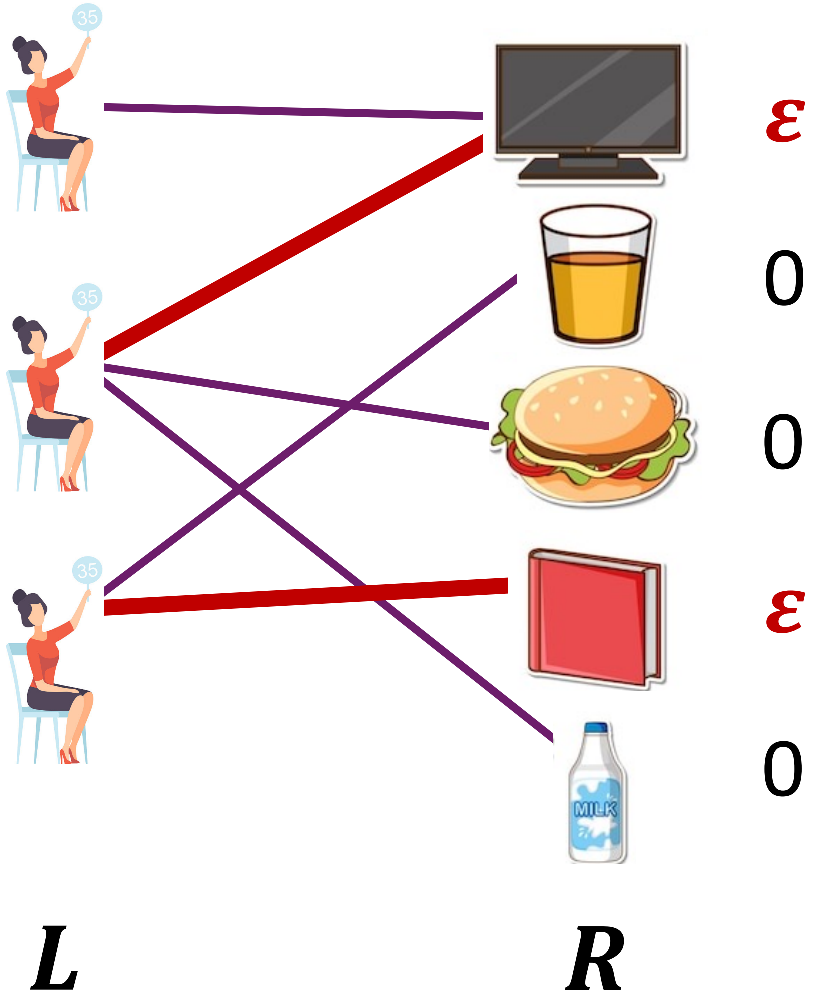

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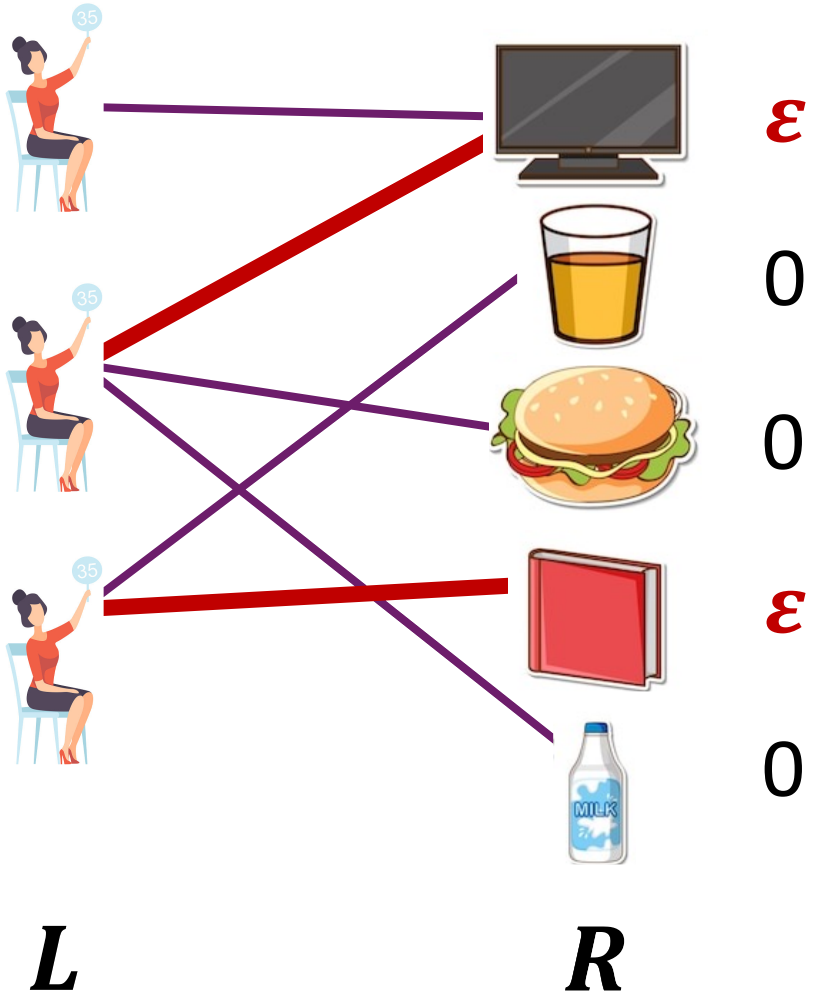

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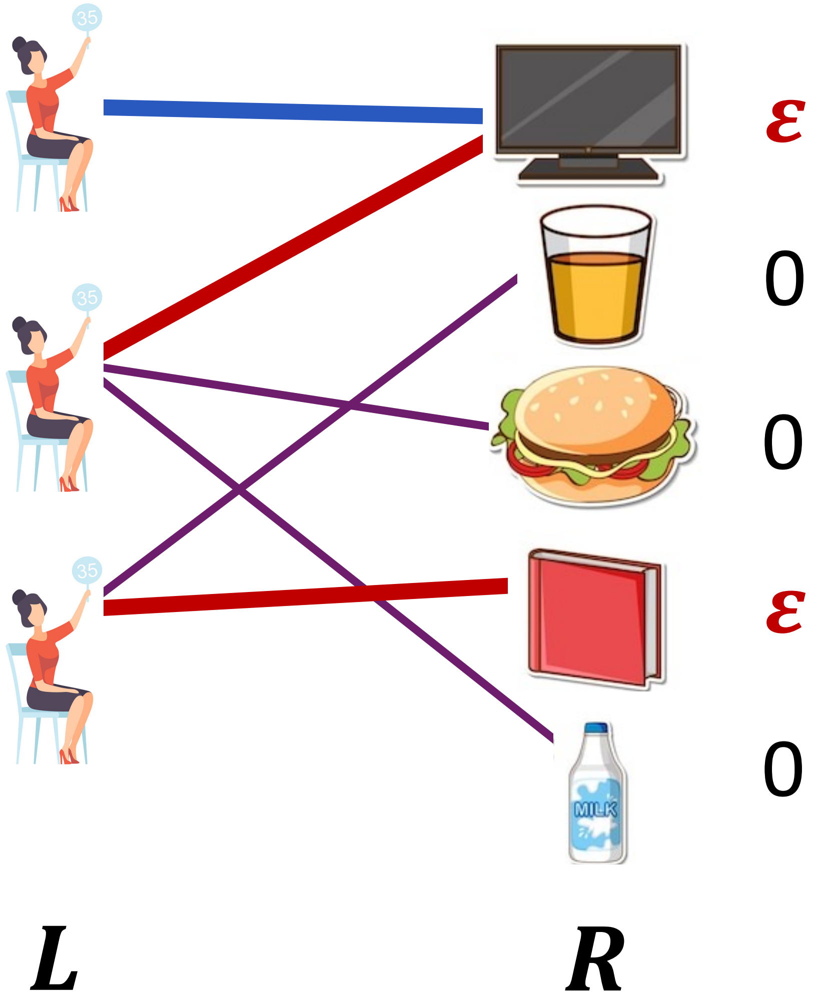

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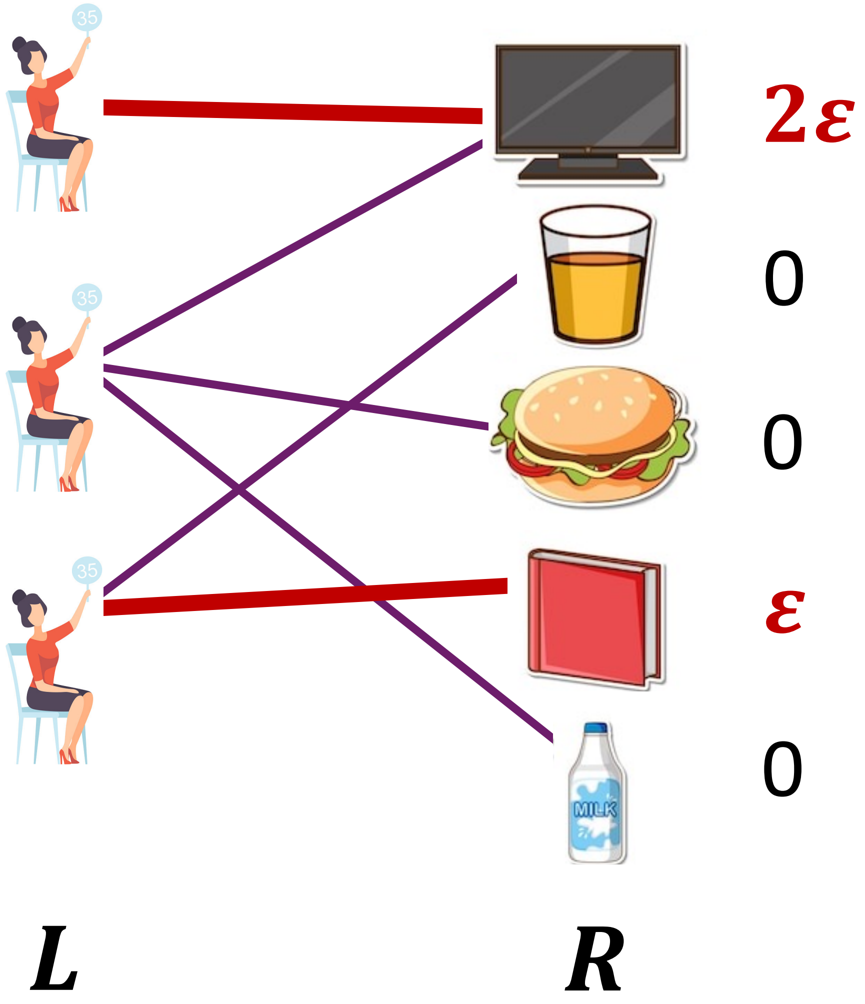
Item goes to new bidder!



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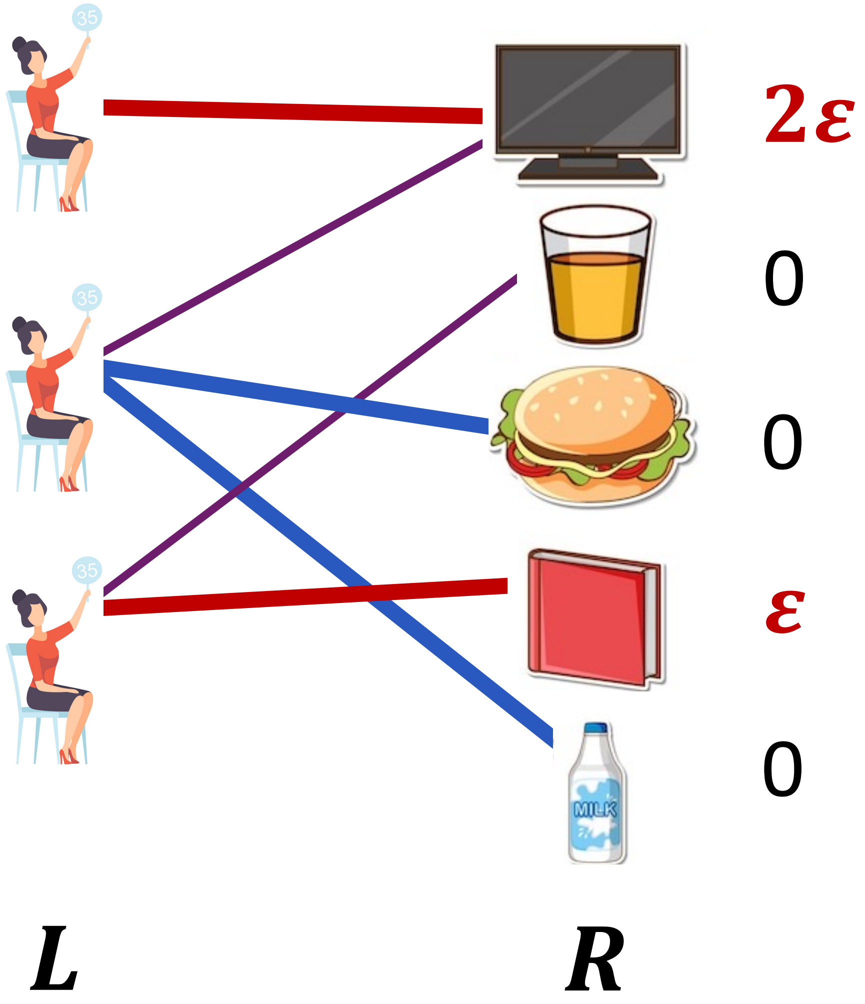

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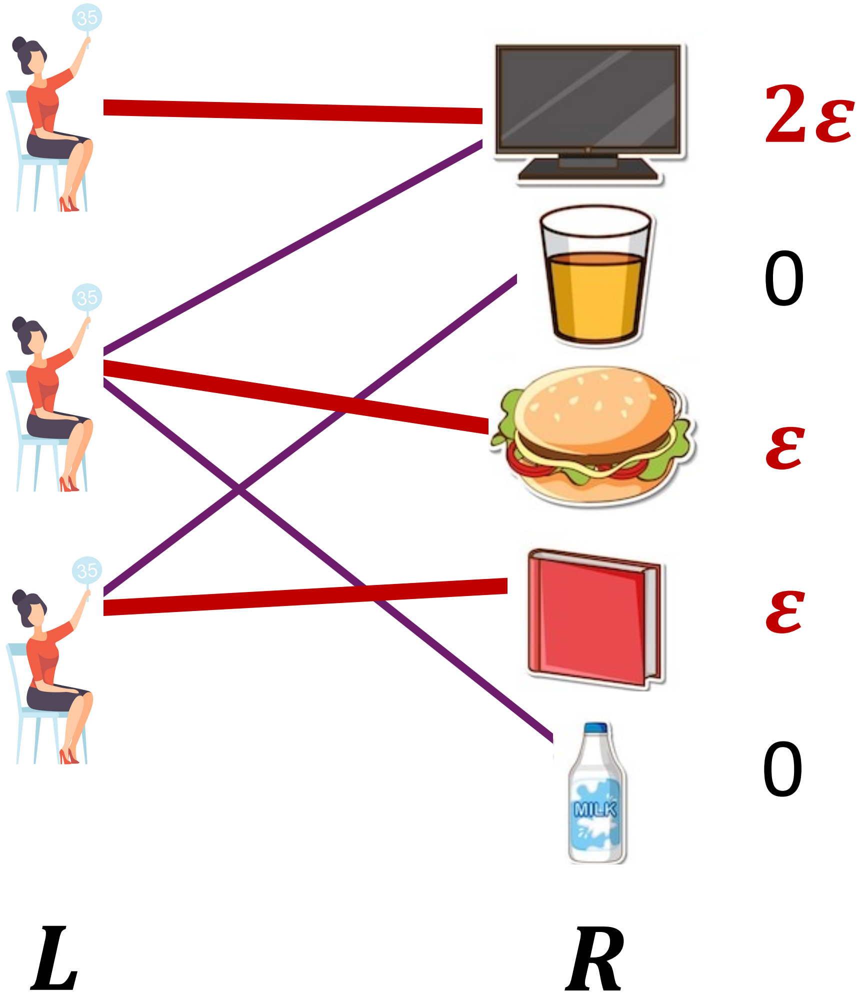

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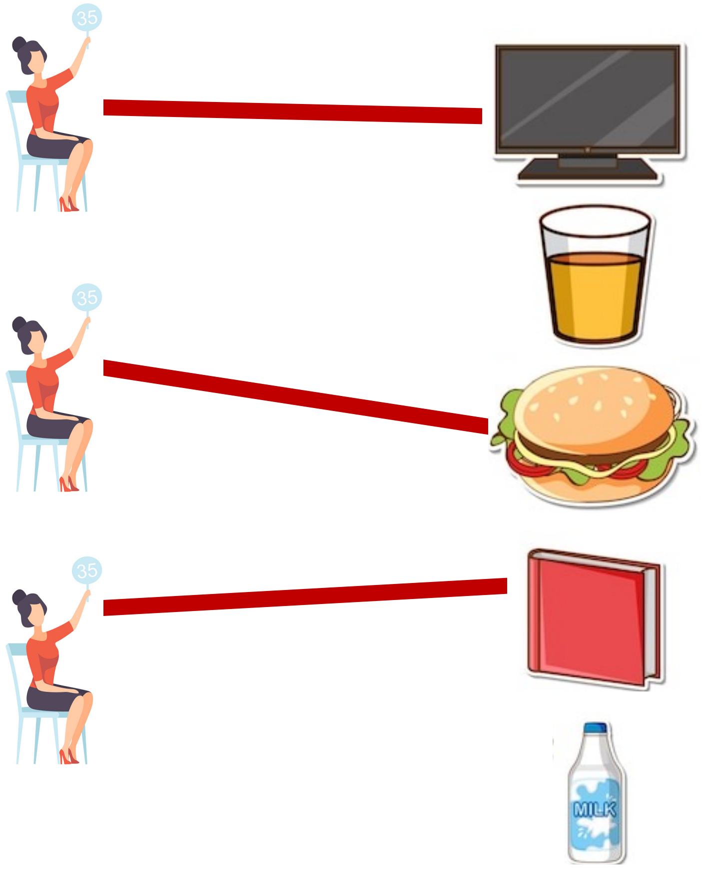
Auction Algorithm of [ALT21]

Final Matching



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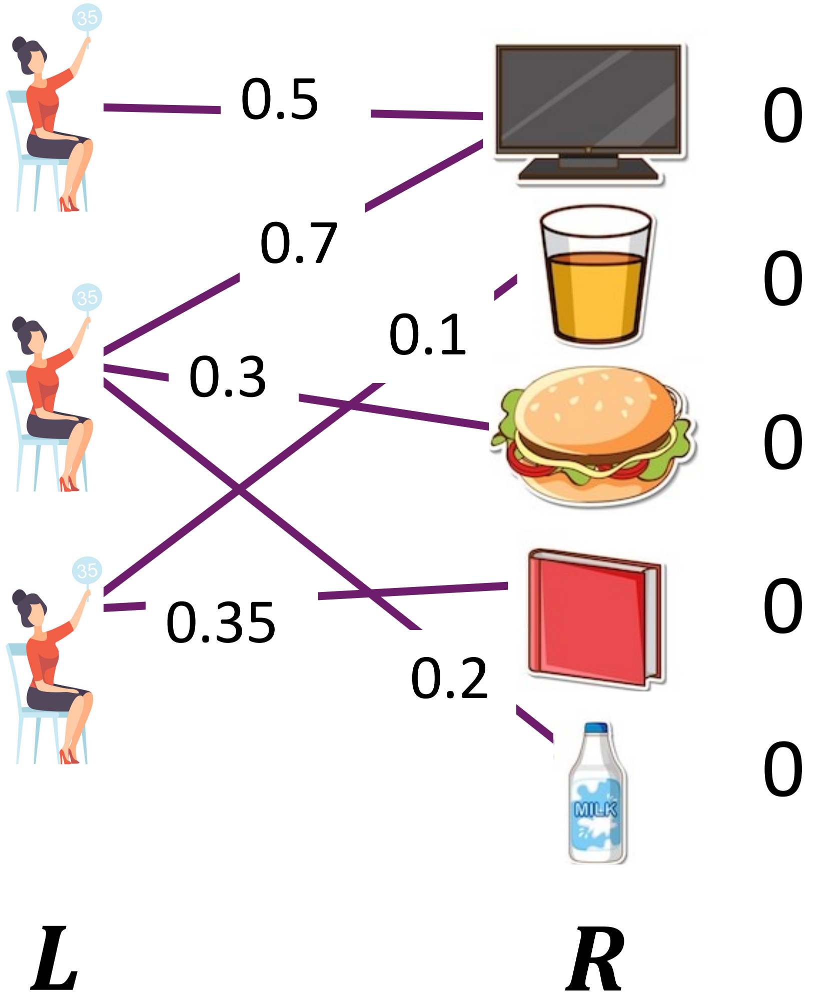
Our Maximum Weight Auction Algorithm

- **Bucket the edges** using buckets based on the weights of the edges
  - **Rescale weights** to **(0, 1]**
  - Edge with weight $\boldsymbol{w} \in (\mathbf{0}, \mathbf{1}]$ is in bucket ***b*** if

$$\boldsymbol{\varepsilon}^{\boldsymbol{b}-\mathbf{1}} \leq \boldsymbol{w} < \boldsymbol{\varepsilon}^{\boldsymbol{b}-\mathbf{2}}$$

Our **Simplified** Maximum Weight Auction Algorithm



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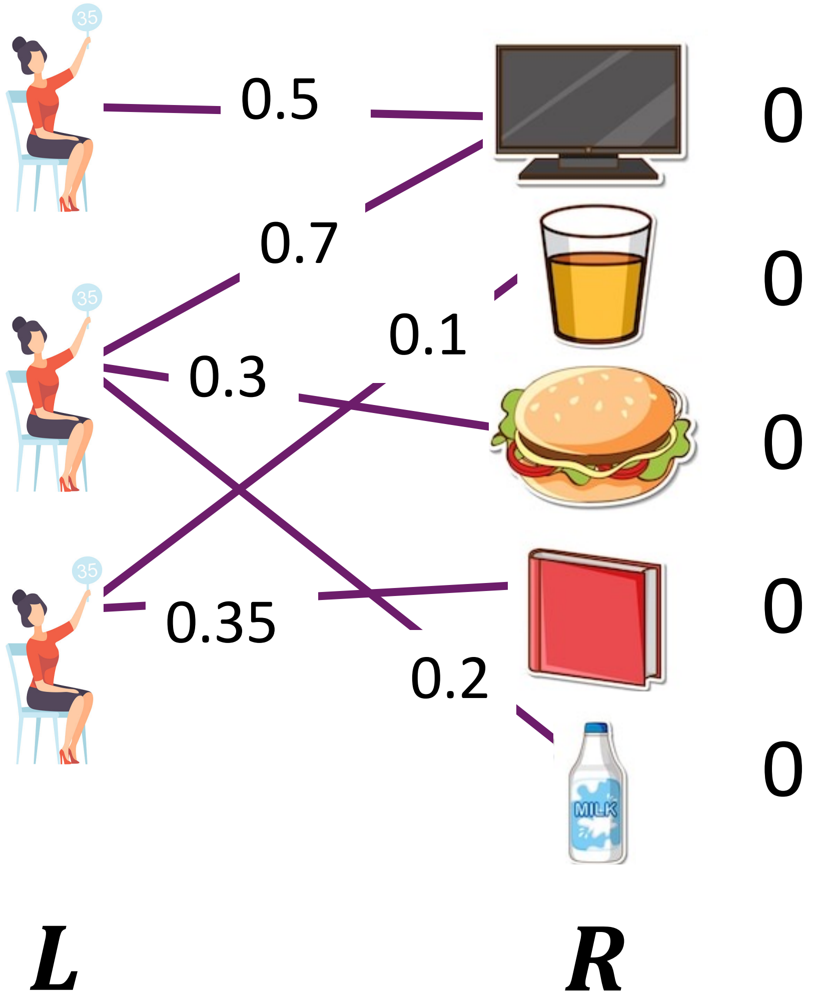
Iteratively, bidders **bid** on all **highest (value – price)** **items**



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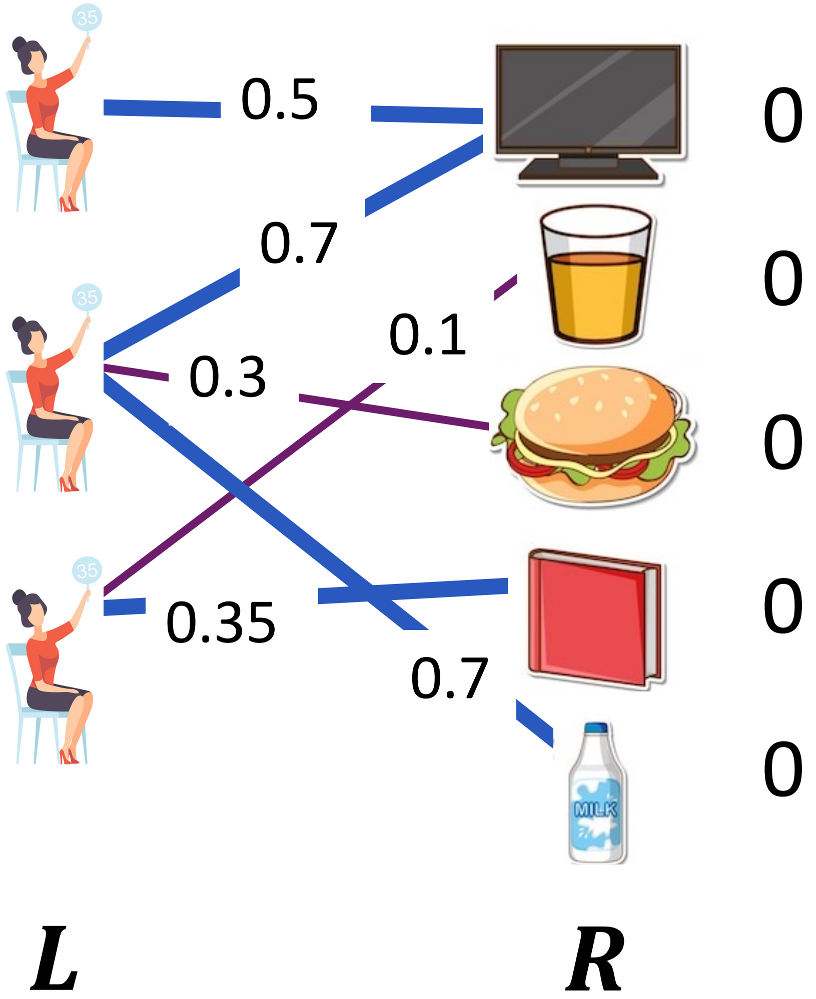

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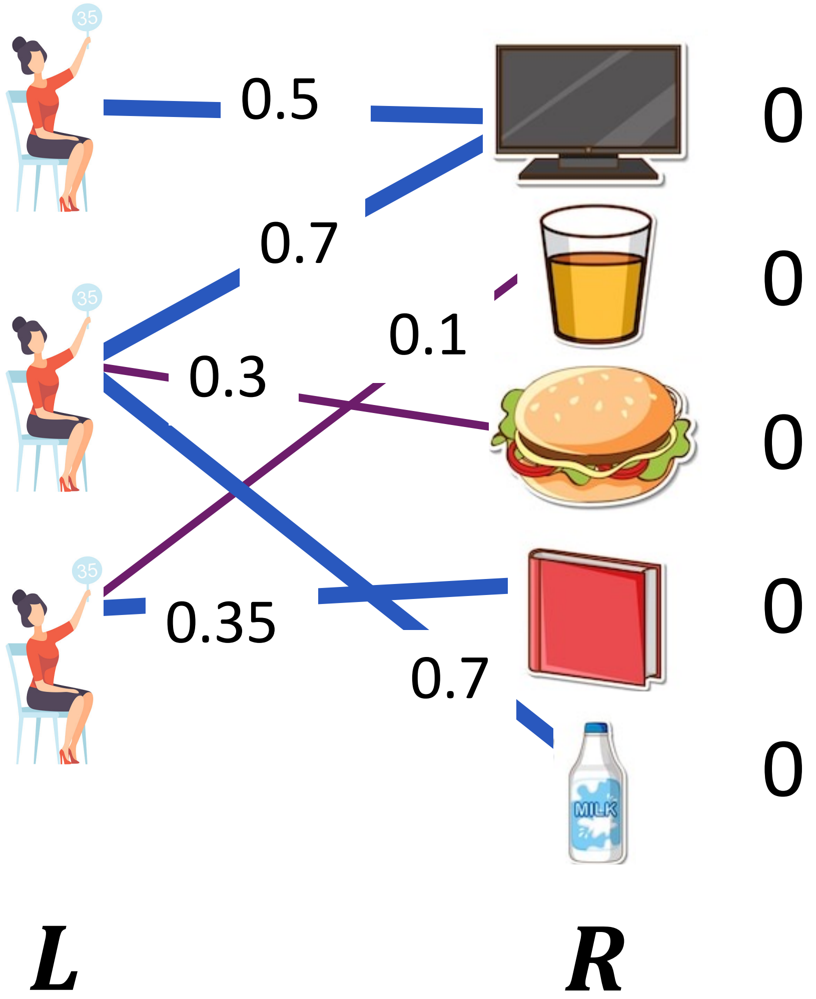
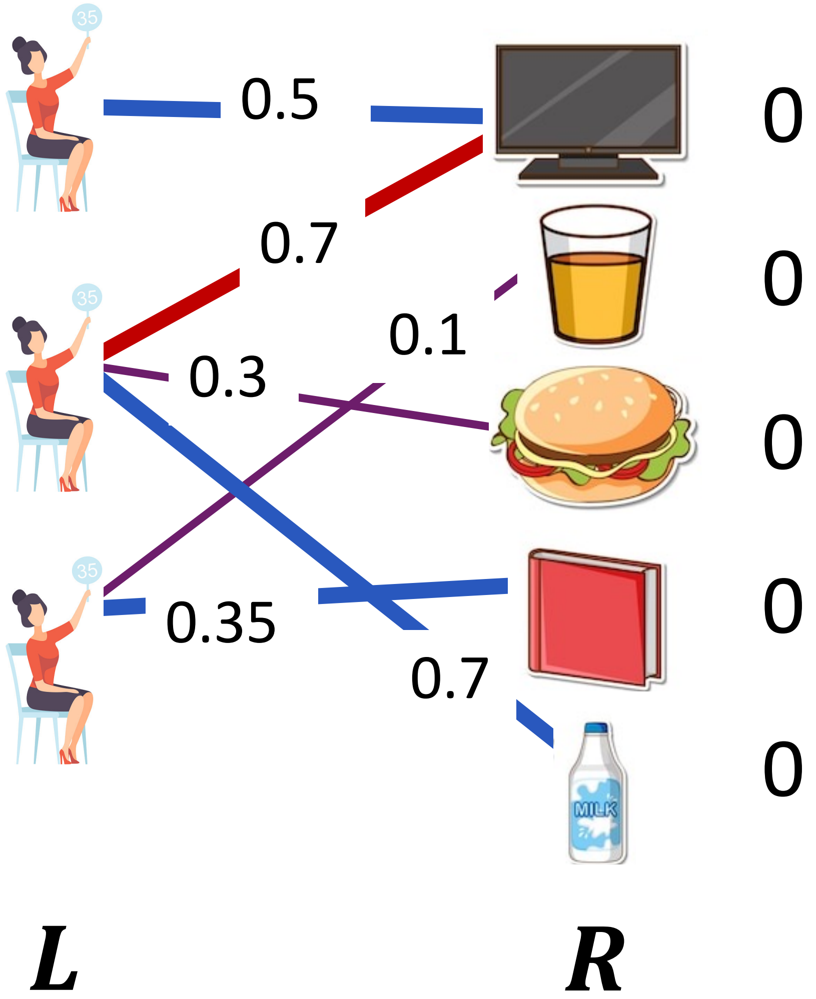
Reason: items **under contention** should be **won by edges with larger weights**



Find **maximal matching** among **induced subgraph of bid items from highest bucket down**

Our **Simplified** Maximum Weight Auction Algorithm

Iteratively, bidders **bid** on all **highest (value – price)** **items**



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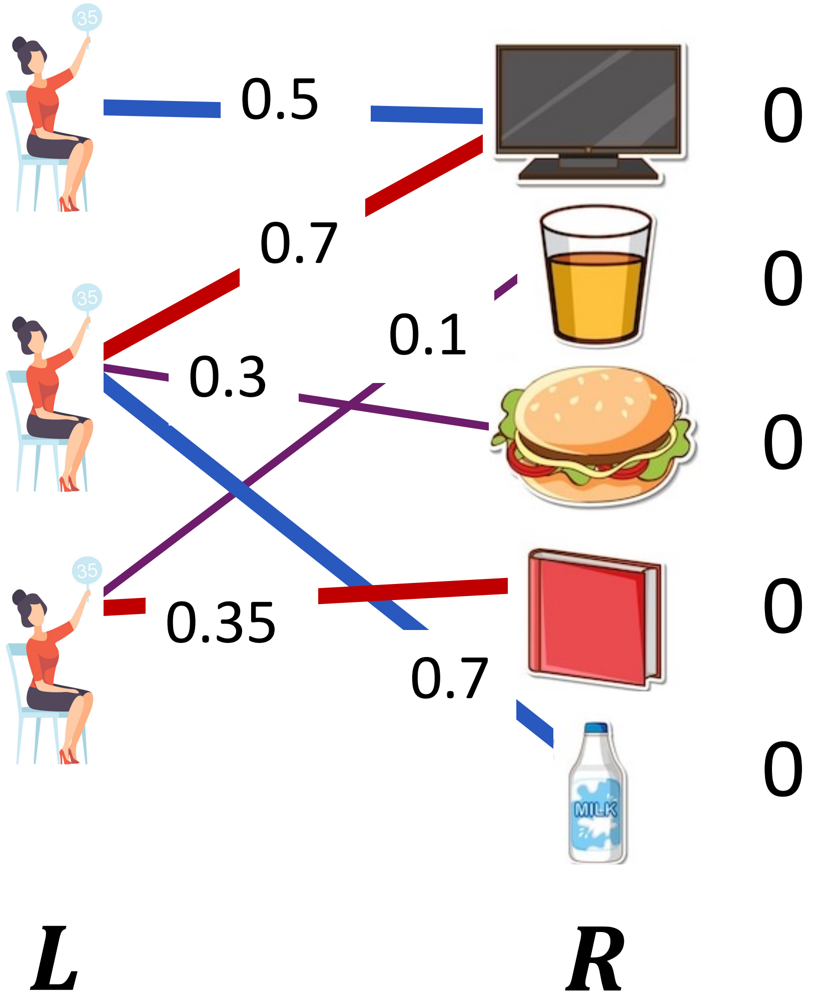

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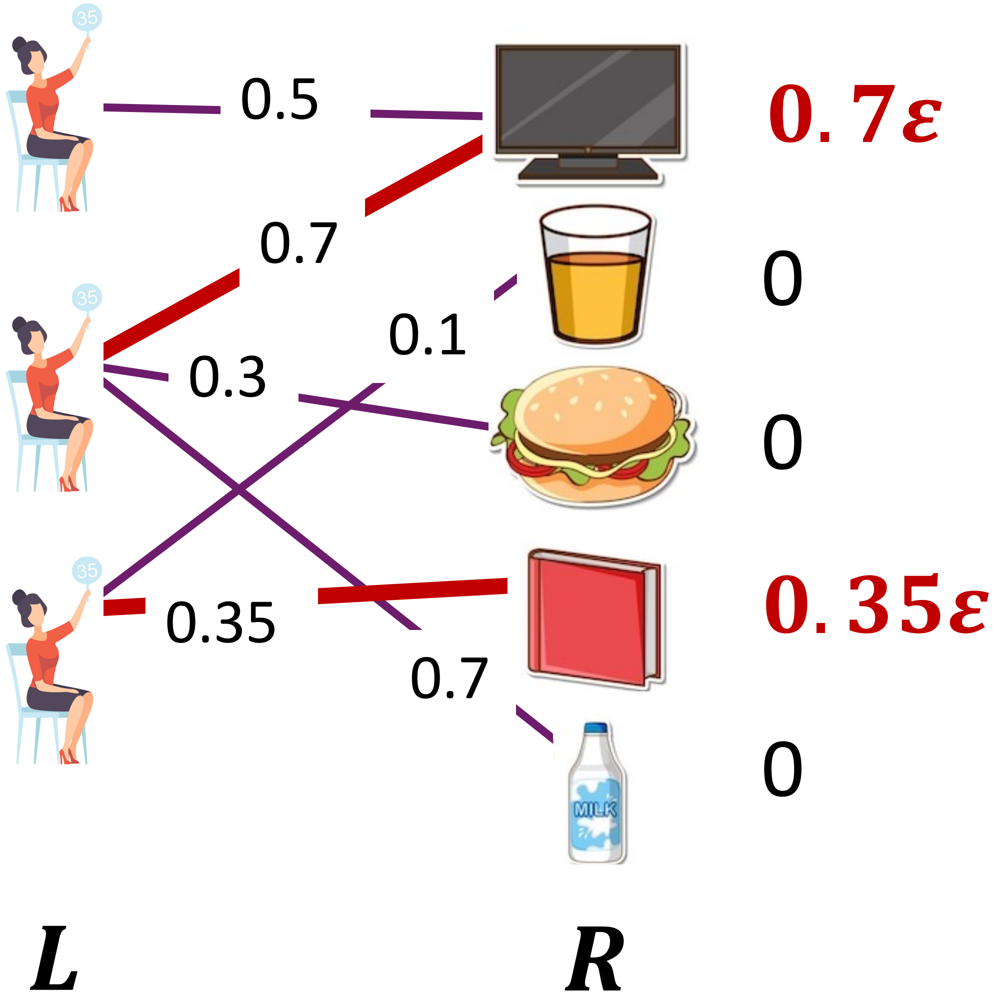
Our **Simplified** Maximum Weight Auction Algorithm

Increase price of items in matching **by $\varepsilon \cdot w$** and **maintain current matching**



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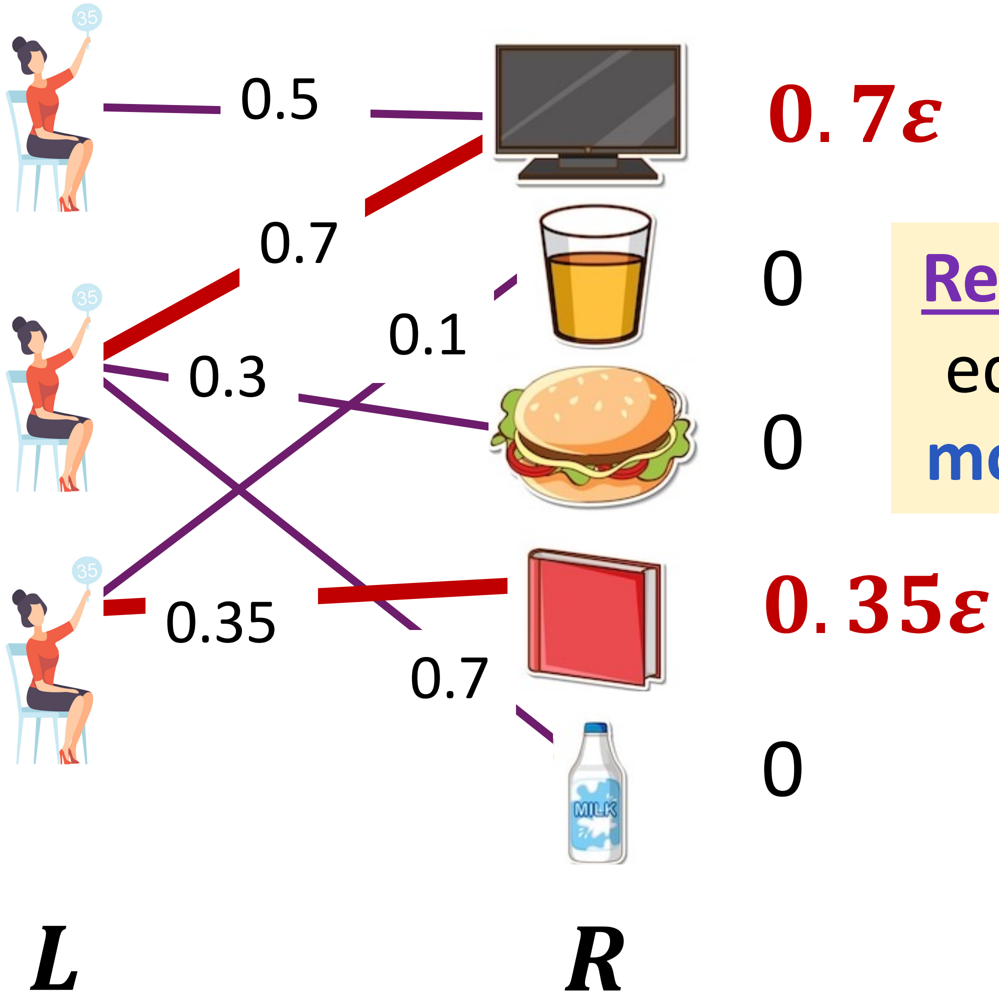

Reason: higher weight edges will contribute **more to the matching**

Our **Simplified** Maximum Weight Auction Algorithm

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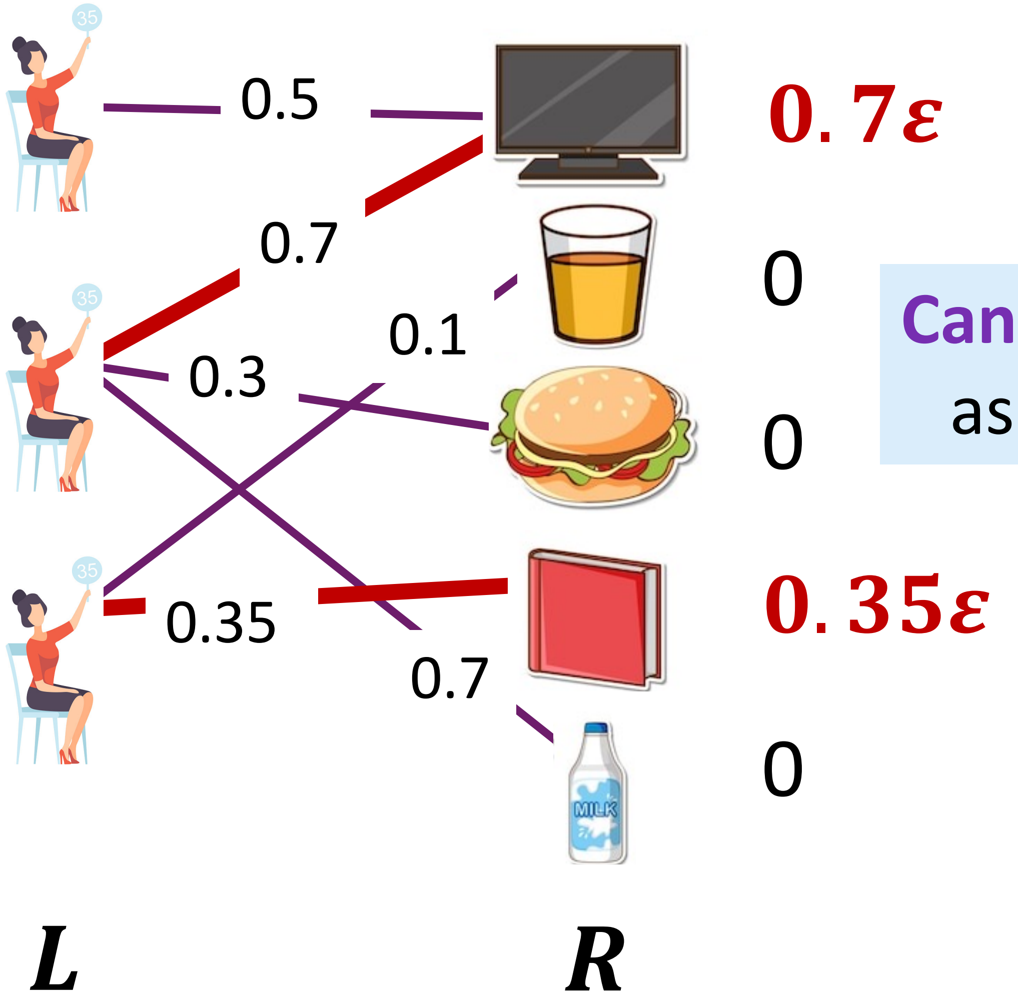
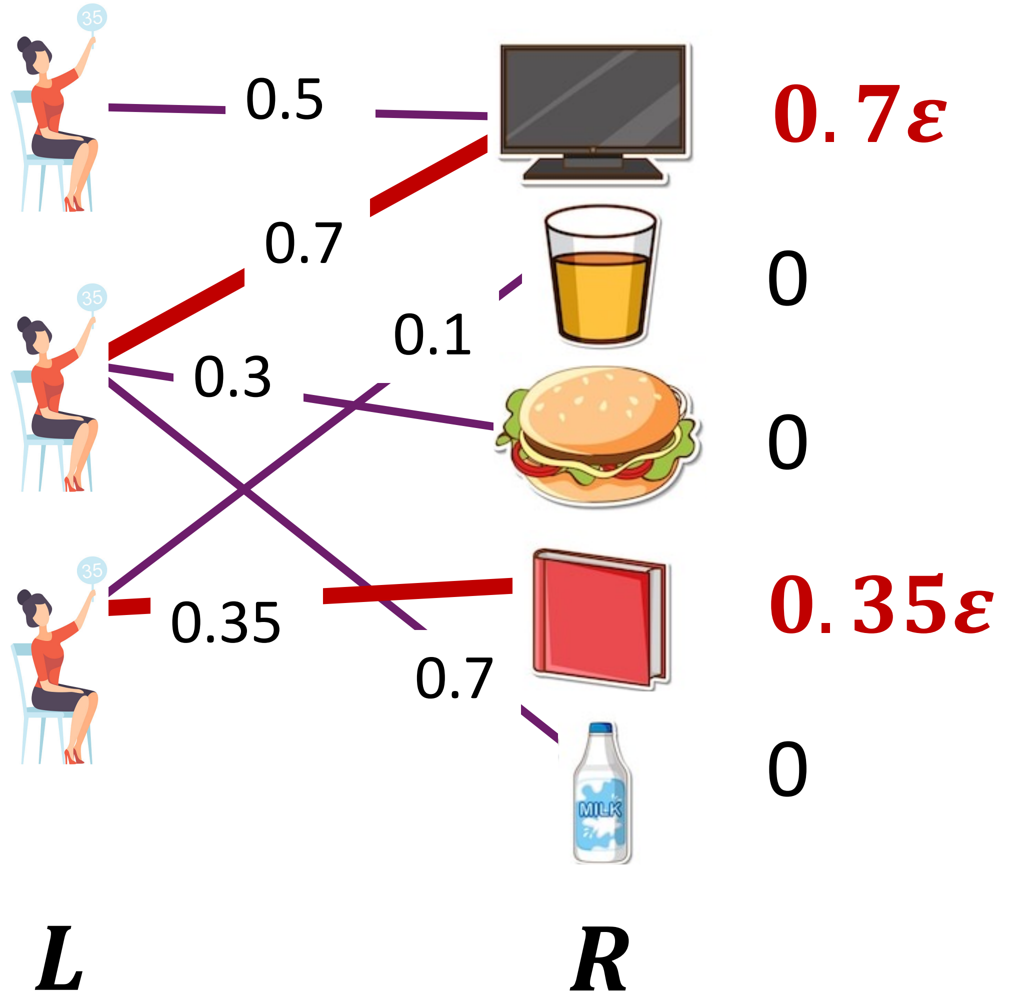


Can bid on item as long as **weight - price > 0**

Our **Simplified** Maximum Weight Auction Algorithm

Iterate for $\left\lceil \frac{\log^2(n)}{\varepsilon^4} \right\rceil$ **iterations**



Our **Simplified** Maximum Weight Auction Algorithm

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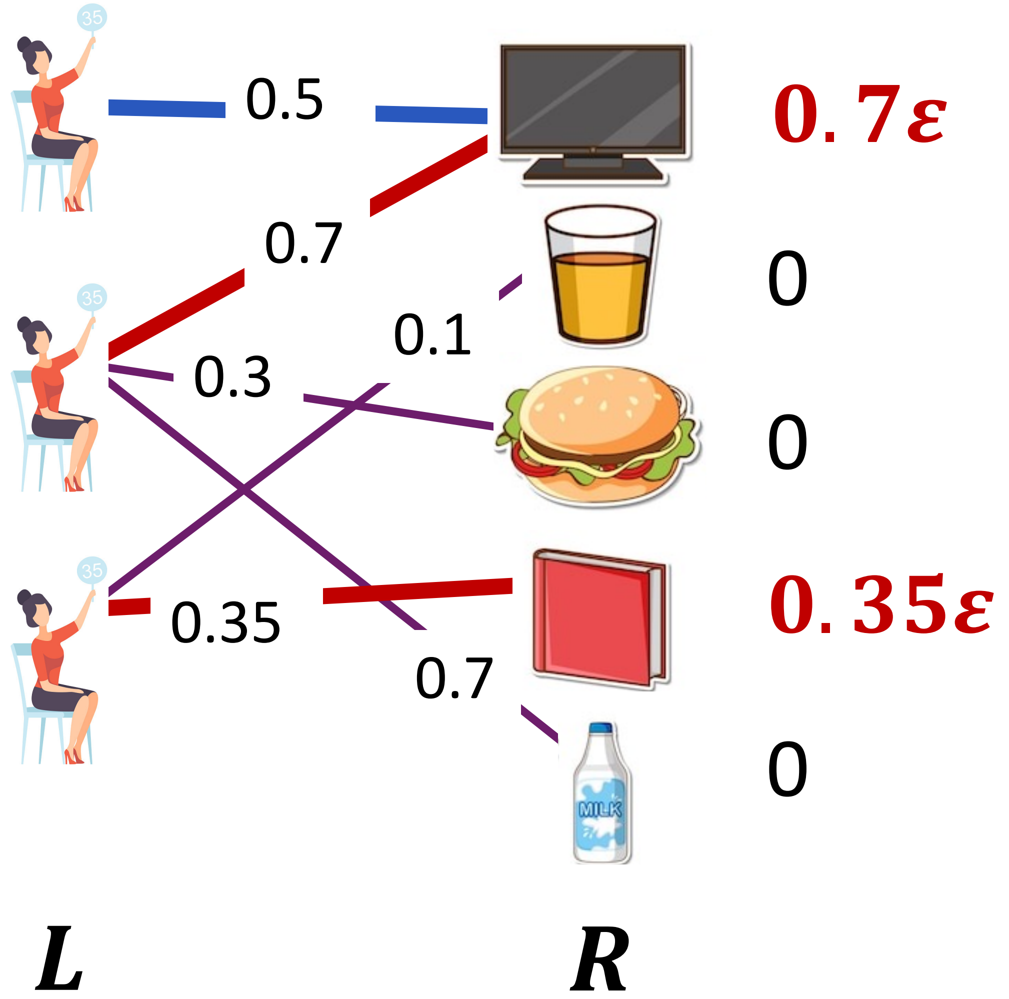
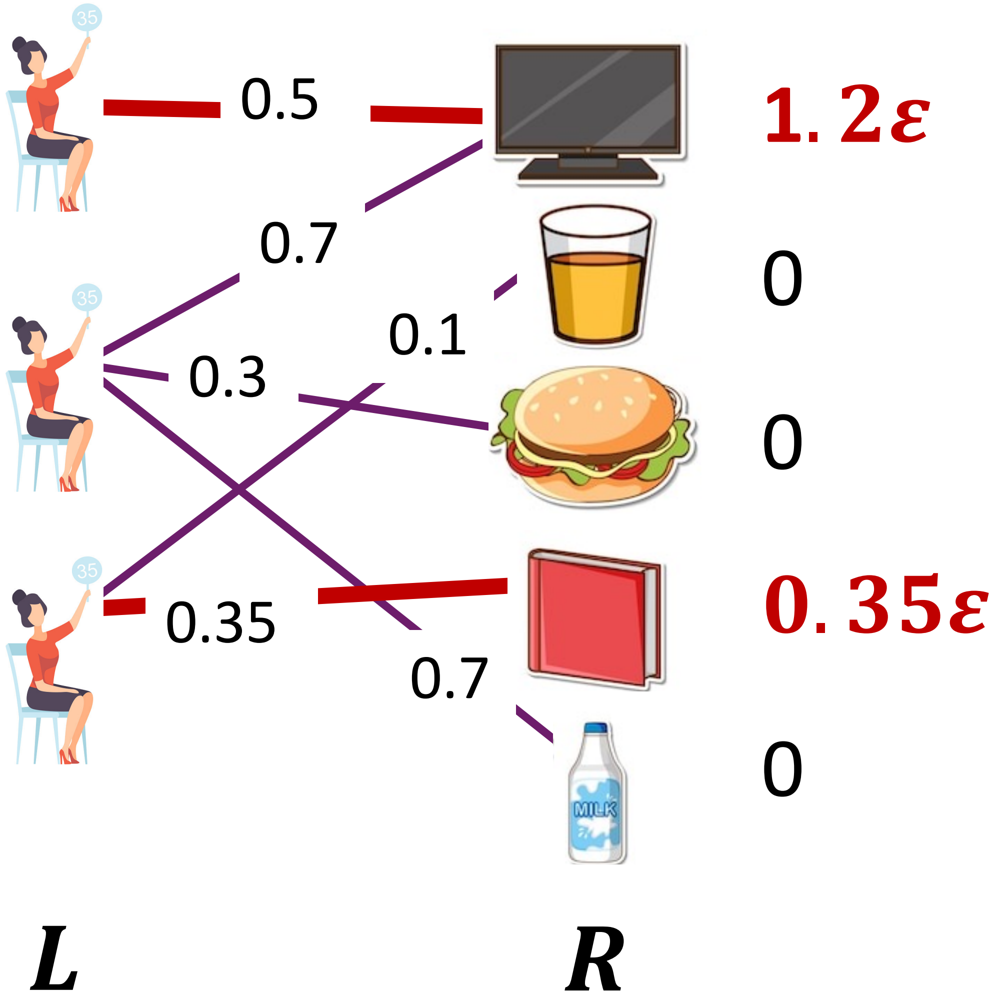
Each **unmatched** bidder bids



Our **Simplified** Maximum Weight Auction Algorithm

Iterate for $\left\lceil \frac{\log^2(n)}{\varepsilon^4} \right\rceil$ **iterations**

Item goes to new bidder!



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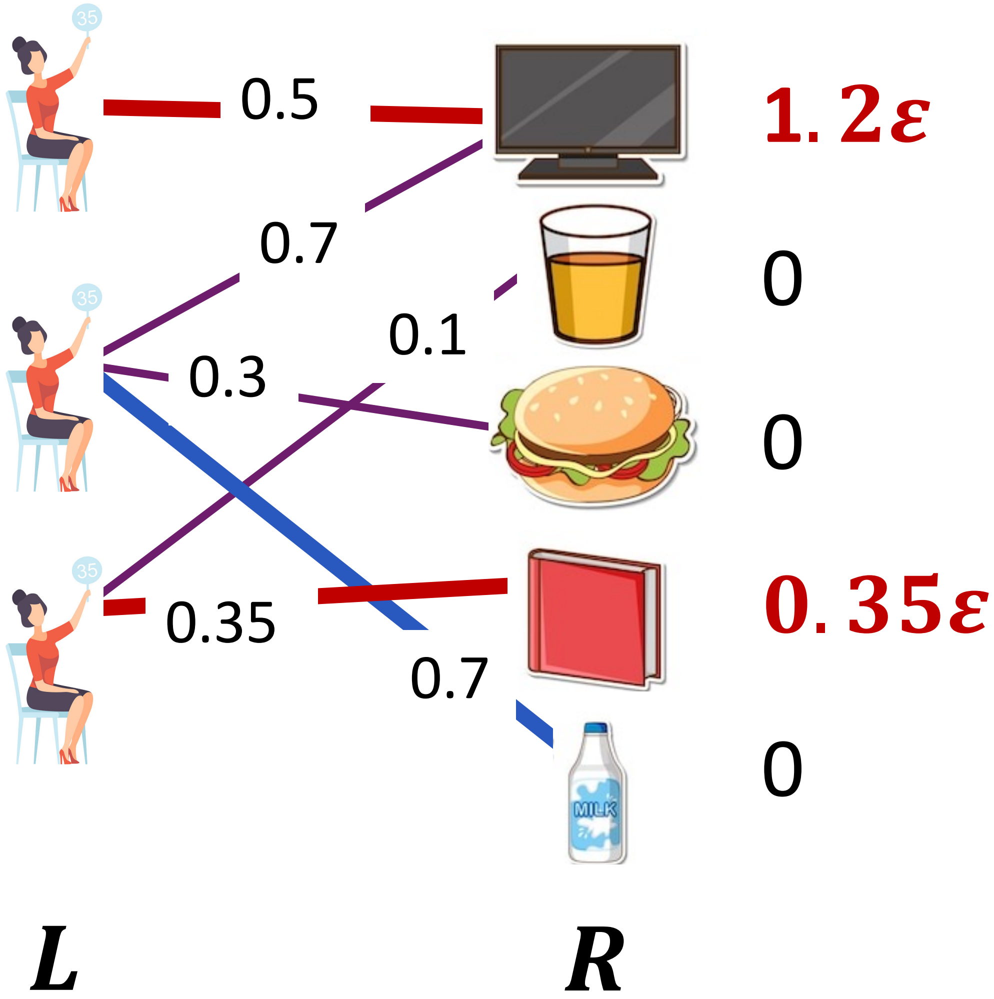

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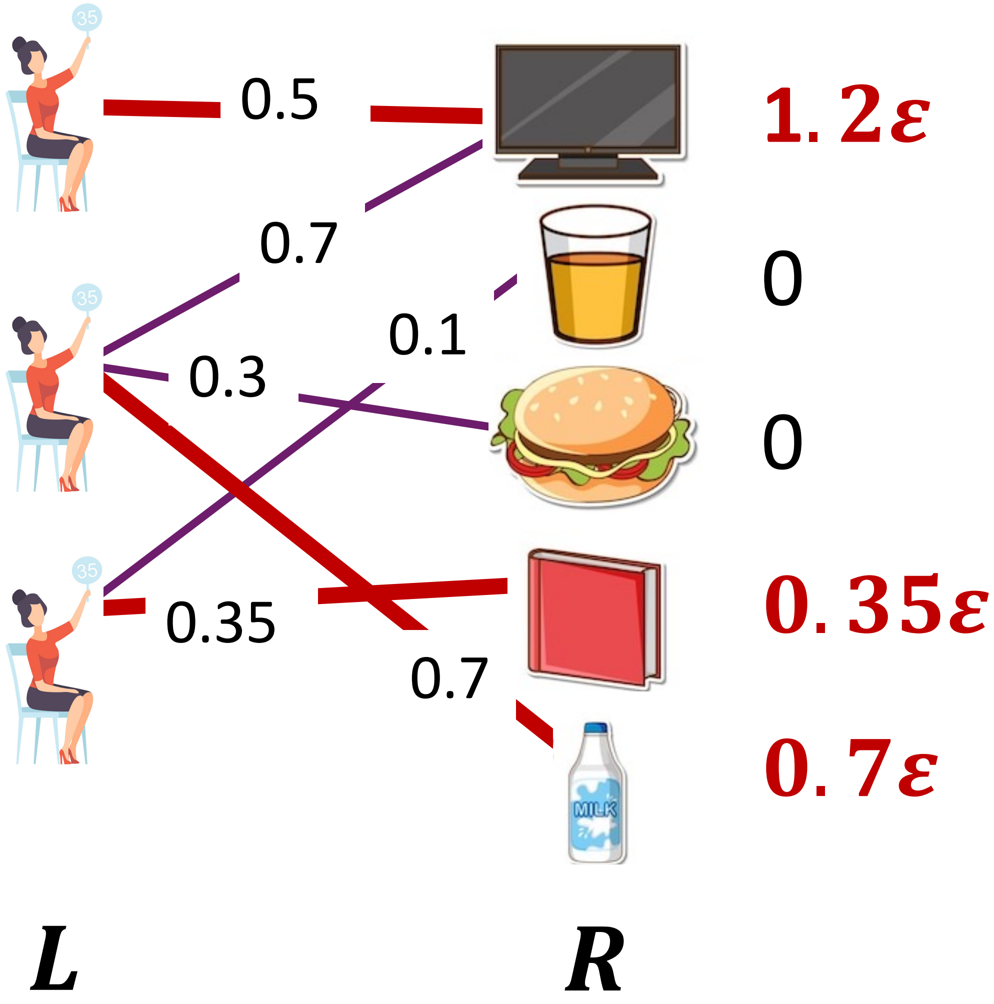
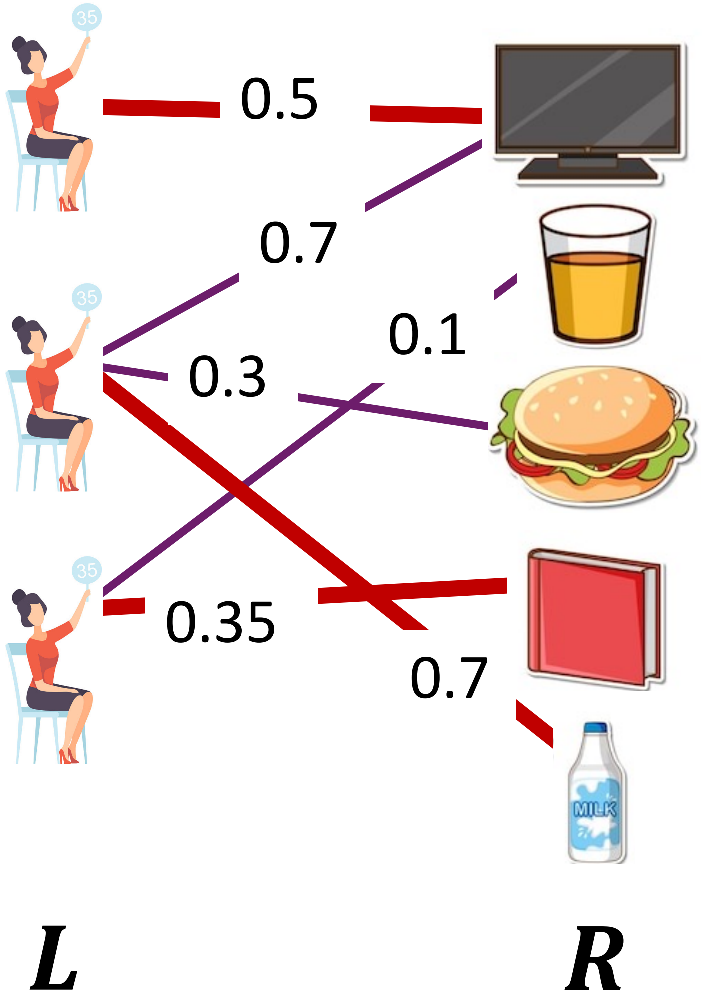


Our **Simplified** Maximum Weight Auction Algorithm

Final Matching



Minimizing Dependence on $\log(W)$

- Modified Gupta-Peng '13 transformation
  - Partition edges into levels based on edge weight
  - Each level contains multiple buckets
  - Omit certain buckets to prevent too large ratio in weights
- **Ratio of weights in each level is bounded by $\varepsilon^{-O\left(\frac{1}{\varepsilon}\right)}$**

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Iterate for $\left\lceil \frac{\log^2(n)}{\varepsilon^4} \right\rceil$ **iterations** → Iterate for $\left\lceil \frac{\log(n)}{\varepsilon^7} \right\rceil$ **iterations**

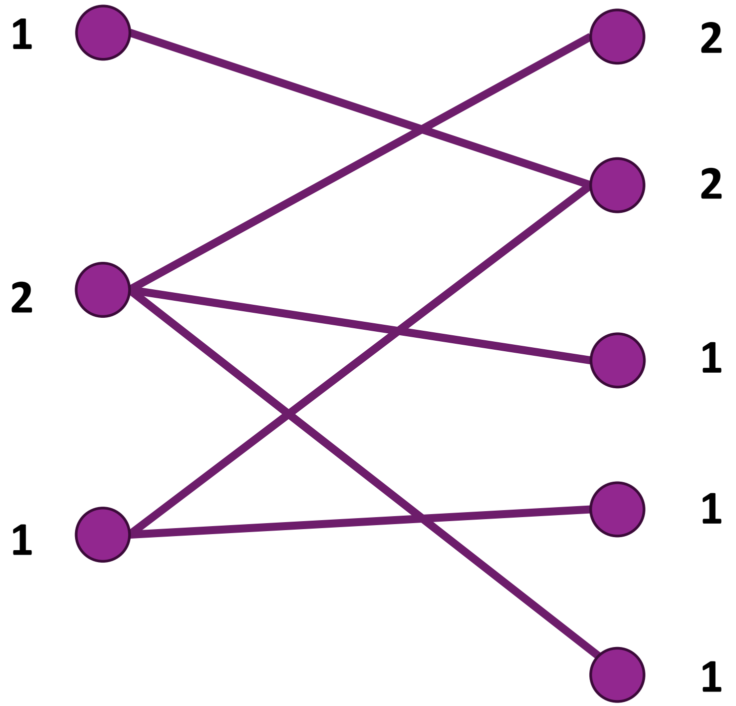
Outline

- Auction Algorithm of [ALT21] for maximum cardinality matching
- Our auction algorithm for maximum weighted matching
  - Algorithm description
  - Minimizing dependence on $\log(W)$
- Our auction algorithm for maximum *b*-matching

Very brief!

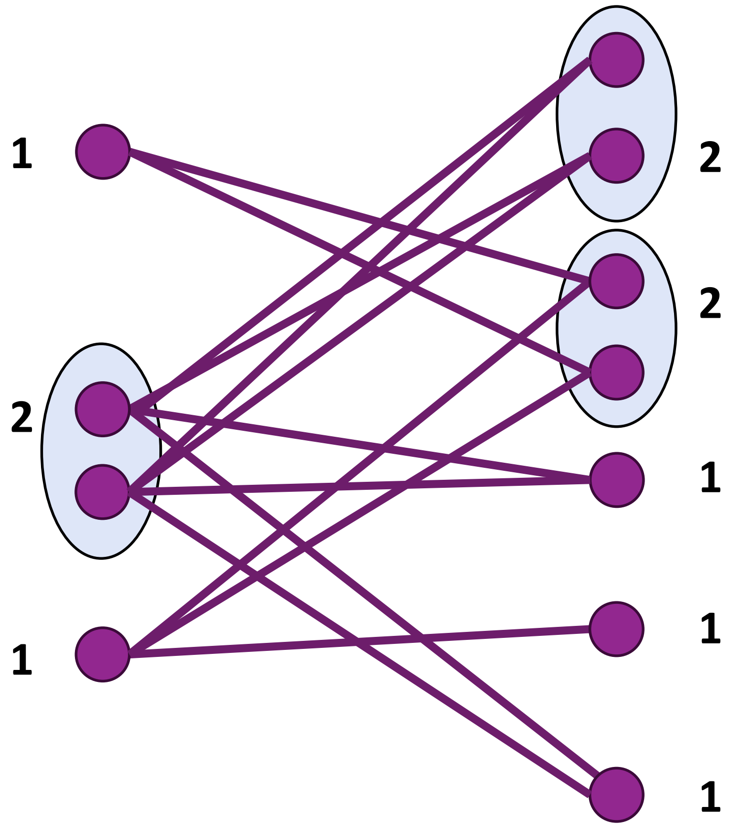
Very Simplified Maximum *b*-Matching Algorithm

Create a copy for each bidder and item equal to their *b* value



Very Simplified Maximum *b*-Matching Algorithm

Create a biclique between copies representing bidder and item



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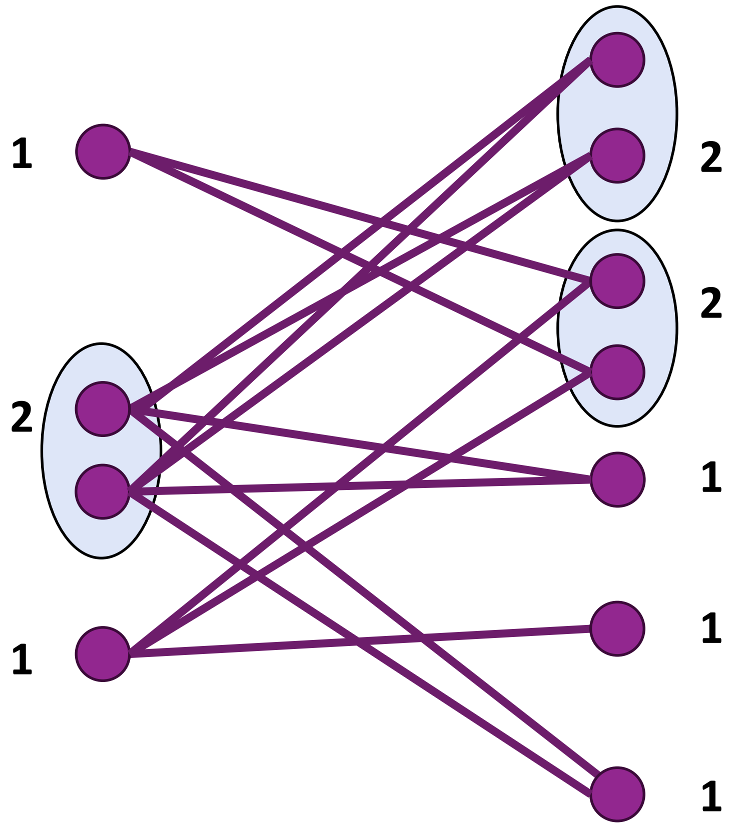

Make sure match only one copy!

Very Simplified Maximum *b*-Matching Algorithm

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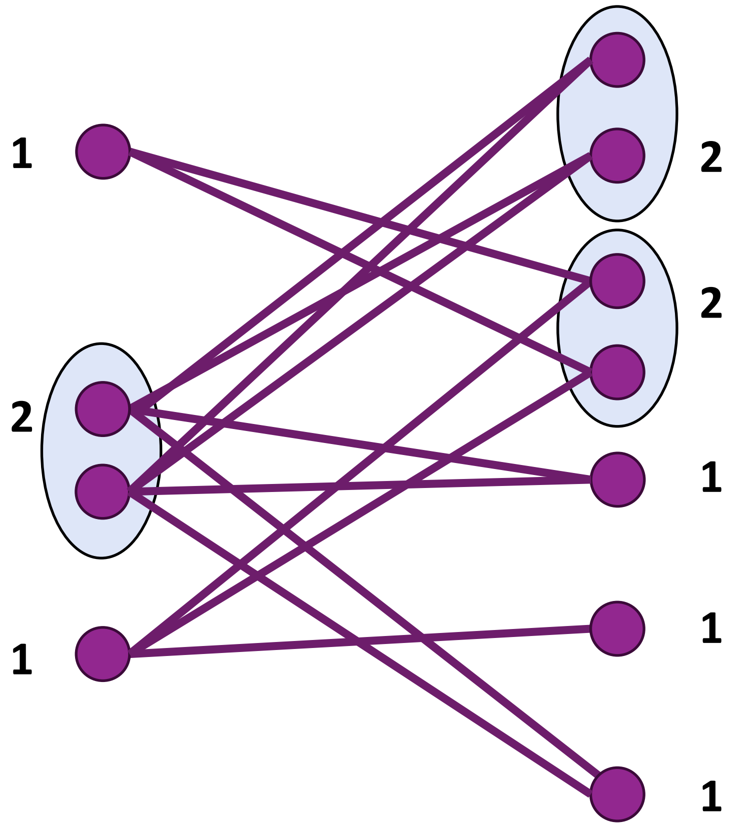

Make sure match only one copy!

Solution: each time price increases, increase the **lowest possible bidding price** for **each unmatched bidder**

Our Results

MWM = Maximum Weighted Matching
MCBM = Maximum Cardinality *b*-Matching

Model		Previous Results		Our Results	
Blackboard Distributed	MWM	$\Omega(n \log n)$ (trivial)	[DNO14]	$O\left(\frac{n\log^3(n)}{\varepsilon^8}\right)$	Theorem 3.9
	MCBM	$\Omega(nb \log n)$	trivial	$O\left(\frac{nb\log^2 n}{\varepsilon^2}\right)$	Theorem 4.8
Streaming	MWM	$O\left(\frac{\log(1/\varepsilon)}{\varepsilon^2}\right)$ pass $O\left(\frac{n\log n}{\varepsilon^2}\right)$ space	[AG11]	$O\left(\frac{1}{\varepsilon^8}\right)$ pass $O(n \cdot \log n \cdot \log(1/\varepsilon))$ space	Theorem 3.11
	MCBM	$O(\log n/\varepsilon^3)$ pass $\widetilde{O}\left(\frac{\sum_{i\in L\cup R} b_i}{\varepsilon^3}\right)$ space	[AG18]	$O\left(\frac{1}{\varepsilon^2}\right)$ pass $O\left(\left(\sum_{i\in L} b_i + \lvert R\rvert\right)\log(1/\varepsilon)\right)$ space	Theorem 4.10
MPC	MWM	$O_\varepsilon(\log\log n)$ rounds $O_\varepsilon(n \operatorname{poly}(\log n))$ space p.m.	[GKMS19] (general)	$O\left(\frac{\log\log n}{\varepsilon^7}\right)$ rounds $O(n \cdot \log_{(1/\varepsilon)}(n))$ space p.m.	Theorem 3.15
Parallel	MWM	$O(m \cdot \operatorname{poly}(1/\varepsilon, \log n))$ work* $O(\operatorname{poly}(1/\varepsilon, \log n))$ depth*	[HS22] (general)	$O\left(\frac{m\log(n)}{\varepsilon^7}\right)$ work $O\left(\frac{\log^3 n}{\varepsilon^7}\right)$ depth	Theorem 3.13
	MCBM	N/A	N/A	$O\left(\frac{m\log n}{\varepsilon^2}\right)$ work $O\left(\frac{\log^3 n}{\varepsilon^2}\right)$ depth	Theorem 4.11

"Universal" solution across many different scalable models!