Scalable and Efficient Graph Algorithms and Analysis Techniques for Modern Machines

Quanquan C. Liu
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Thesis Defense

August 31, 2021

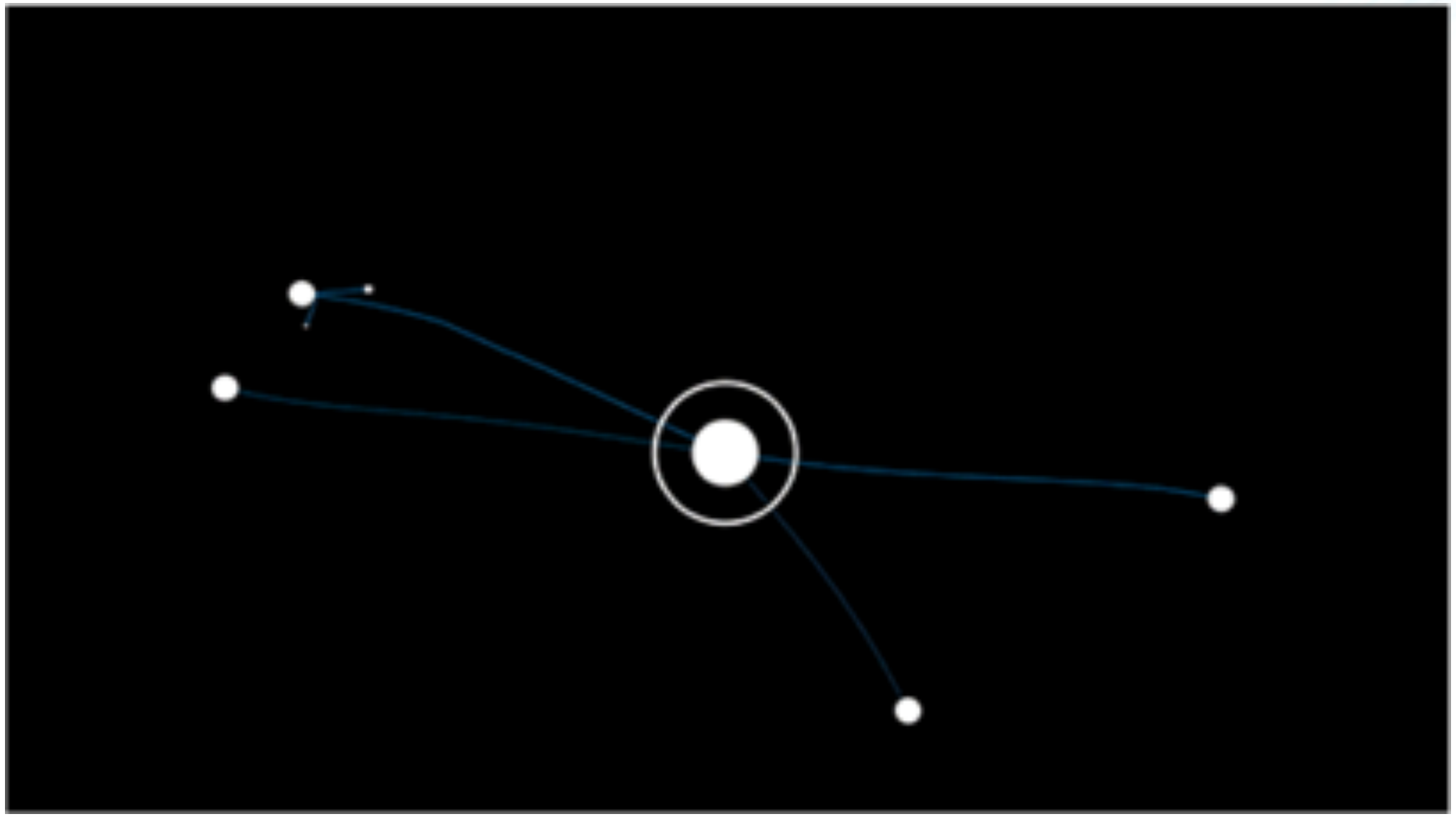
Large, Dynamic Networks

facebook ~ 92.5 million edges

friendster ~ 1.8 billion edges

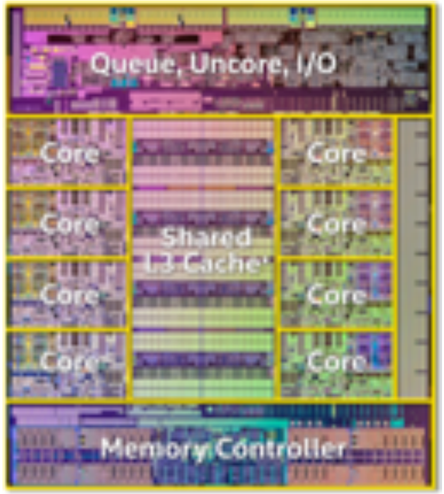
~ 2 billion edges

Common Crawl ~ 128 billion edges



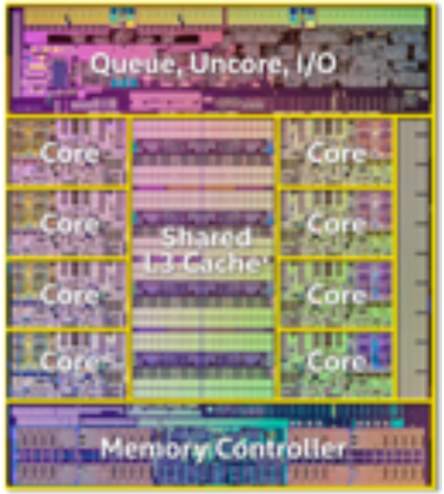
Modern Machines

- **Multicore systems**: shared-memory (virtual) parallel machines since the early 2000s
  - Multiprocessing systems and multi-core processors



Modern Machines

- **Multicore systems**: shared-memory (virtual) parallel machines since the early 2000s
  - Multiprocessing systems and multi-core processors
- **Massively parallel systems**
  - Distributed cluster of multiple machines



Modern Machines

E2 standard machine types

Iowa (us-central1) | Monthly | Hourly

Machine type	Virtual CPUs	Memory	Price (USD)	Preemptible price (USD)
e2-standard-2	2	8GB	$0.067006	$0.020102
e2-standard-4	4	16GB	$0.134012	$0.040204
e2-standard-8	8	32GB	$0.268024	$0.080408
e2-standard-16	16	64GB	$0.536048	$0.160816
e2-standard-32	32	128GB	$1.072096	$0.32163
Custom machine type	If your ideal machine shape is in between two predefined types, using a custom E2 machine type could save you as much as 40%. For more information, see E2 custom vCPUs and memory.			

Google Cloud

aws

Instance name	On-Demand hourly rate	vCPU	Memory	Storage	[illegible]
m6g.8xlarge	$1.232	32	128 GiB	EBS Only	[illegible]
m6gd.8xlarge	$1.4464	32	128 GiB	1 x 1900 NVMe SSD	10 Gigabit
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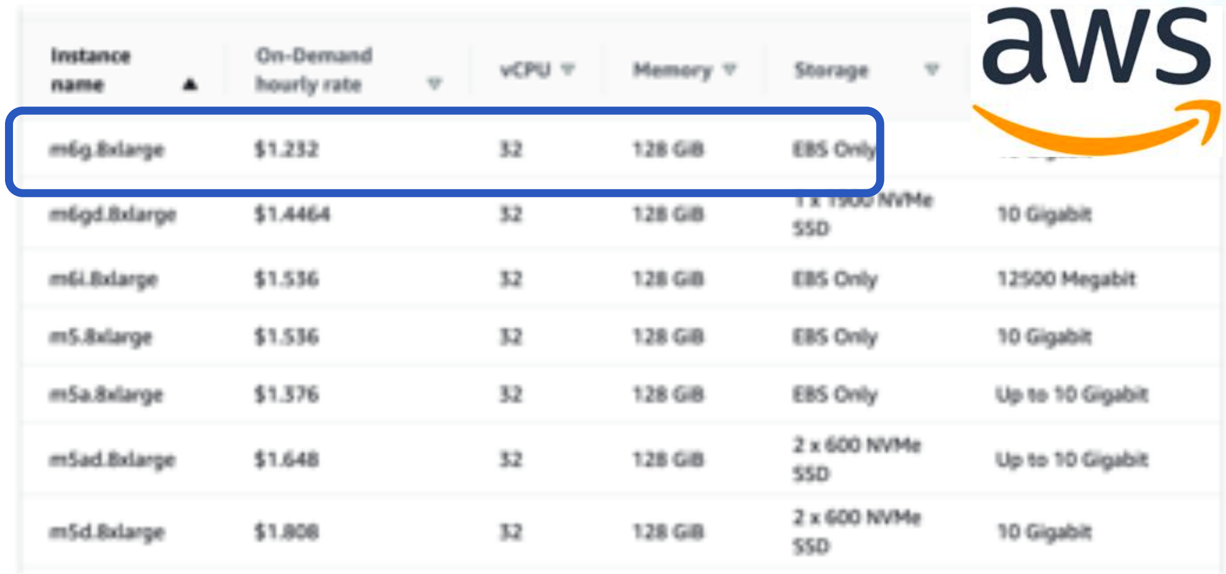

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Some companies using these services: *Netflix, Twitch, LinkedIn, Facebook, BBC, Baidu, Adobe, Twitter, Coinbase, Comcast, Coursera, Disney, Expedia, Harvard Medical School, International Centre for Radio Astronomy Research, Novartis, Pfizer, Reddit, Sage, Samsung, US Department of State, USDA Food and Nutrition Service, Verizon, Lyft, State of Arizona, Topcoder, Autodesk, Georgetown University, many others*

Modern Machines

1 million companies each!

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Commercial Data Centers

Google Kubernetes Engine

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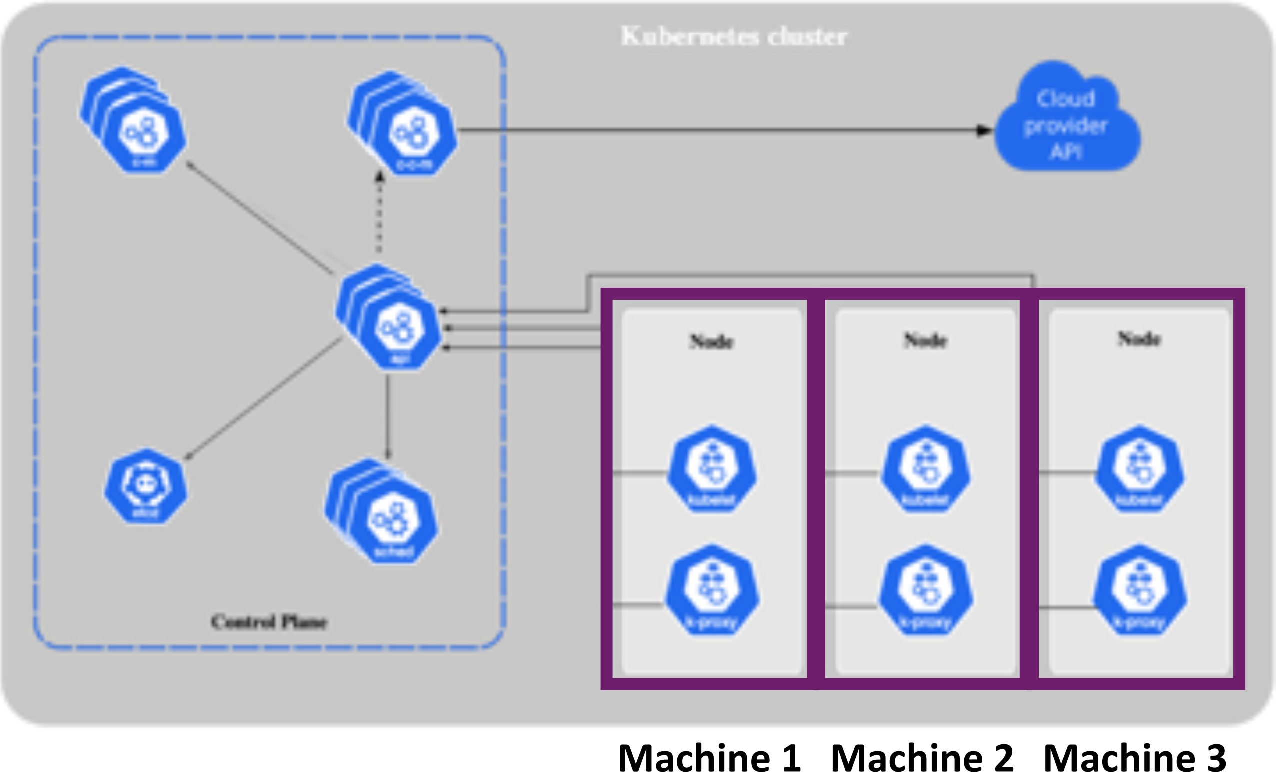
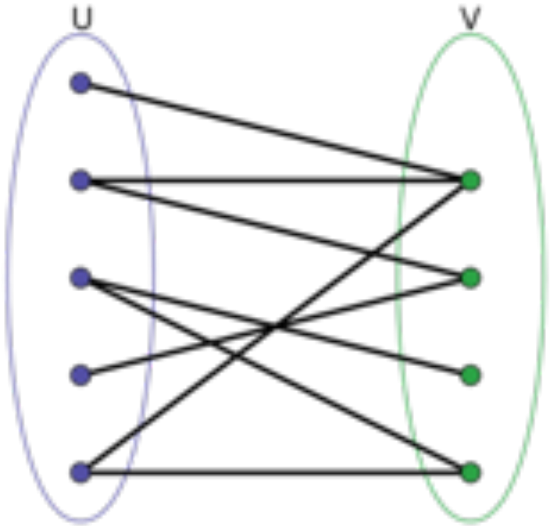
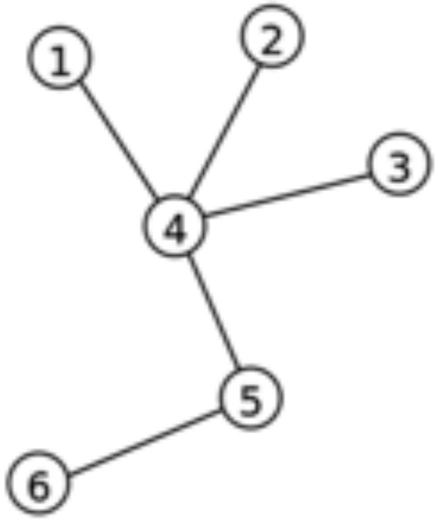


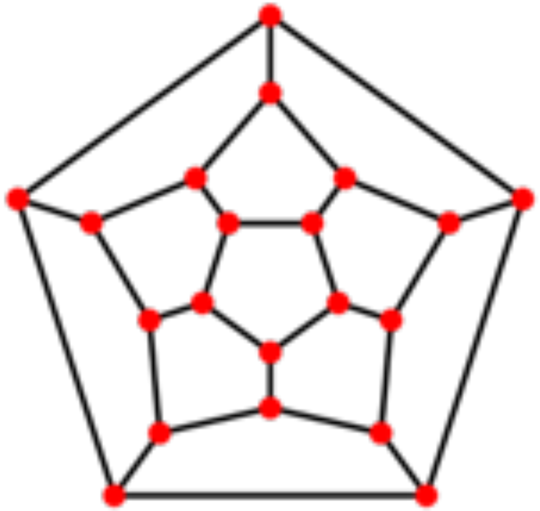
Fast Algorithms on Classes of Graphs



Bipartite Graphs
minimum vertex cover easy, NP-hard in general

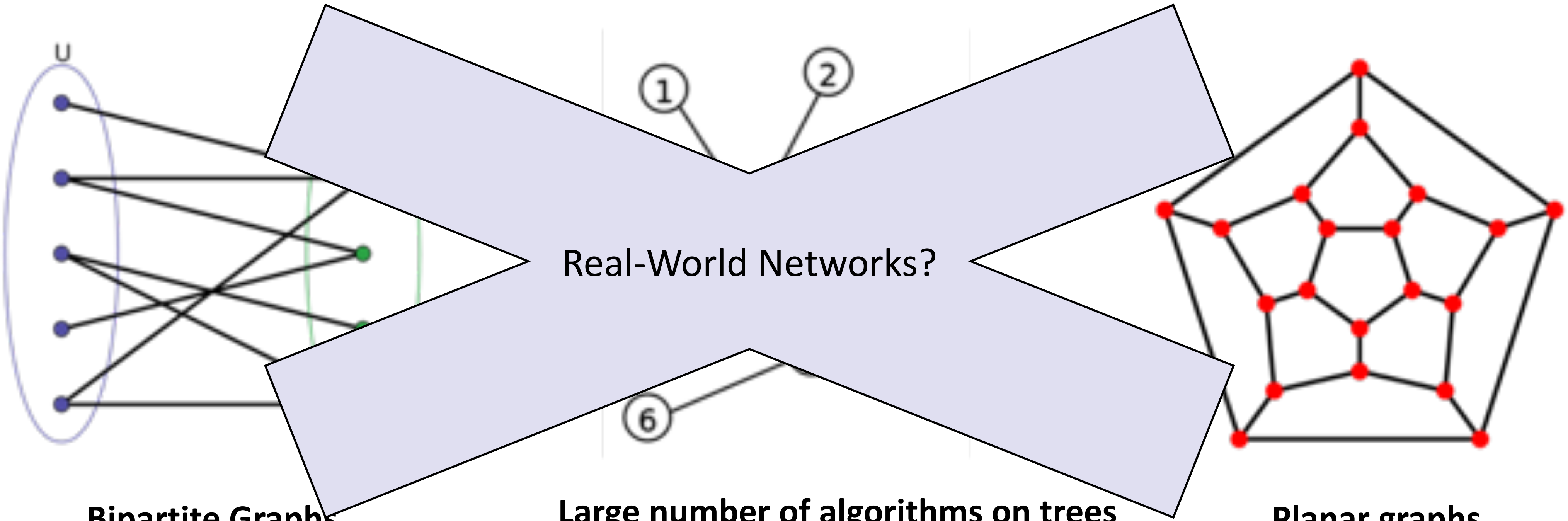


Large number of algorithms on trees
maximum independent set, subgraph isomorphism, dominating set, coloring…etc. all easy



Planar graphs
Single-source shortest path can be linear

Fast Algorithms on Classes of Graphs



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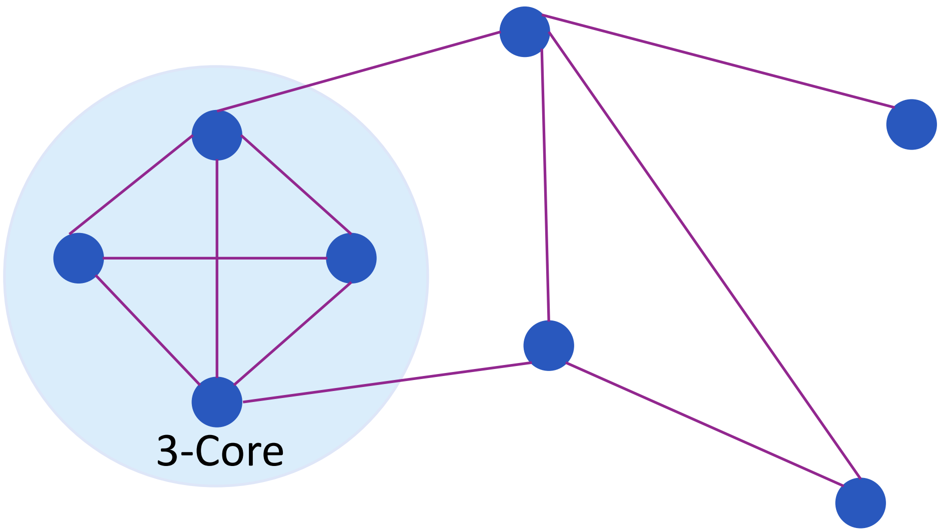
Degeneracy

- A ***k*-core** is a maximal subgraph where each vertex has degree at least *k*
- The **degeneracy** of a graph is equal to the maximum *k* for which a k-core exists in a graph



Properties of Real-World Networks

- Degeneracy: a measure of a graph's sparsity

Graph	Num. Vertices	Num. Edges	Degeneracy (k)
dblp	425,957	2,099,732	101
brain-network	784,262	267,844,669	1200
wikipedia	1,140,149	2,787,967	124
youtube	1,138,499	5,980,886	51
stackoverflow	2,601,977	28,183,518	163
livejournal	4,847,571	85,702,474	329
orkut	3,072,627	234,370,166	253
usa-road	23,072,627	28,854,312	3
twitter	41,652,231	1,202,513,046	2484
friendster	65,608,366	1,806,067,135	304

Properties of Real-World Networks

Factor of $\boxed{O(k)}$ vs. $O(n)$ or $O(m)$

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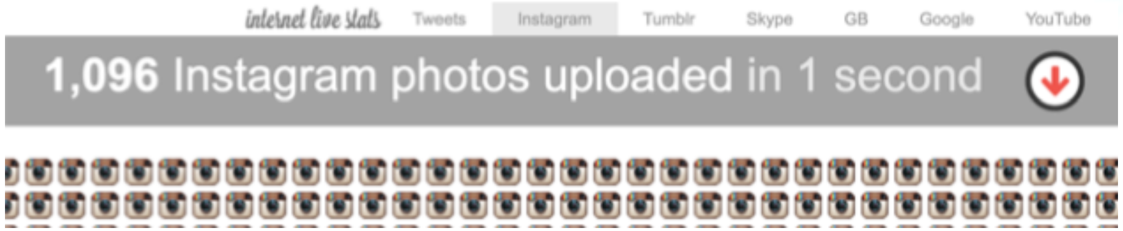
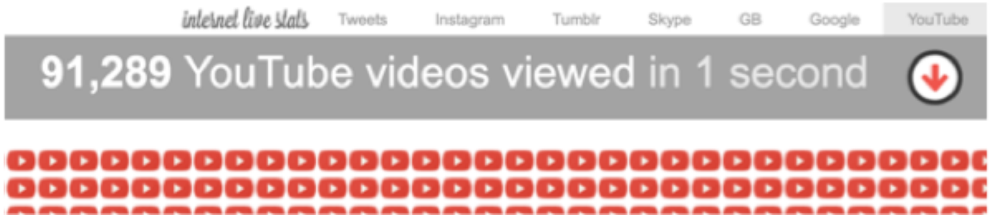
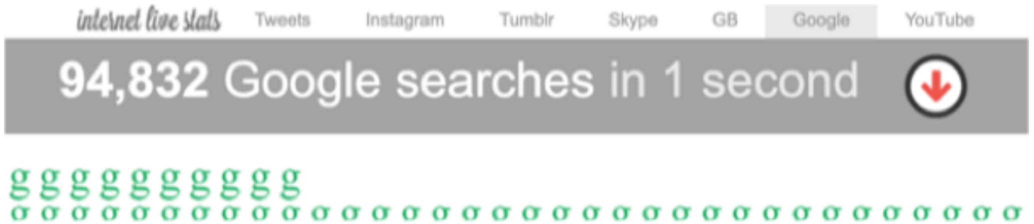
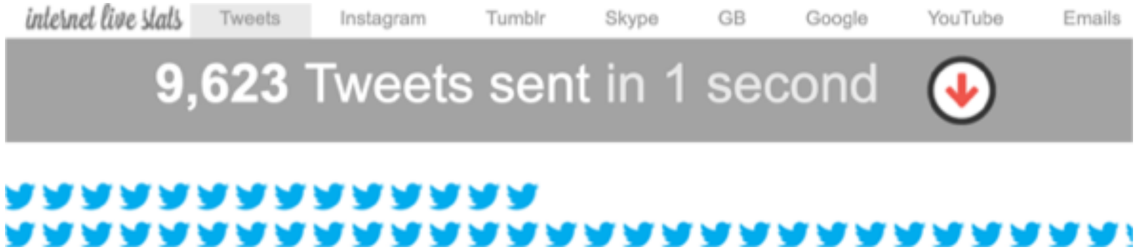
Dynamic Graphs



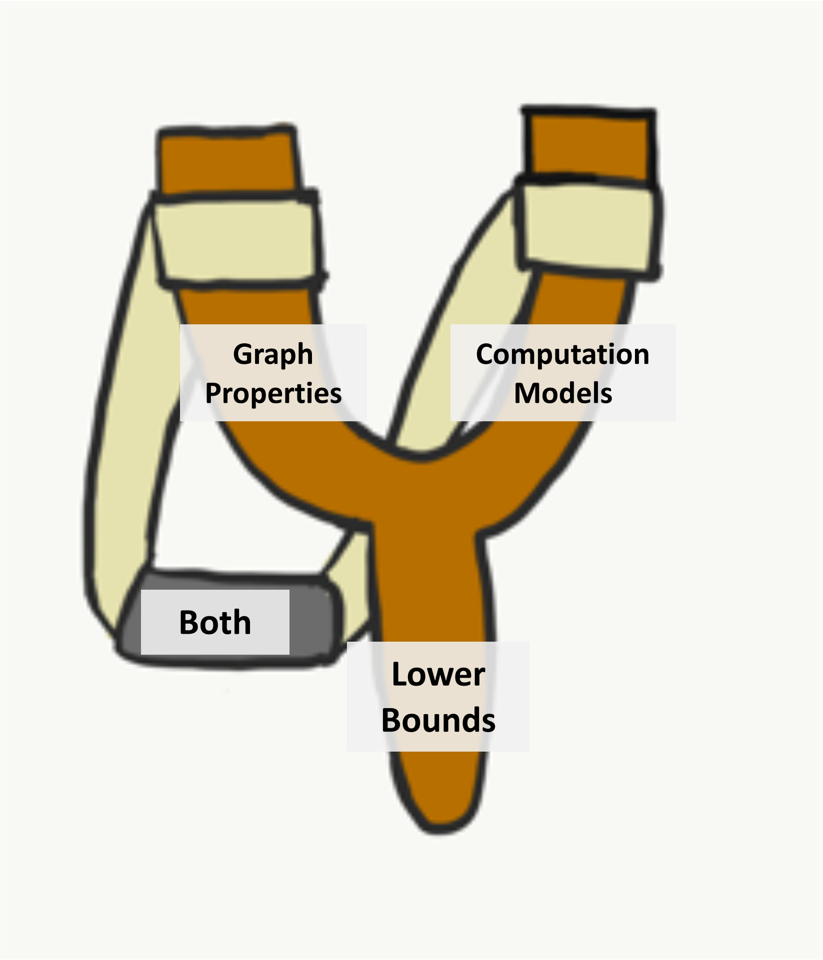






Graph Algorithms for Modern Machines

- Two-pronged approach:
  - Computation models that represent modern computing environments
  - Graph properties exhibited by real-world networks
- Experiments for new algorithms
- Lower bounds



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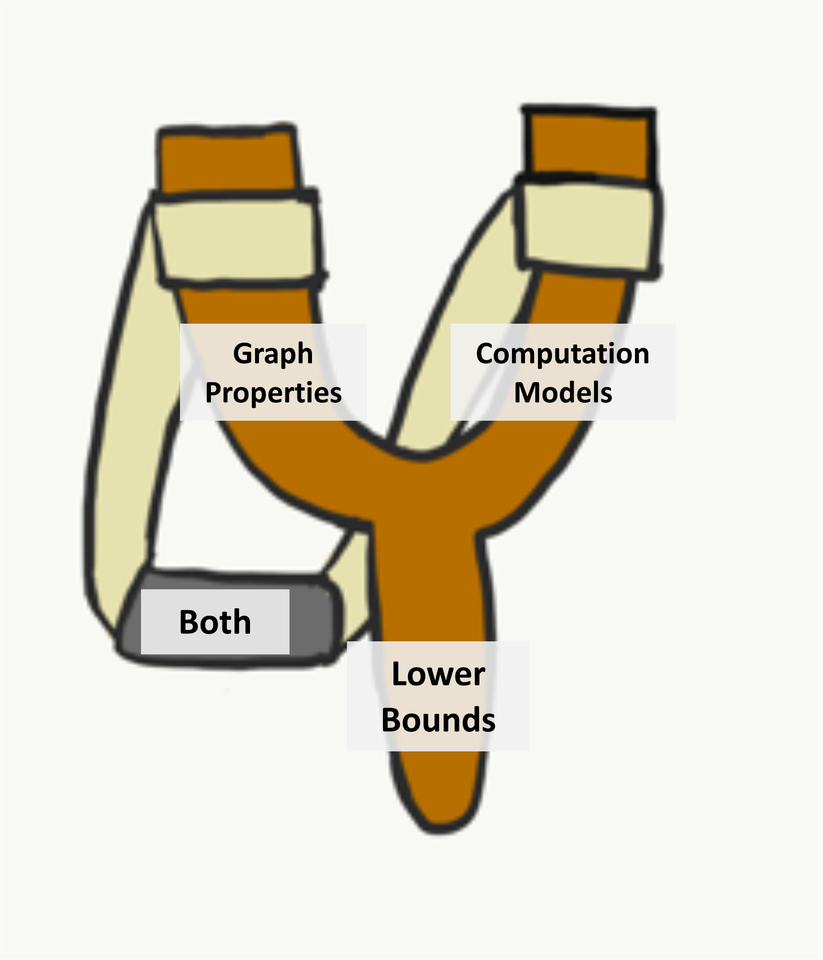

Summary of Results

Static Algorithms

Structural Rounding

Structural Rounding: Approximation Algorithms for Graphs Near an Algorithmically Tractable Class

Erik D. Demaine[1], Timothy D. Goodrich[2], Kyle Kloster[2], Brian Lavallee[2], Quanquan C. Liu[1], Blair D. Sullivan[2], Ali Vakilian[1], and Andrew van der Poel[2]

Scheduling with Communication Delay

Scheduling with Communication Delay in Near-Linear Time

Quanquan C. Liu, Manish Purohit, Zoya Svitkina, Erik Vee, Joshua R. Wang

MPC Algorithms for Subgraph Counting

Parallel Algorithms for Small Subgraph Counting

Amartya Shankha Biswas[*†] Talya Eden[*‡]
Quanquan C. Liu[*] Slobodan Mitrović[*§] Ronitt Rubinfeld[¶]

Dynamic Algorithms

Parallel Dynamic k-Core Decomposition

Parallel Batch-Dynamic k-Core Decomposition

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Shangdi Yu, MIT CSAIL, shangdiy@mit.edu
Laxman Dhulipala, MIT CSAIL, laxman@mit.edu
Julian Shun, MIT CSAIL, jshun@mit.edu

Dynamic Vertex Coloring

Fully Dynamic $(\Delta+1)$-Coloring in Constant Update Time

Sayan Bhattacharya[*] Fabrizio Grandoni[†] Janardhan Kulkarni[‡] Quanquan C. Liu[§]
Shay Solomon[¶]

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Lower Bounds/Constructions

Hardness from Pebbling

Red-Blue Pebble Game: Complexity of Computing the Trade-Off between Cache Size and Memory Transfers

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$\boxed{\text{Parallel}}$ Batch-Dynamic *k*-Core Decomposition

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Work-Depth Model

- **Work:**
  - Total time of performing all operations executed by algorithm

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Work-Depth Model

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  - Total time of performing all operations executed by algorithm
  - ***Work-efficient***: work asymptotically the same as *best-known sequential algorithm*
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  - Longest chain of sequential dependencies in algorithm
- **Shared Memory:**
  - Processes can concurrently read/write to the same shared memory

Parallel Batch-Dynamic k-Core Decomposition

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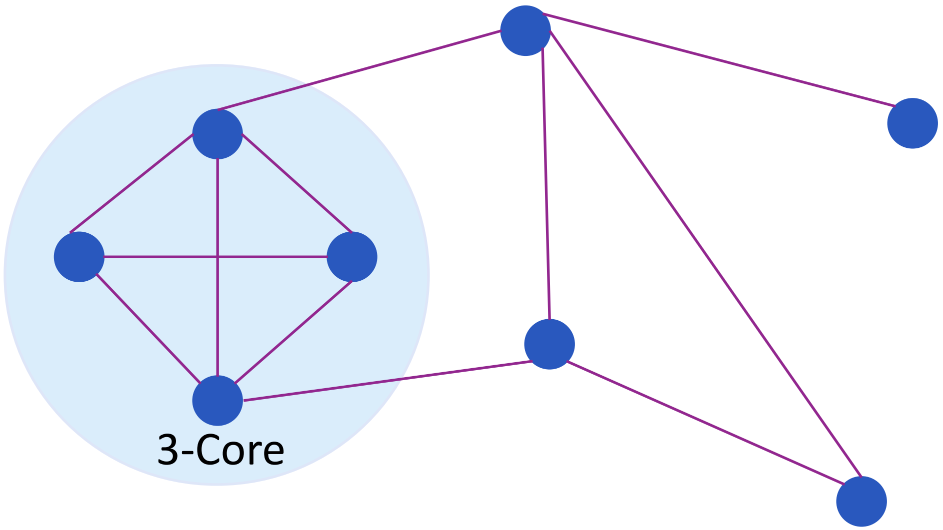
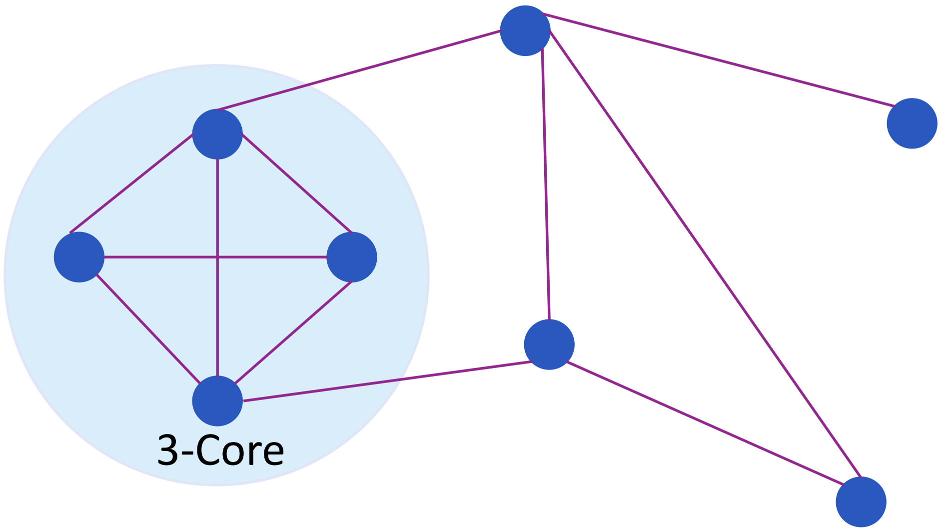
k-Core Decomposition



k-Core Decomposition

Coreness or Core Number of Node v: **Maximum Core Value** of a Core Containing v



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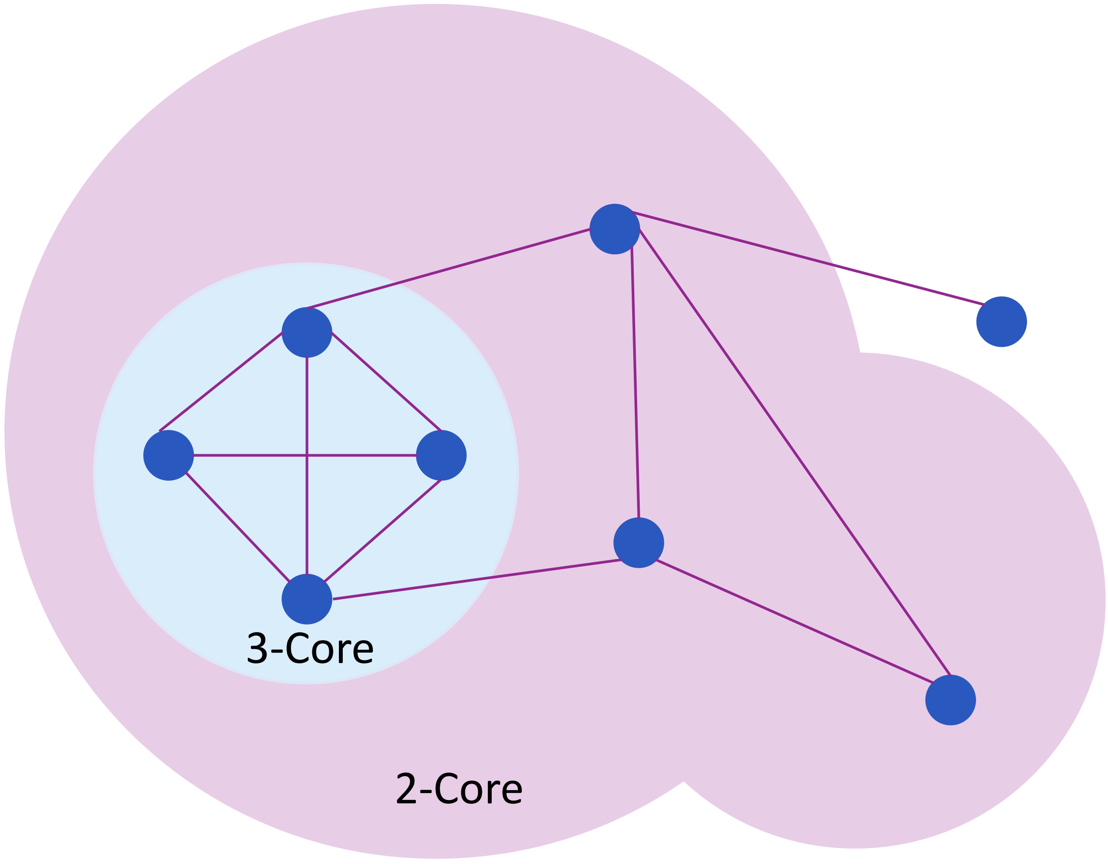

k-Core Decomposition

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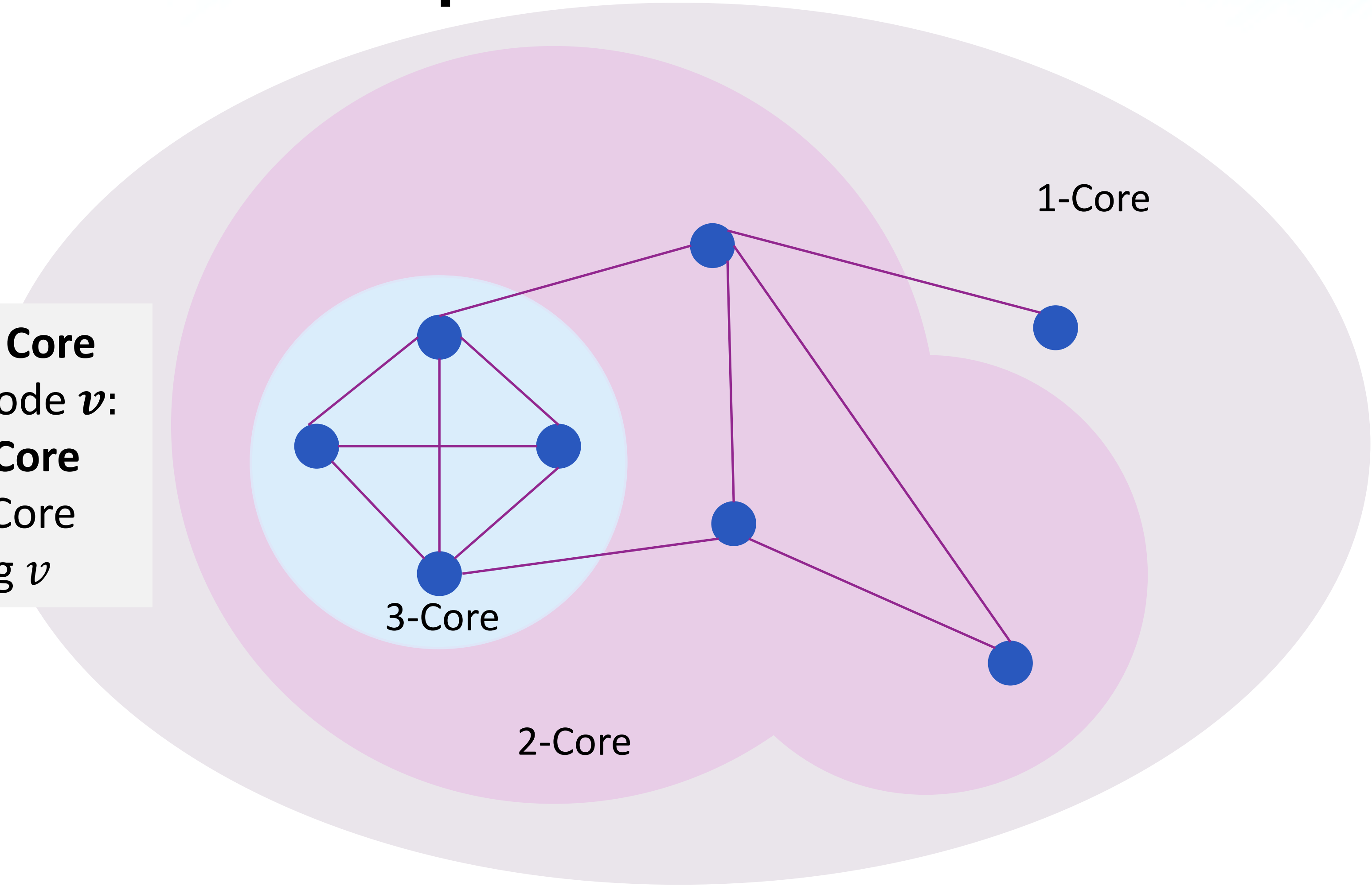
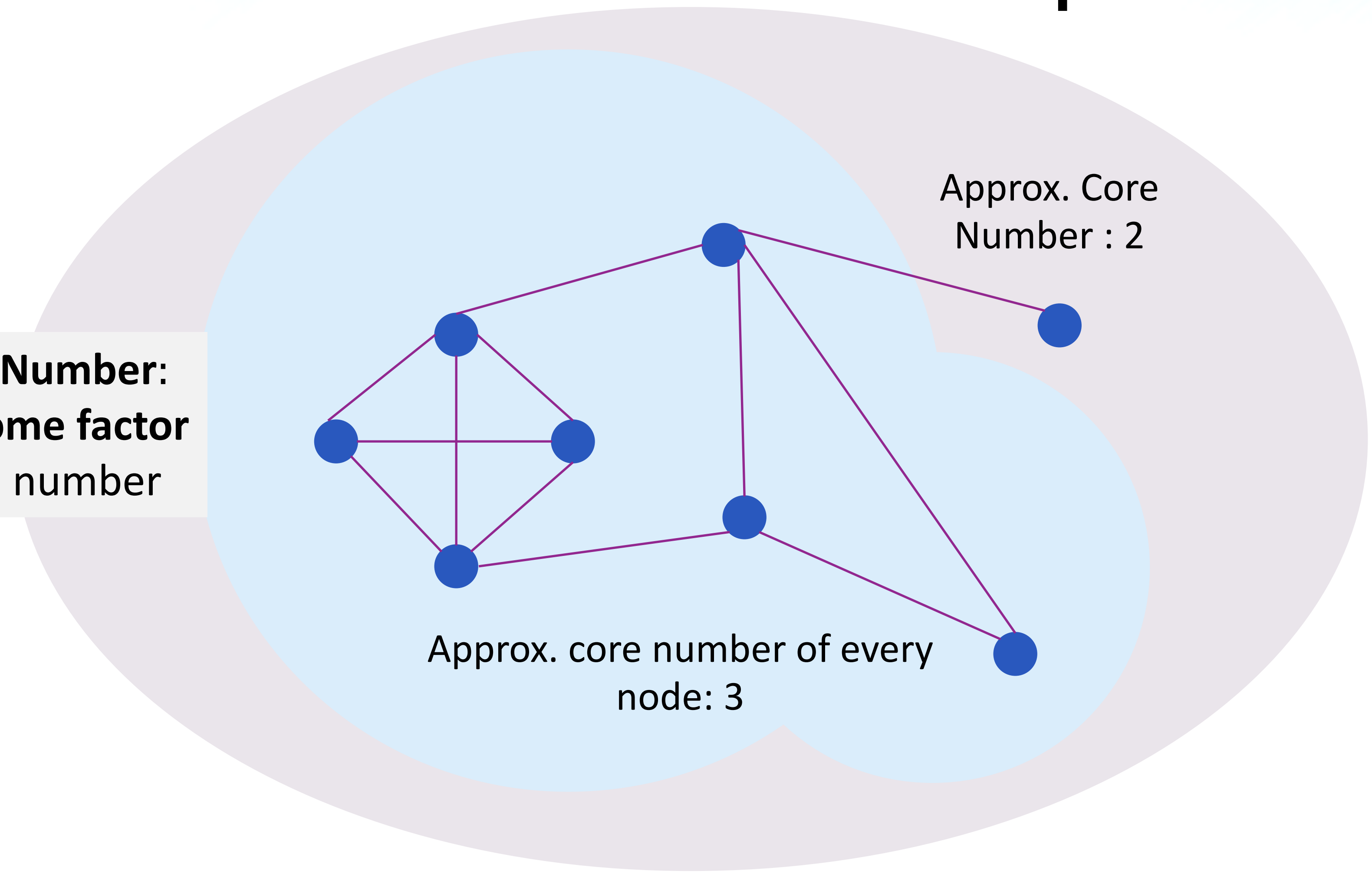


Approximate *k*-Core Decomposition

Approx. Core Number: Value within **some factor** of actual core number



Approximate *k*-Core Decomposition

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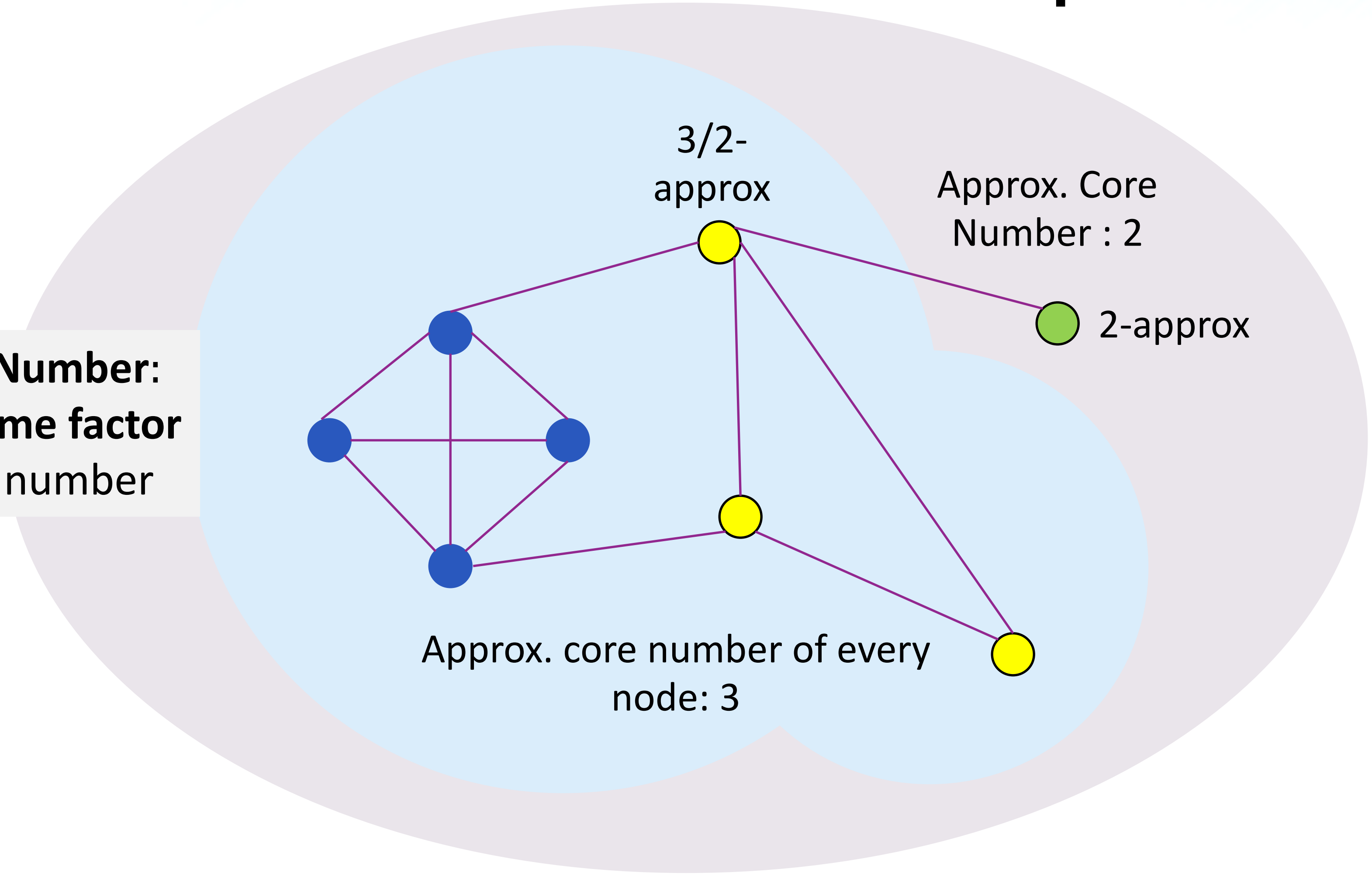

Applications of *k*-Core Decomposition

- Graph clustering
- Community detection
- Graph visualizations
- Protein network analysis
- Approximating network centrality measures
- Much interest in the machine learning, database, graph analytics, and other communities

Parallel Batch-Dynamic *k*-Core Decomposition

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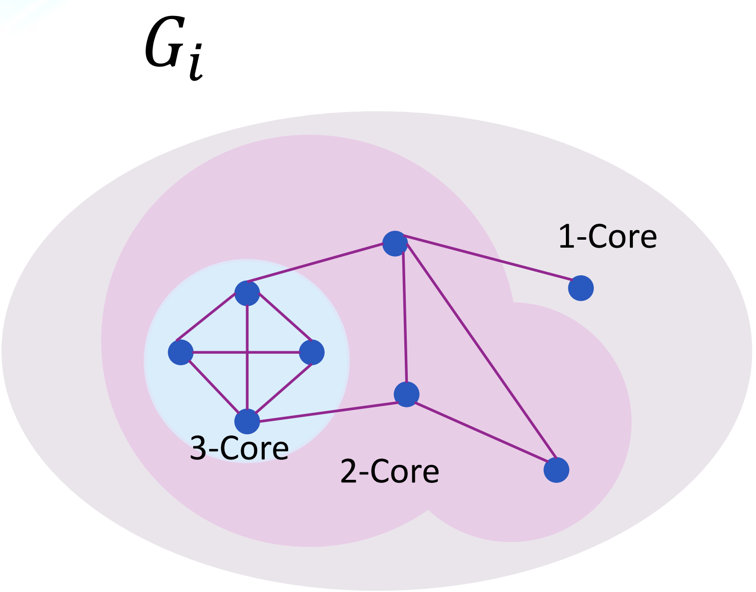
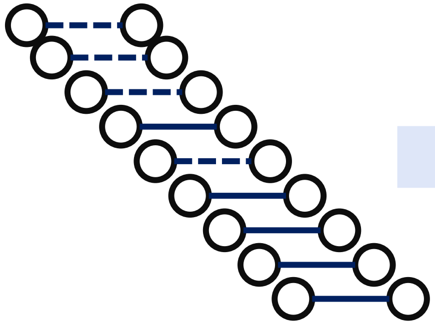
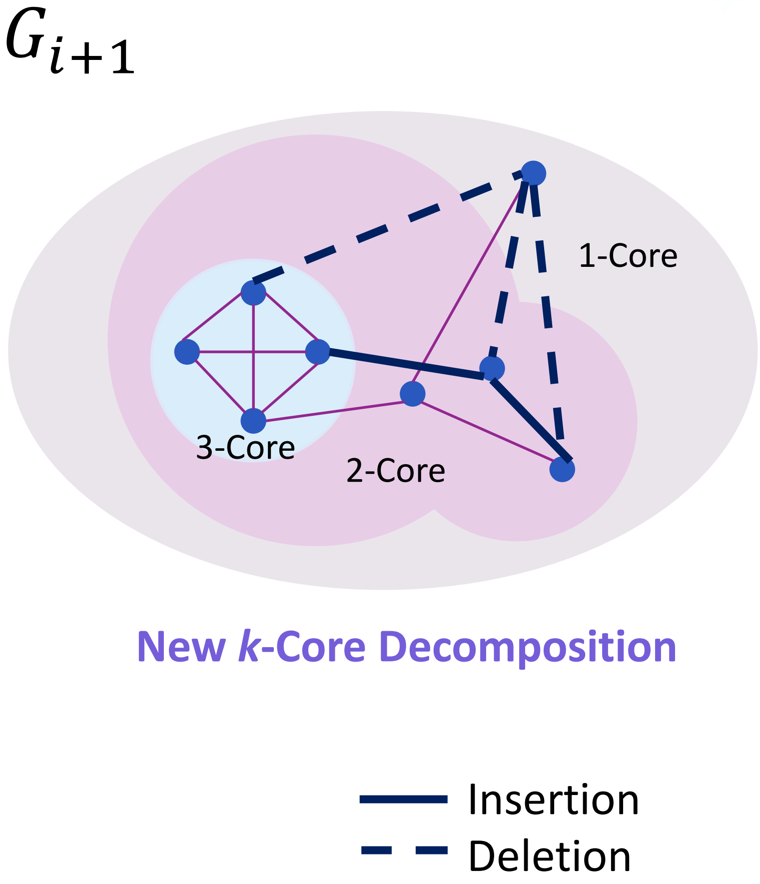
Batch-Dynamic Model Definition [BW09, DDKPSS20]



Initial *k*-Core Decomposition



***B* Edge Insertions/Deletions**



New *k*-Core Decomposition

Parallel Batch-Dynamic *k*-Core Decomposition

Parallel Batch-Dynamic k-Core Decomposition and Low Out-Degree Orientation

Quanquan C. Liu, Jessica Shi, Shangdi Yu, Laxman Dhulipala, Julian Shun

There exists a parallel batch-dynamic algorithm that outputs a $(2+\epsilon)$-**approximation** for coreness of each node in $O(B \cdot \log^2 n)$ **amortized work** and $O(\log^2 n \log\log n)$ **depth w.h.p., batch size** B.

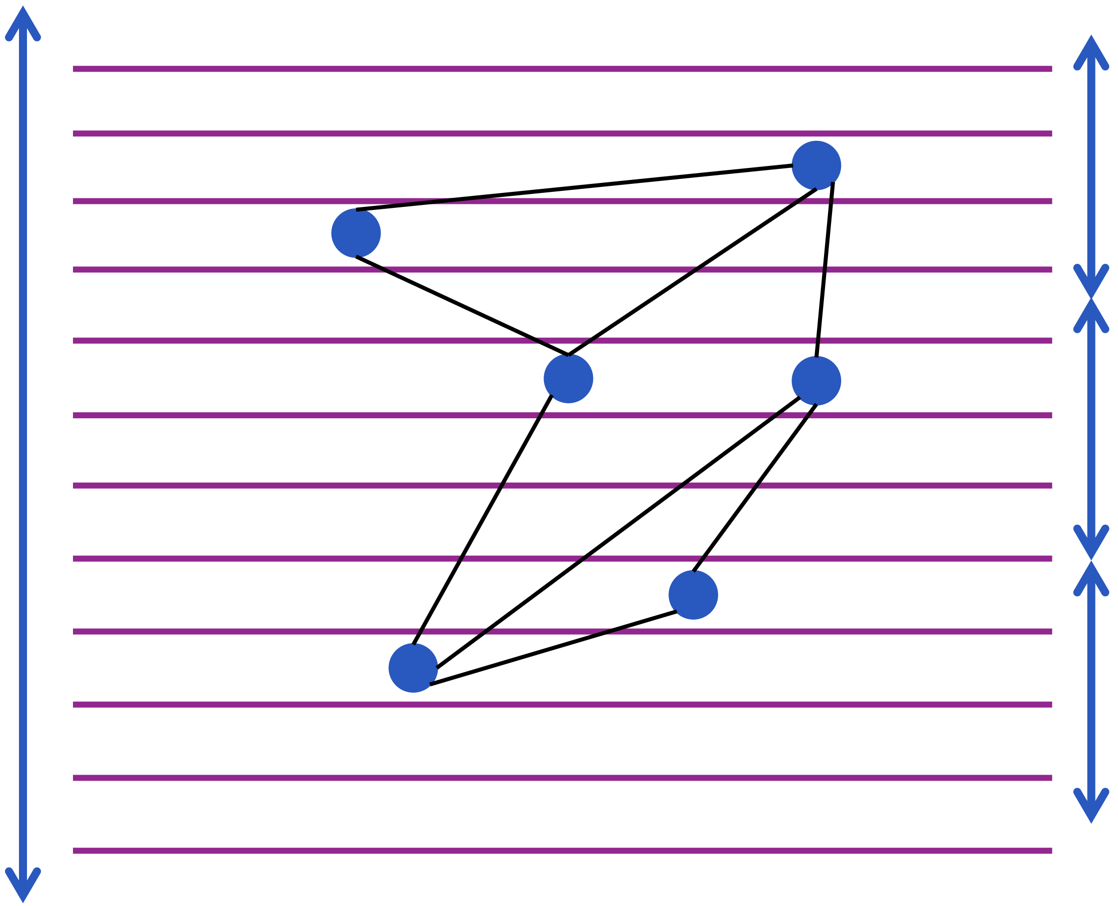
Level Data Structures

- Maximal Matching [Baswana-Gupta-Sen 18, Solomon 16]
- $(\Delta + 1)$-Coloring [Bhattacharya-Chakrabarty-Henzinger-Nanongkai 18, Bhattacharya-Grandoni-Kulkarni-Liu-Solomon 19]
- Clustering [Wulff-Nilsen 12]
- Low out-degree orientation [Solomon-Wein 20, Henzinger-Neumann-Weiss 20]
- Densest subgraph [Bhattacharya-Henzinger-Nanongkai-Tsourakakis 15]

Sequential Level Data Structure (LDS)

$O(\log^2 n)$

Vertices partitioned into levels



Group of $O(\log n)$ levels

Cut-off: $(1+\epsilon)^i$

Group of $O(\log n)$ levels

Cut-off: $(1+\epsilon)^{i-1}$

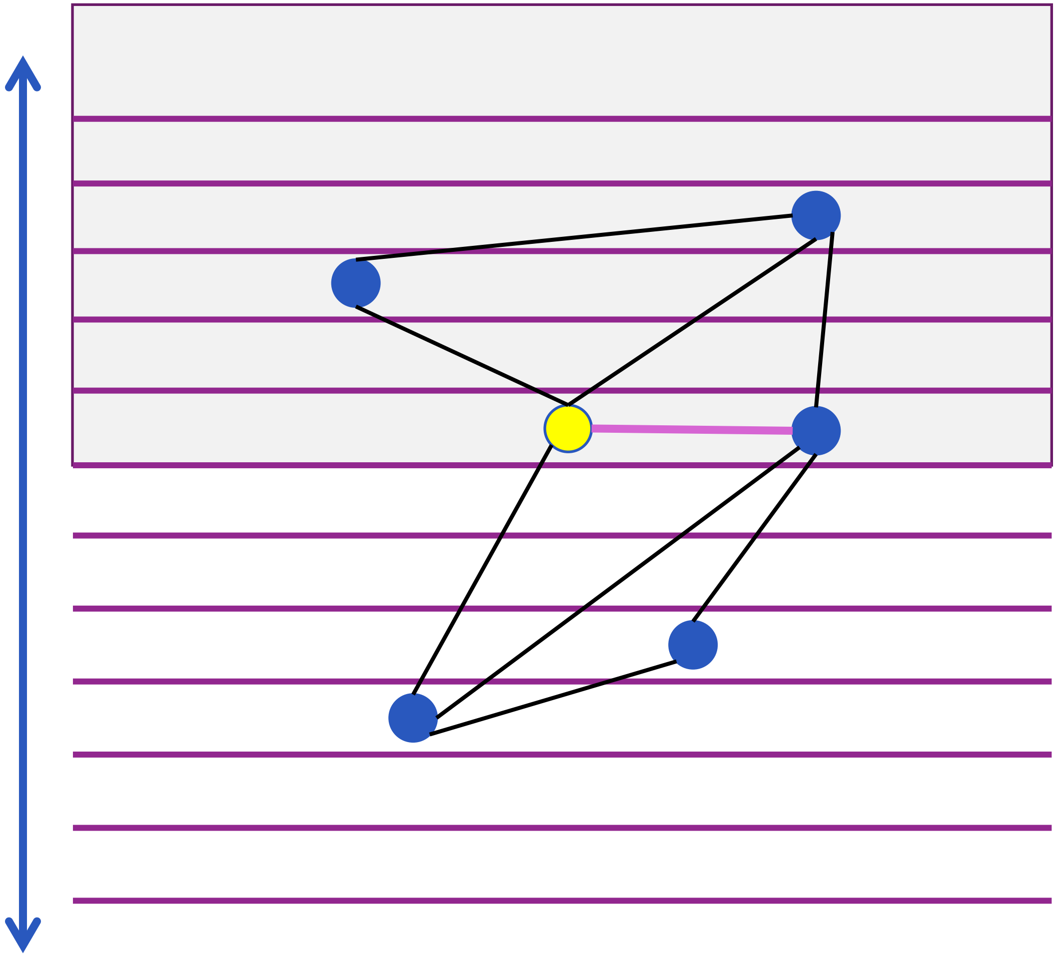
Group of $O(\log n)$ levels

Cut-off: $(1+\epsilon)^{i-2}$

...

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

Sequential Level Data Structure (LDS)



$O(\log^2 n)$

Vertices partitioned into levels

neighbors: $> 2.1(1+\epsilon)^i$

= edge insertion

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

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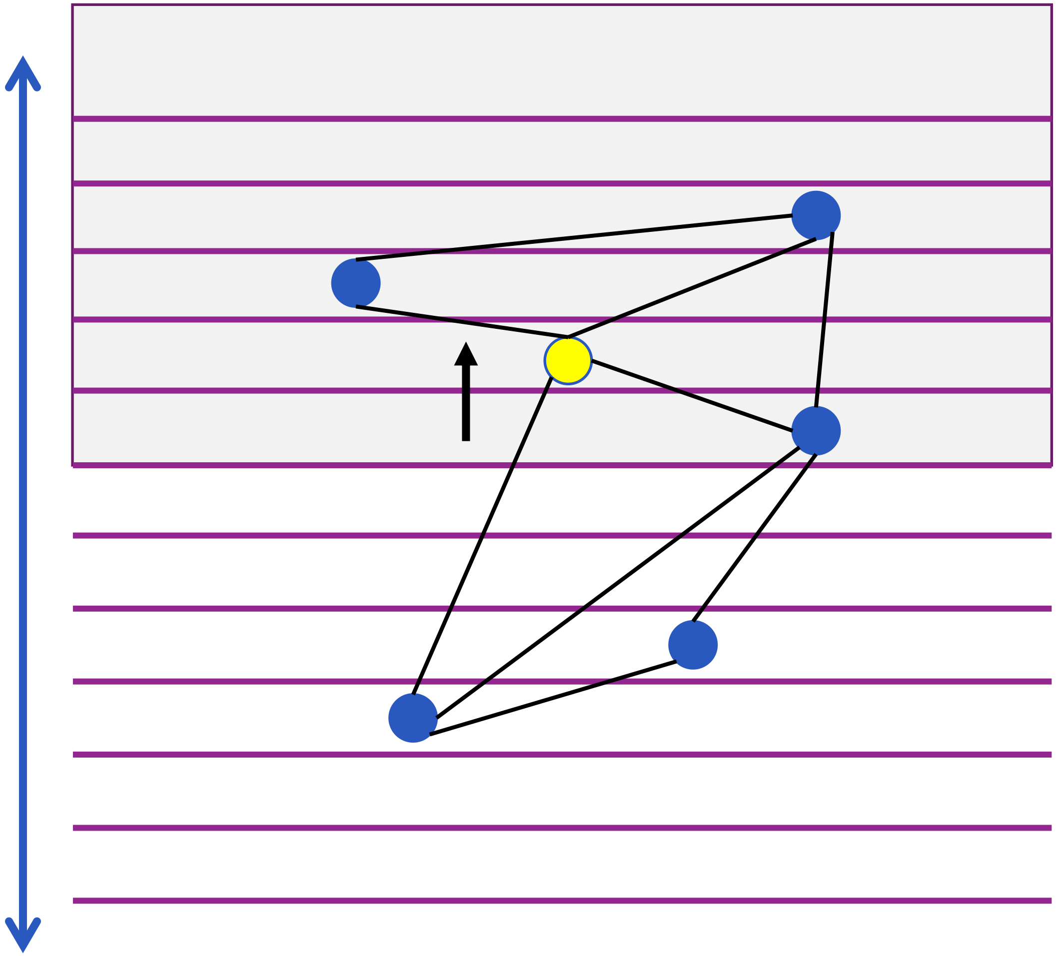

\# neighbors: $> 2.1(1+\epsilon)^i$

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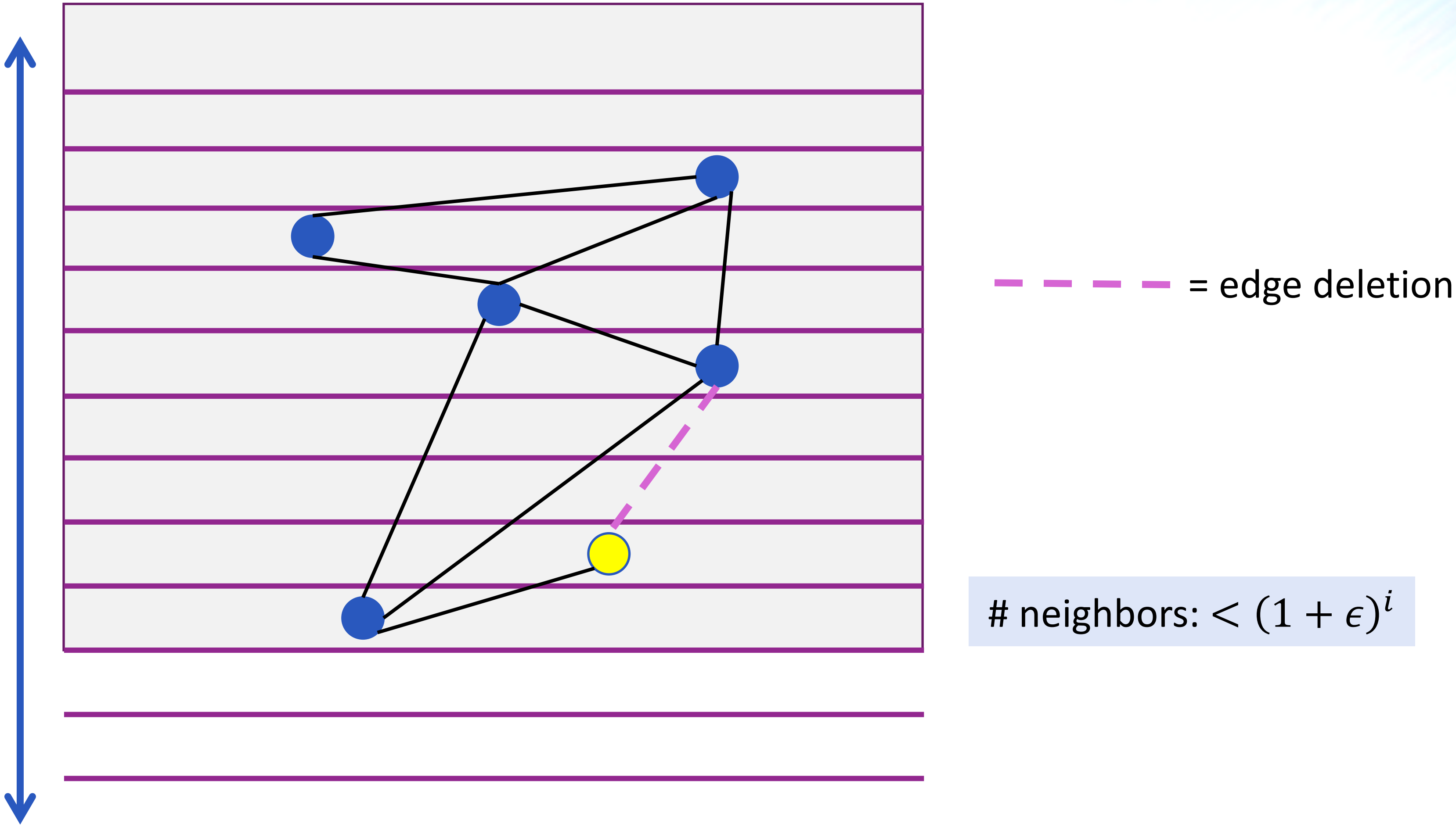

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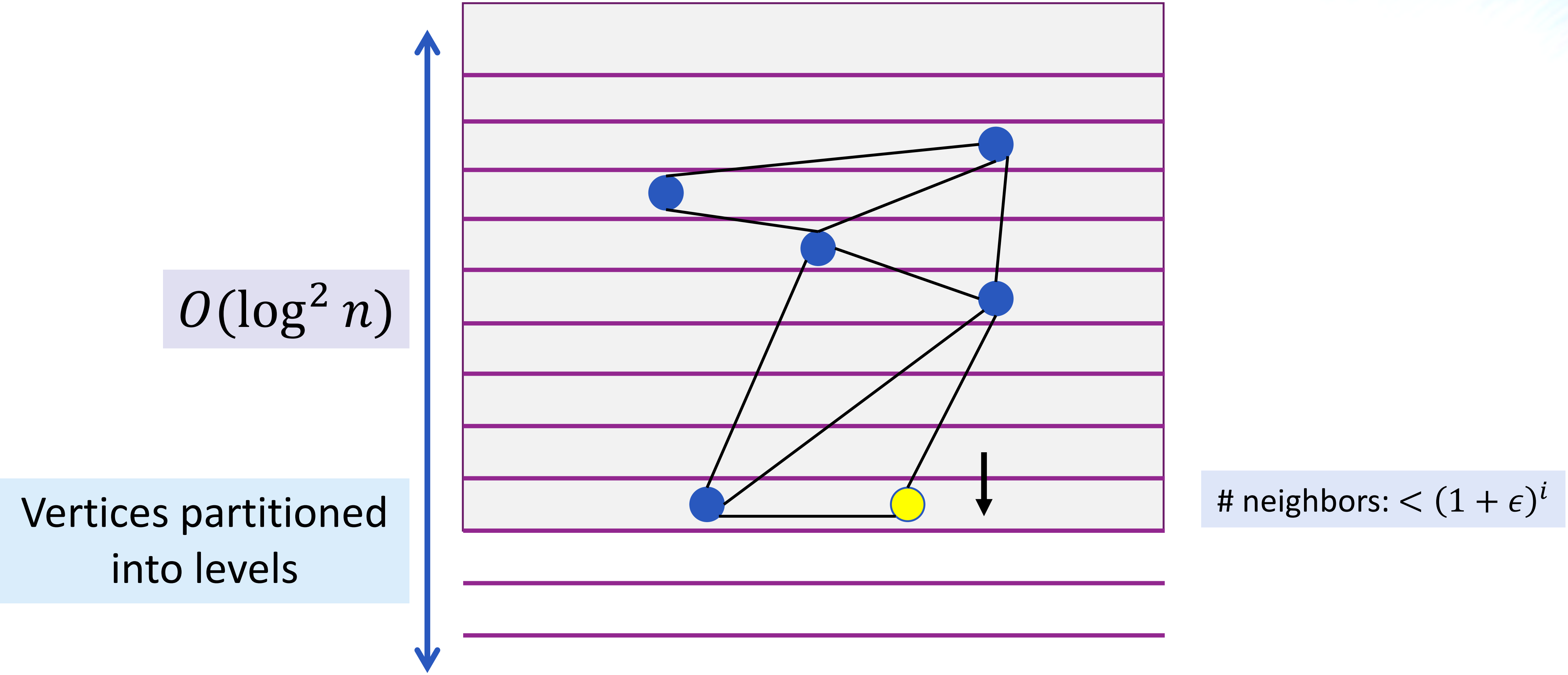

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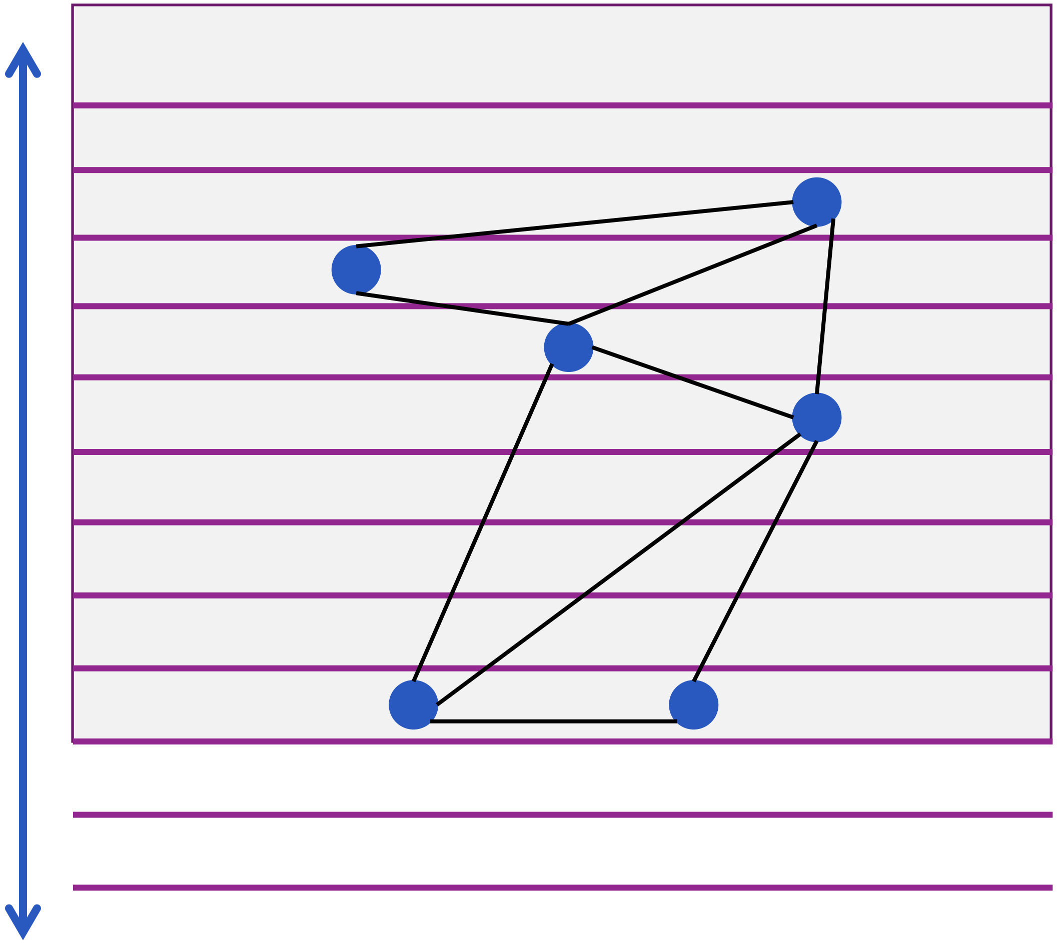

$O(\log^2 n)$

Vertices partitioned into levels

v is stored at a level where
1. Number of neighbors at or above same level is $\leq 2.1(1+\epsilon)^i$
2. Number of neighbors at or above level below v is $\geq (1+\epsilon)^i$

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

Sequential Level Data Structure (LDS)

Vertices partitioned into levels

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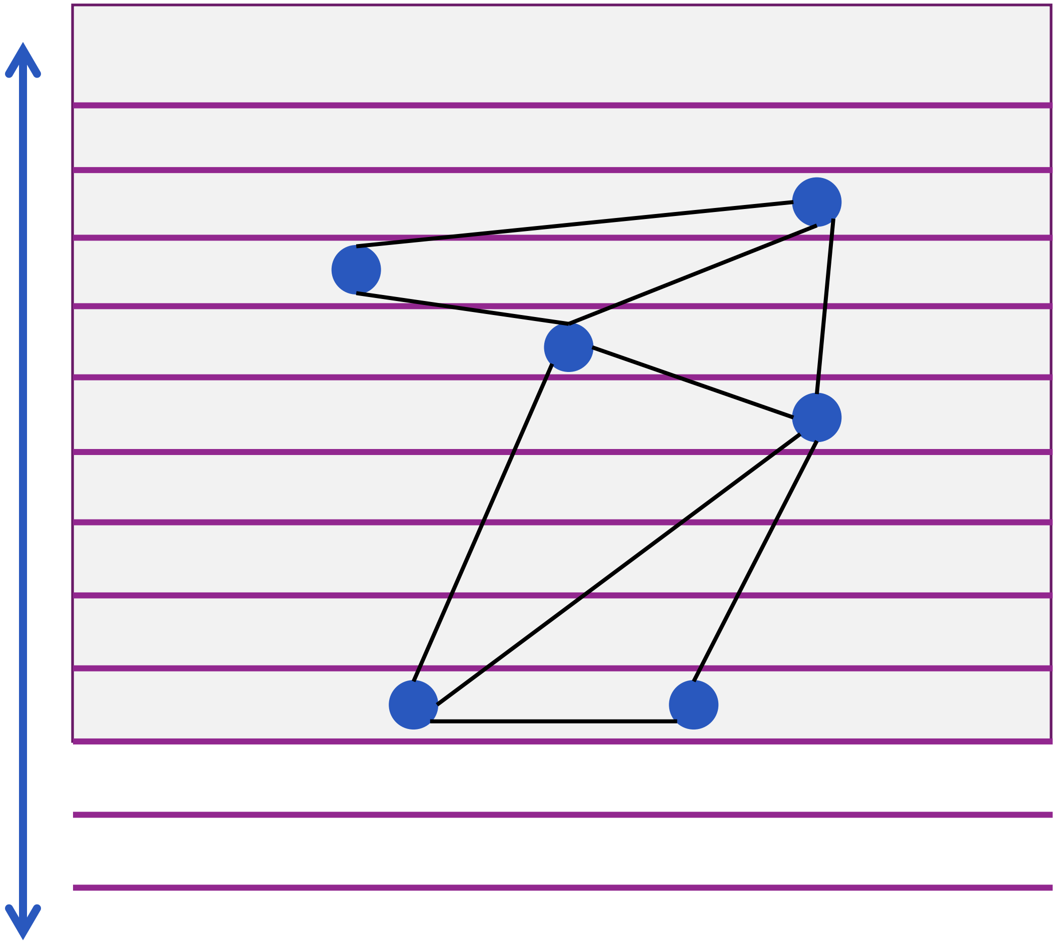

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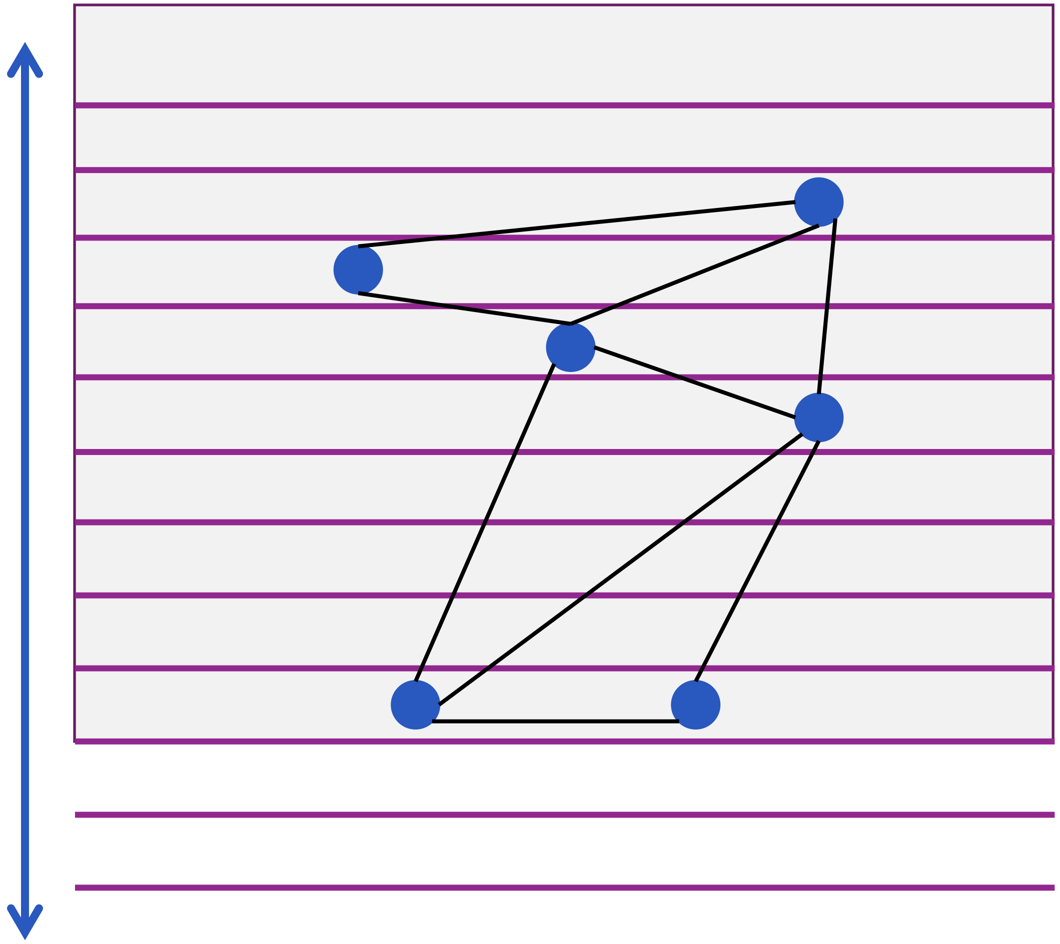

v is stored at a level where

1. Number of neighbors at or above same level is $\leq \boxed{2.1}(1+\epsilon)^i$
2. Number of neighbors at or above level below v is $\boxed{\geq (1+\epsilon)^i}$

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

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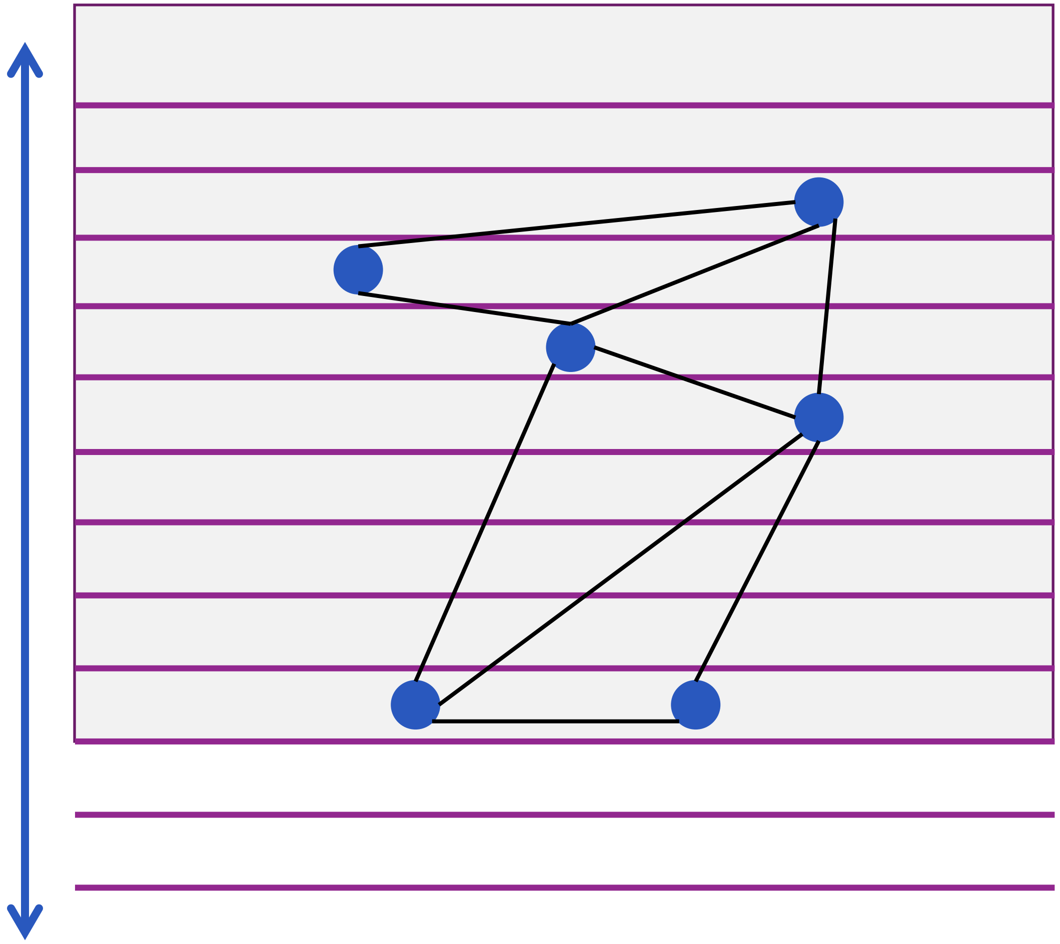

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Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

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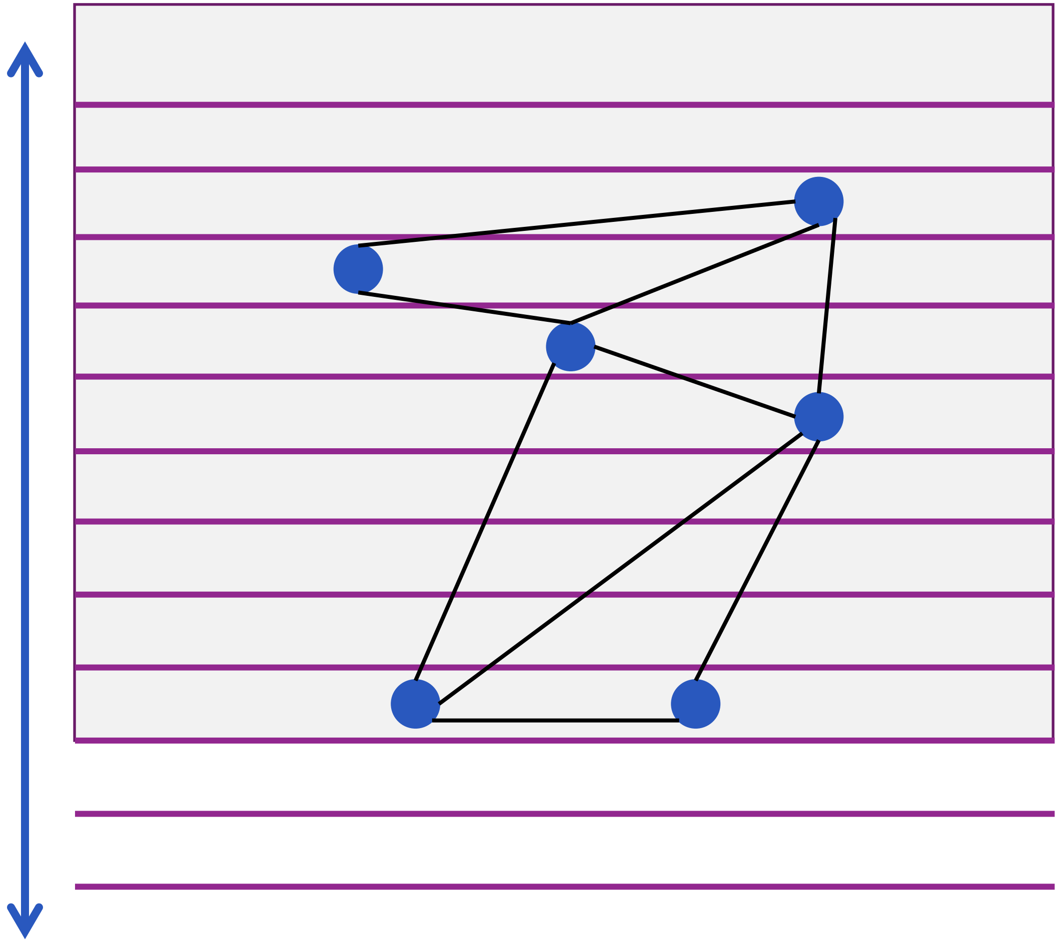

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Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

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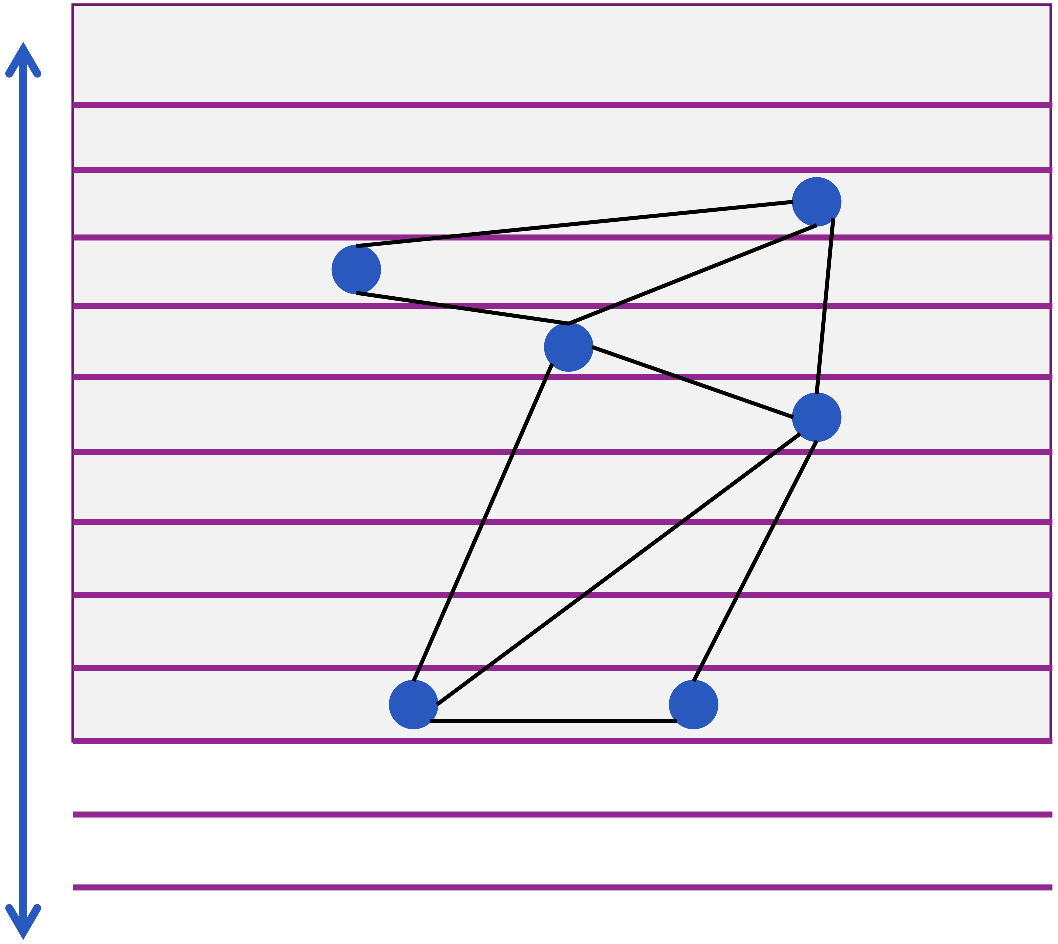


v is stored at a level where

1. **Upper Bound Invariant:**

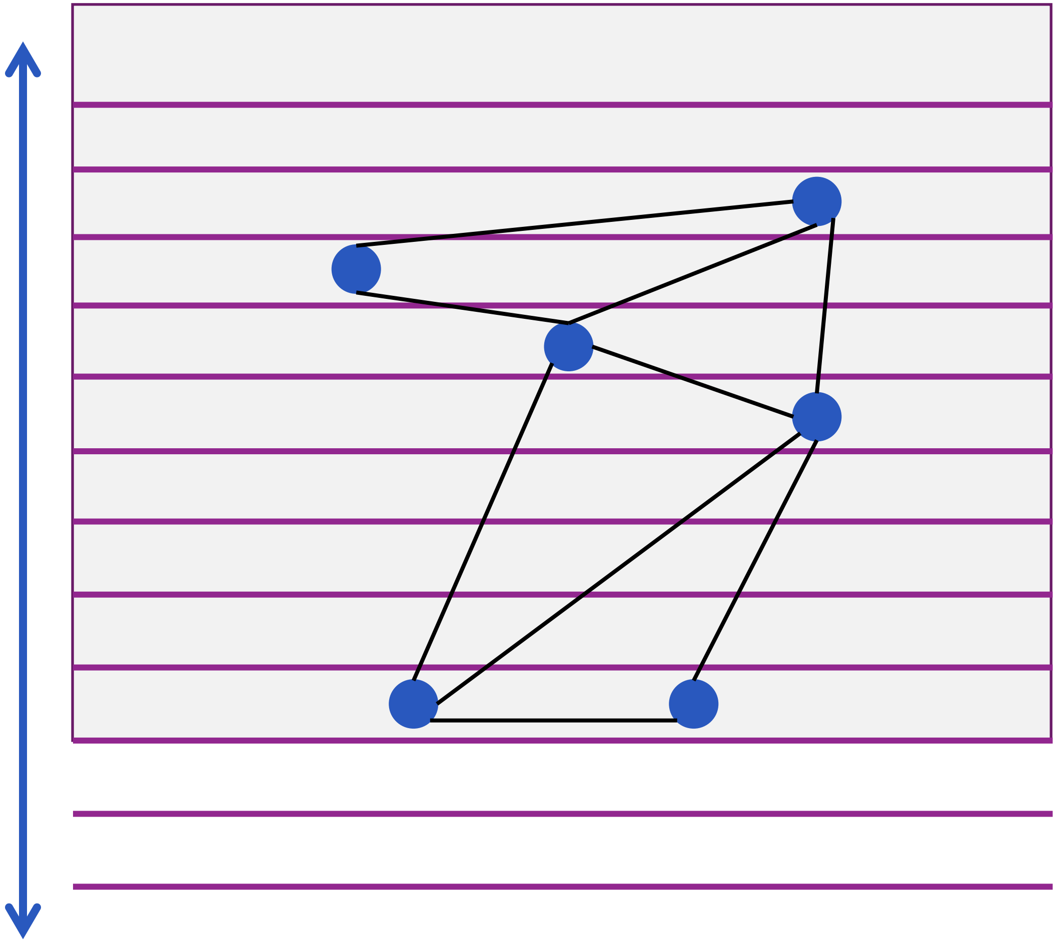
   $\leq 2.1(1+\epsilon)^i$

2. **Lower Bound Invariant:**

   $\geq (1+\epsilon)^i$

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

Sequential Level Data Structure (LDS)



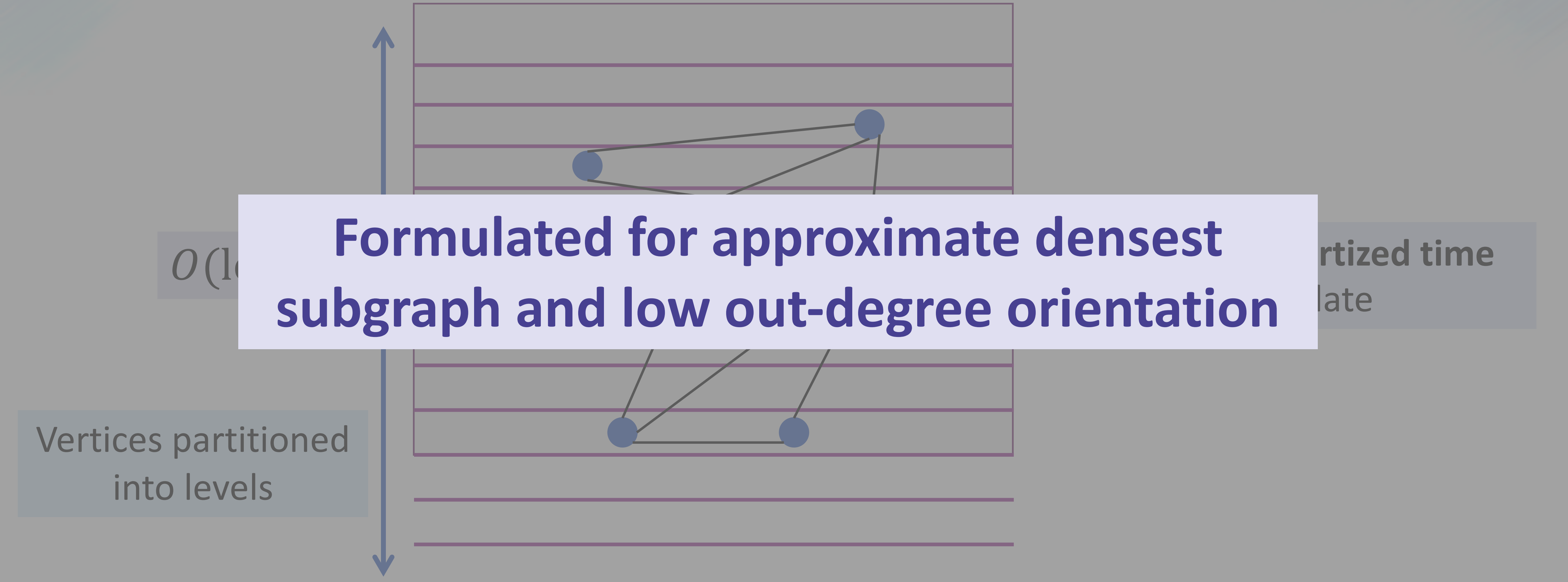
$O(\log^2 n)$

Vertices partitioned into levels

$O(\log^2 n)$ **amortized time** per update

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

Sequential Level Data Structure (LDS)

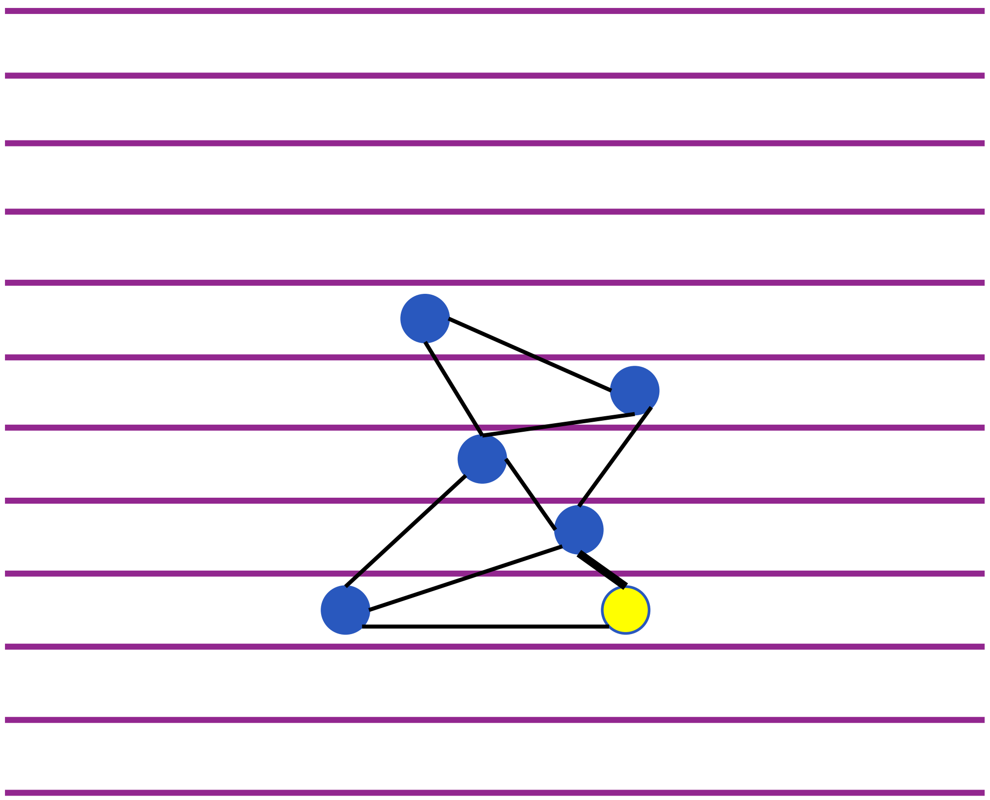


Formulated for approximate densest subgraph and low out-degree orientation

Bhattacharya-Henzinger-Nanongkai-Tsourakakis STOC 2015 and Henzinger-Neumann-Weiss 2020

Difficulties with Parallelization

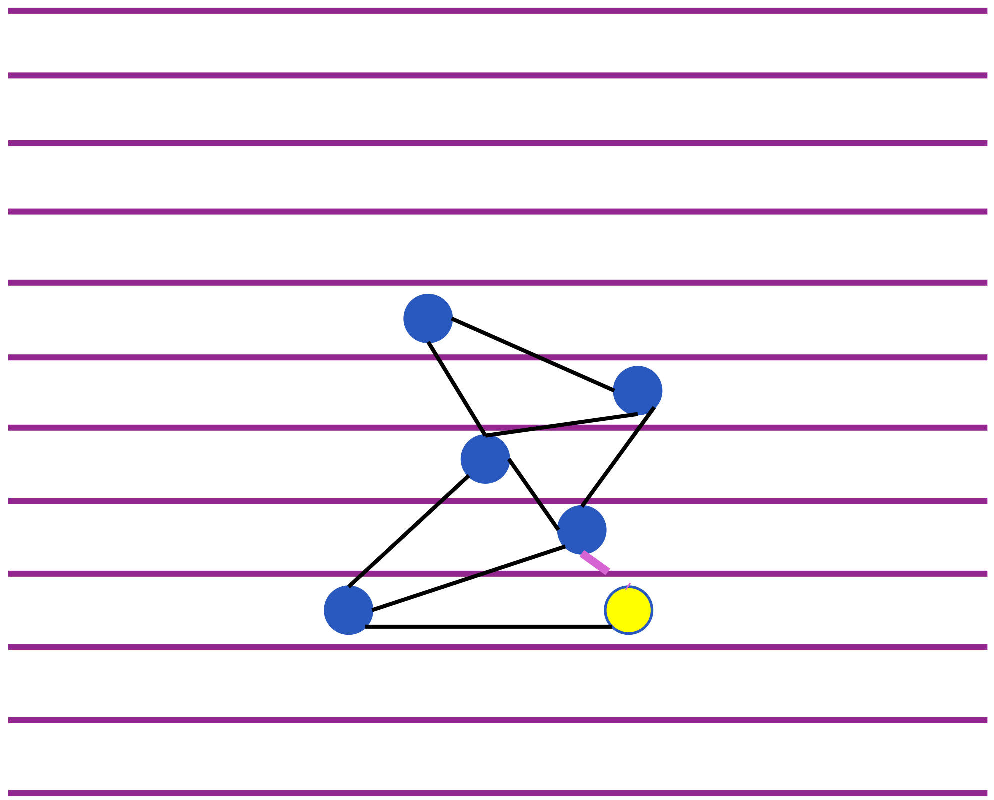
Large sequential dependencies



Large depth

Difficulties with Parallelization

Large sequential dependencies



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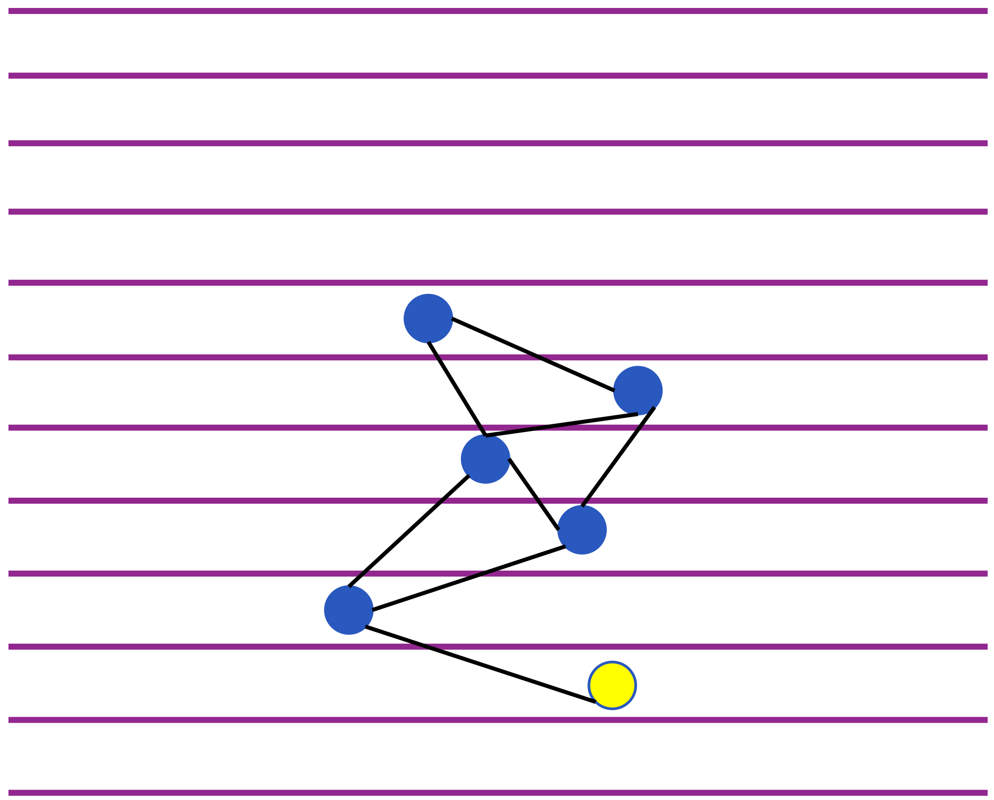

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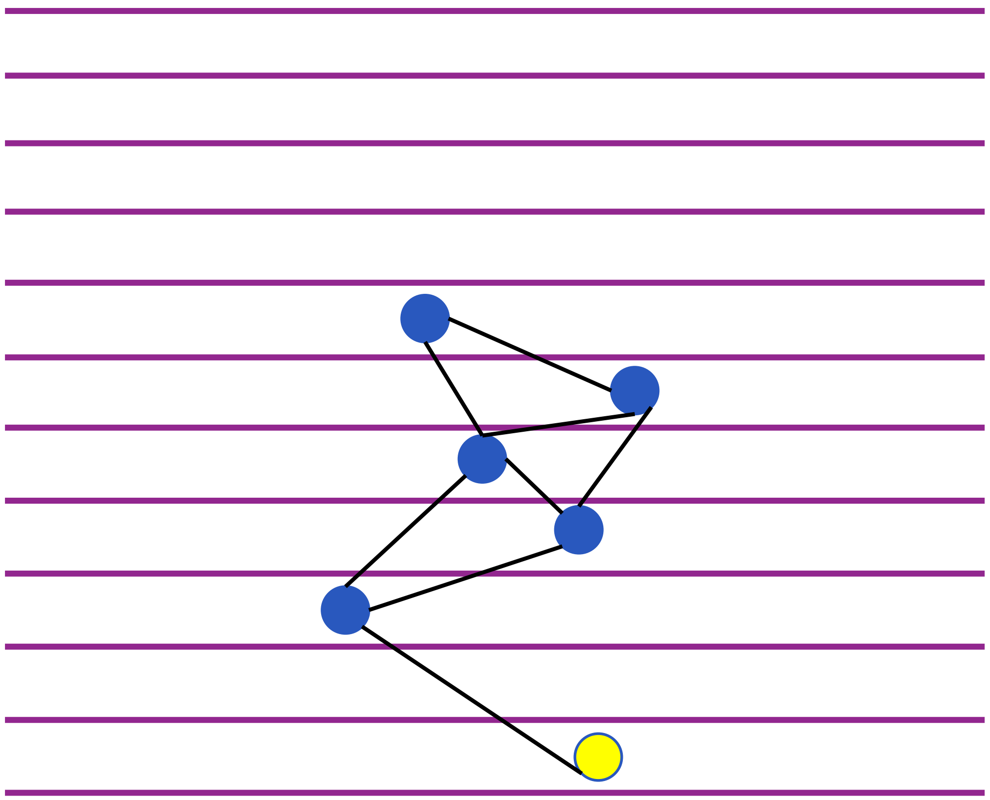

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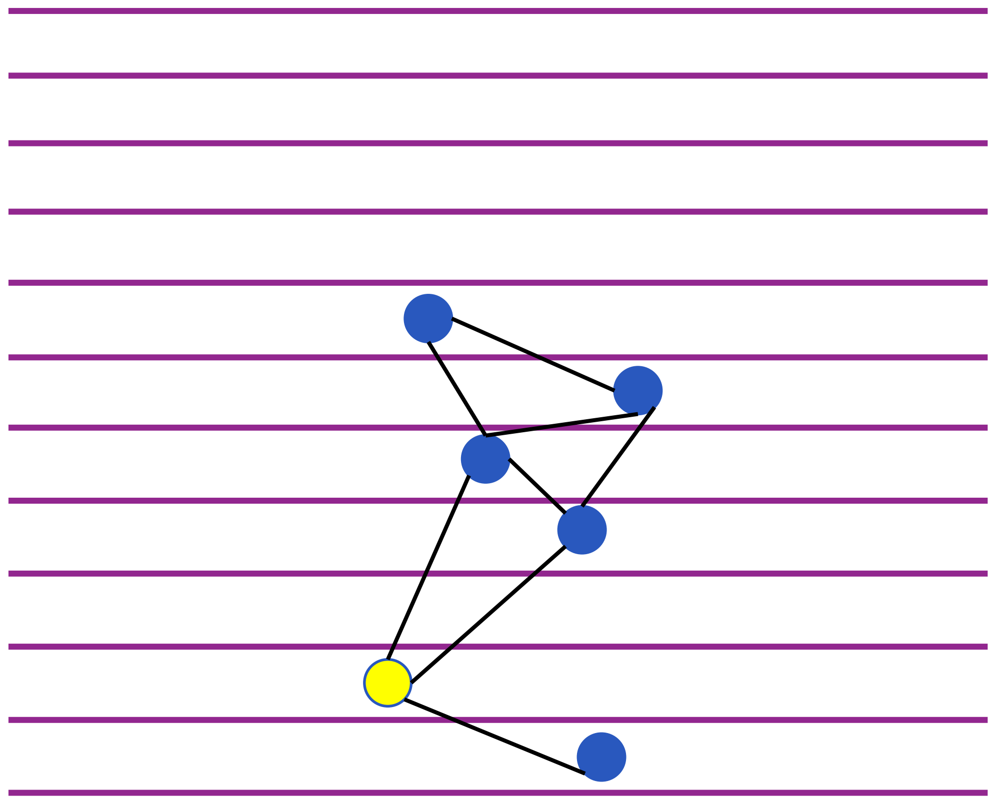

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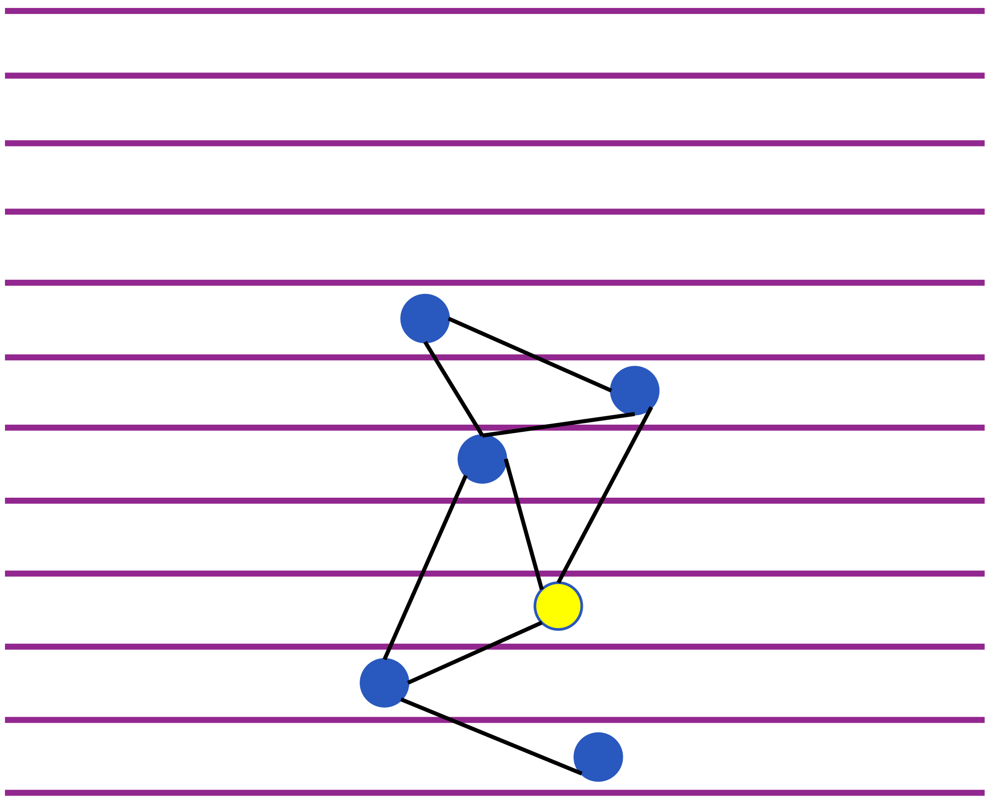

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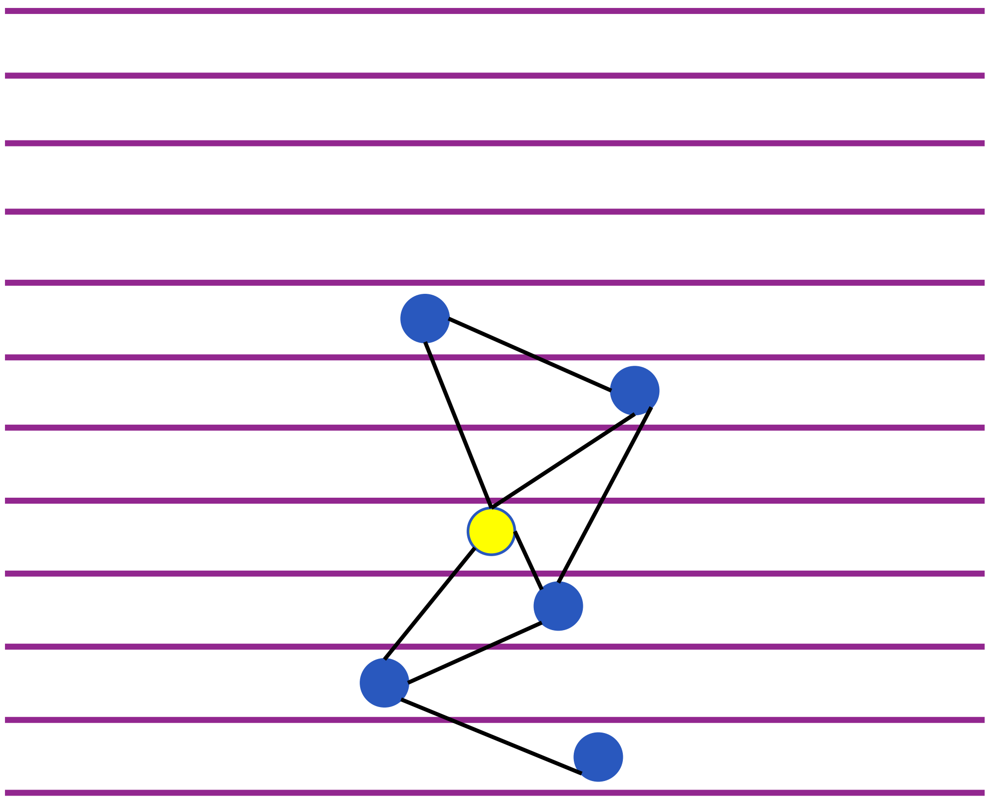

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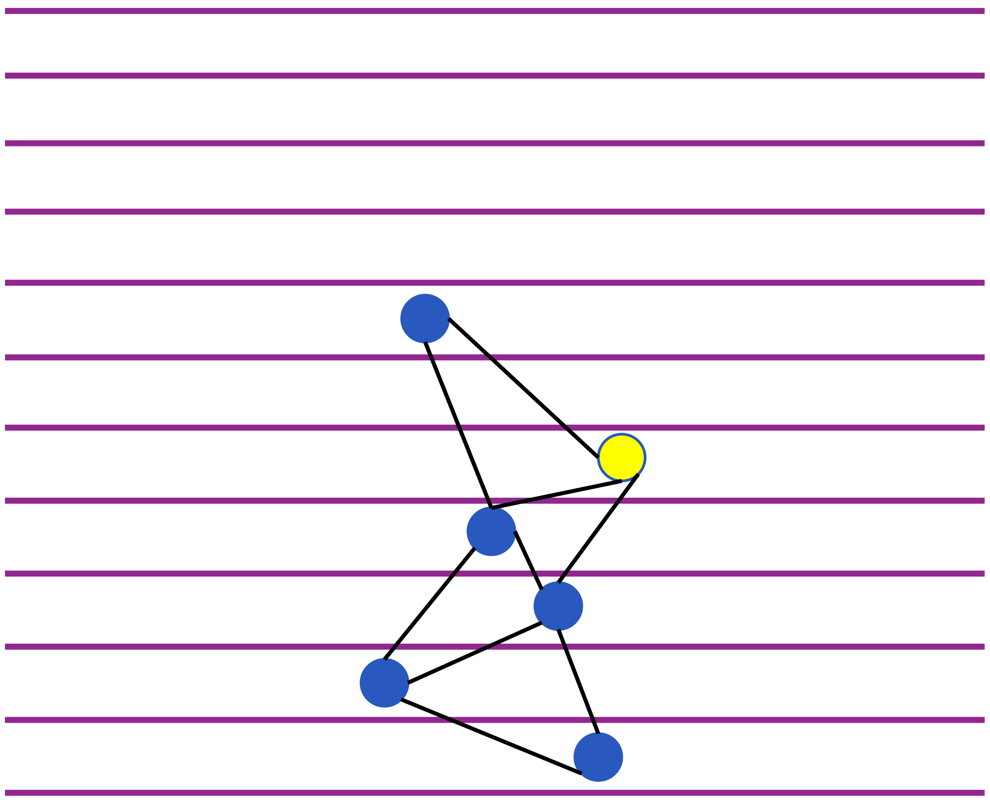

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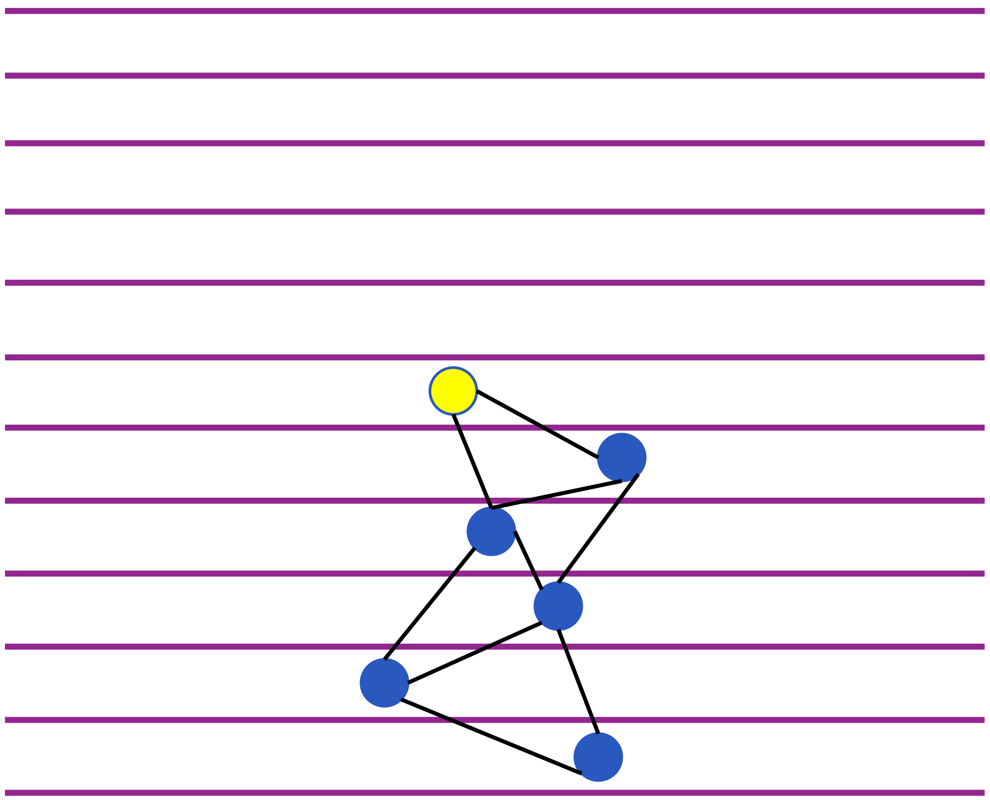
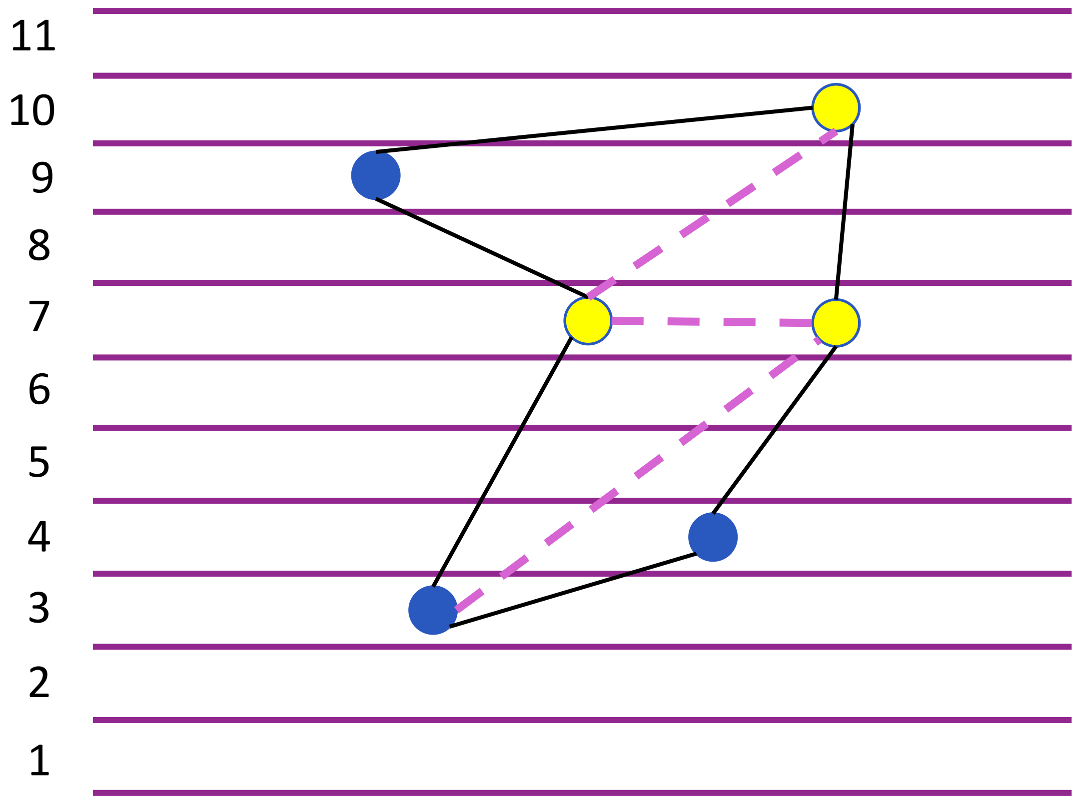


Large depth

Our Parallel Batch-Dynamic Level Data Structure (PLDS)

Deletions



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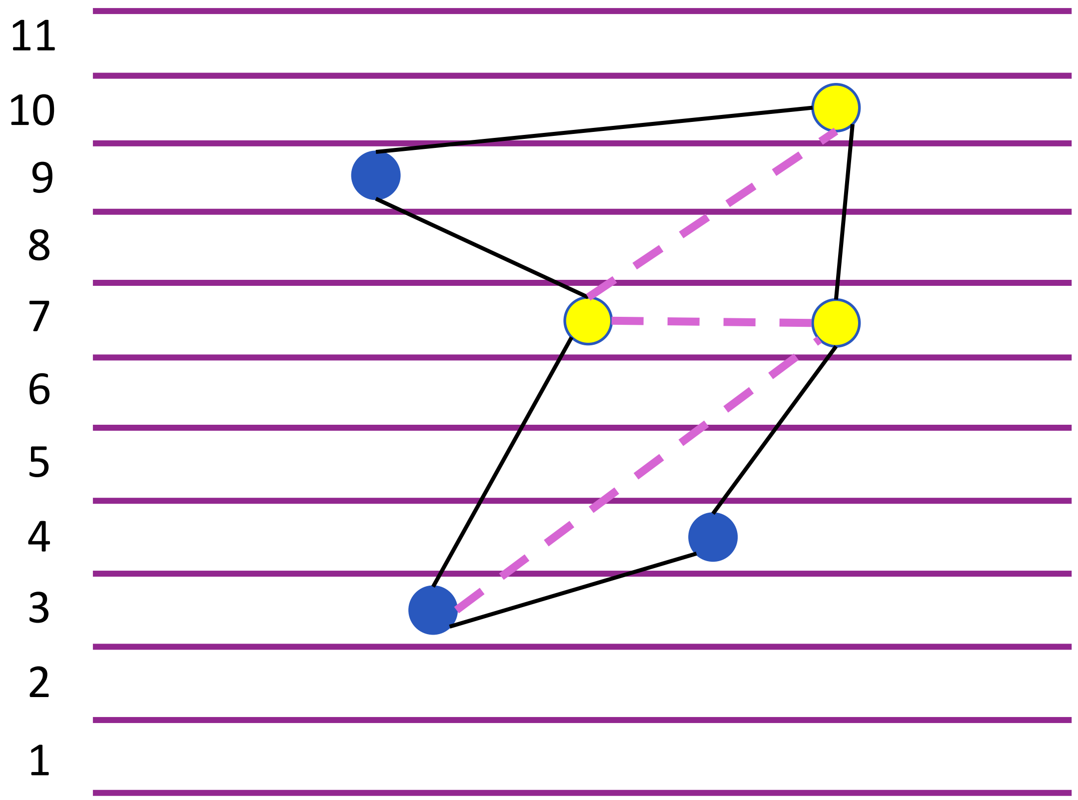
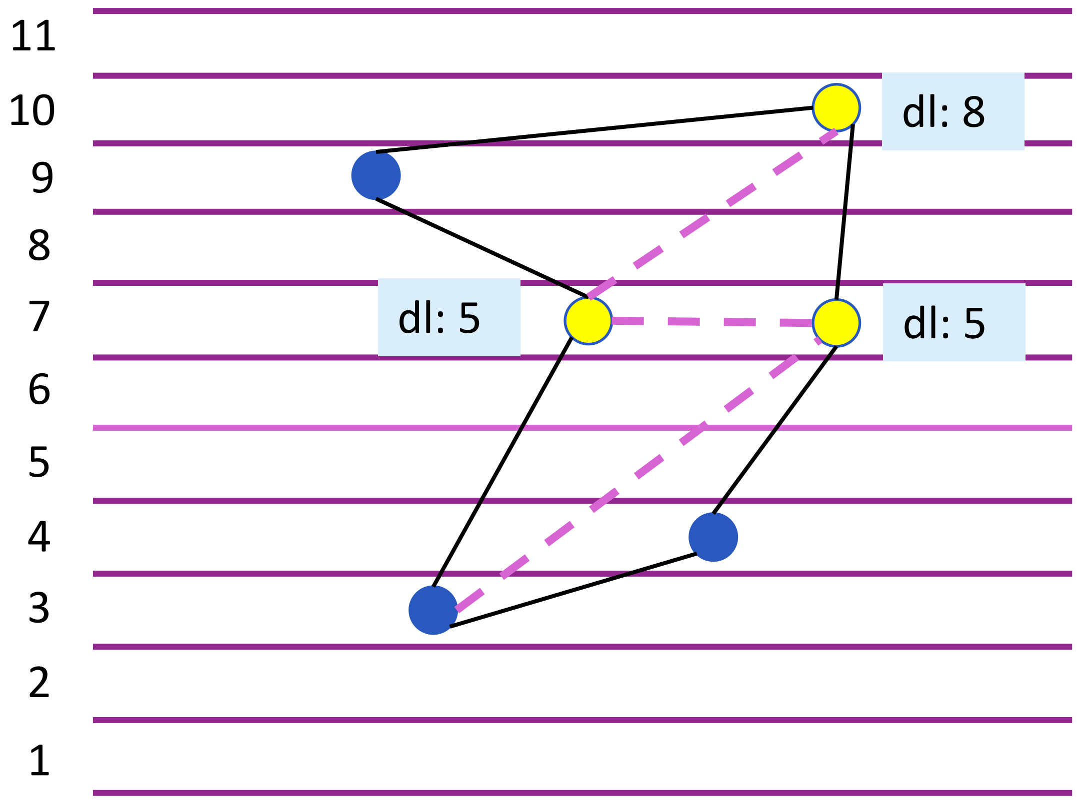


Only lower bound invariant, $(1+\epsilon)^i$, ever violated.

Our Parallel Batch-Dynamic Level Data Structure

Deletions

Calculate *desire-level*: **closest level that satisfies invariants**



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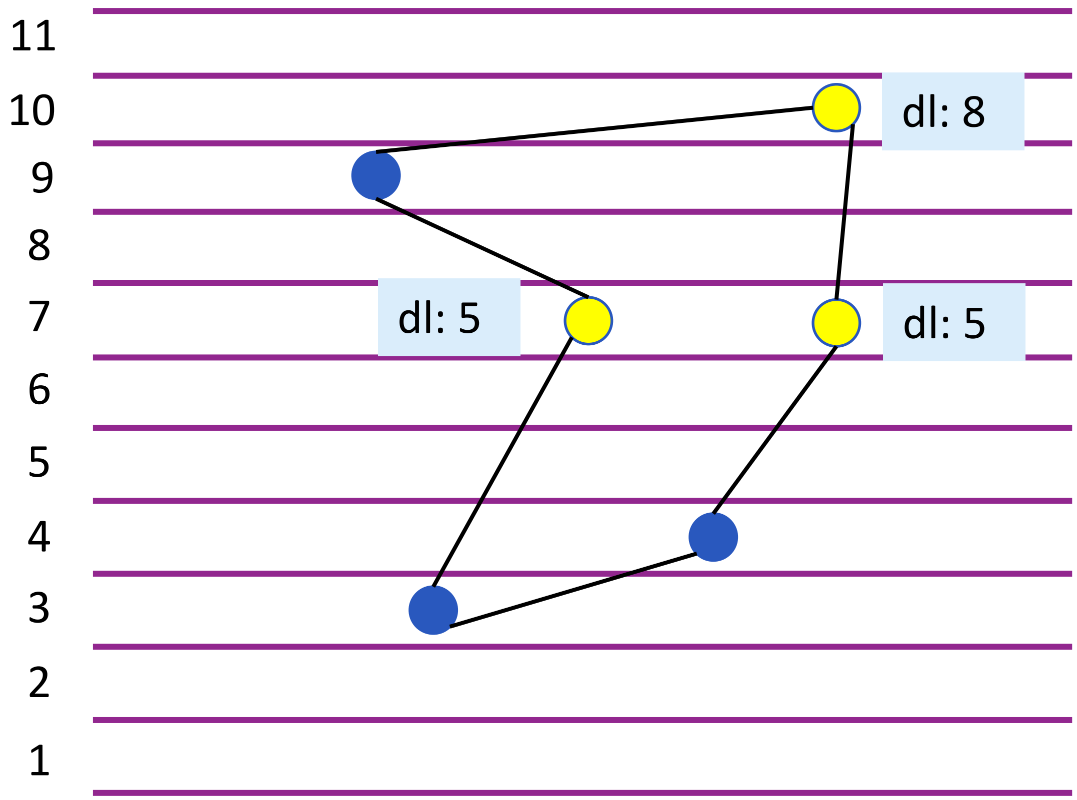

Iterate from **bottommost level to top level** and move vertices to desire-level

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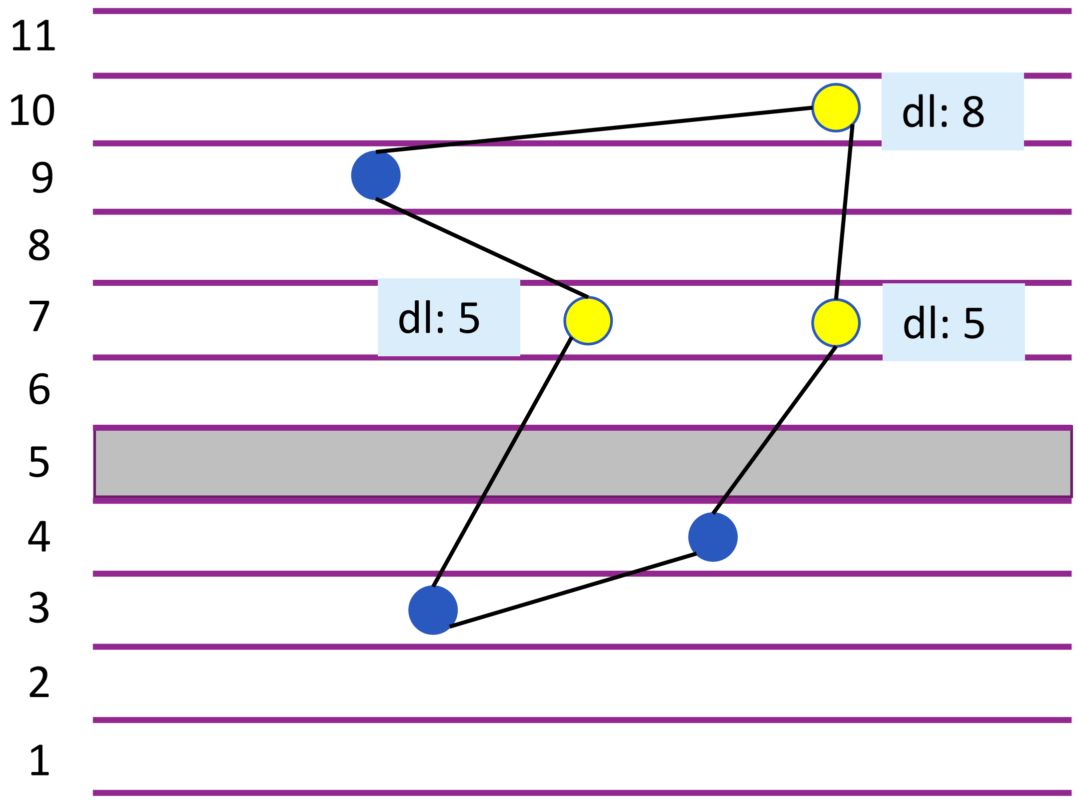

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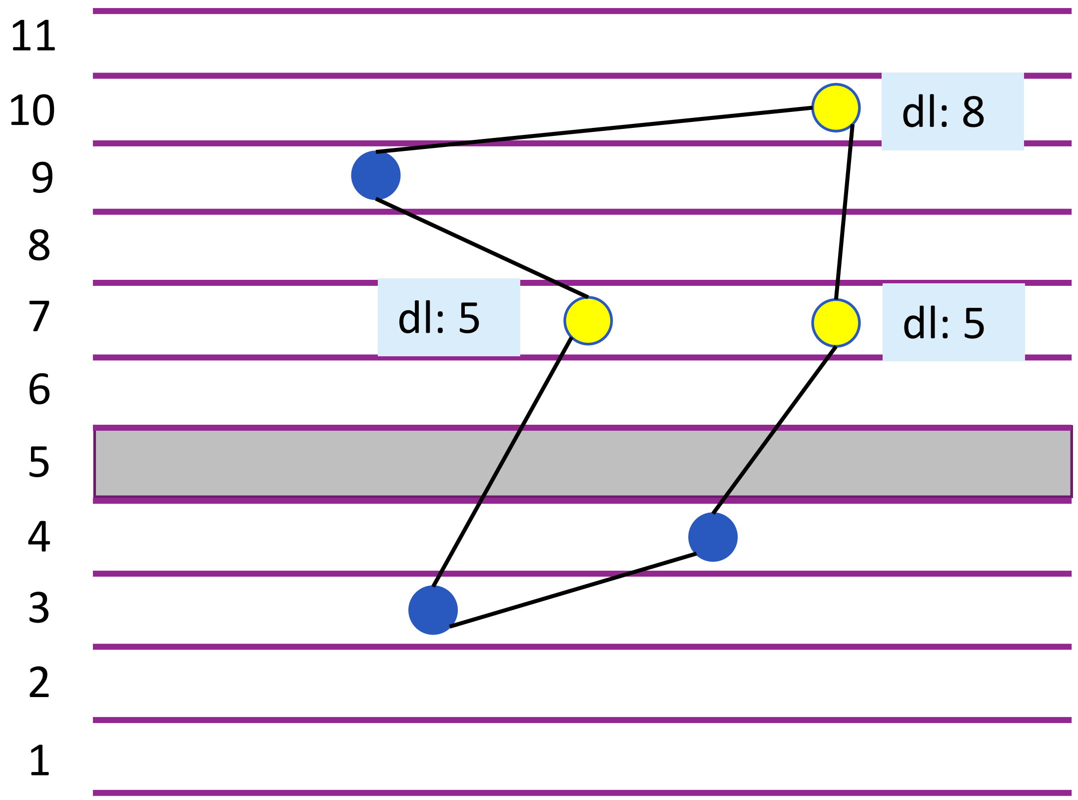

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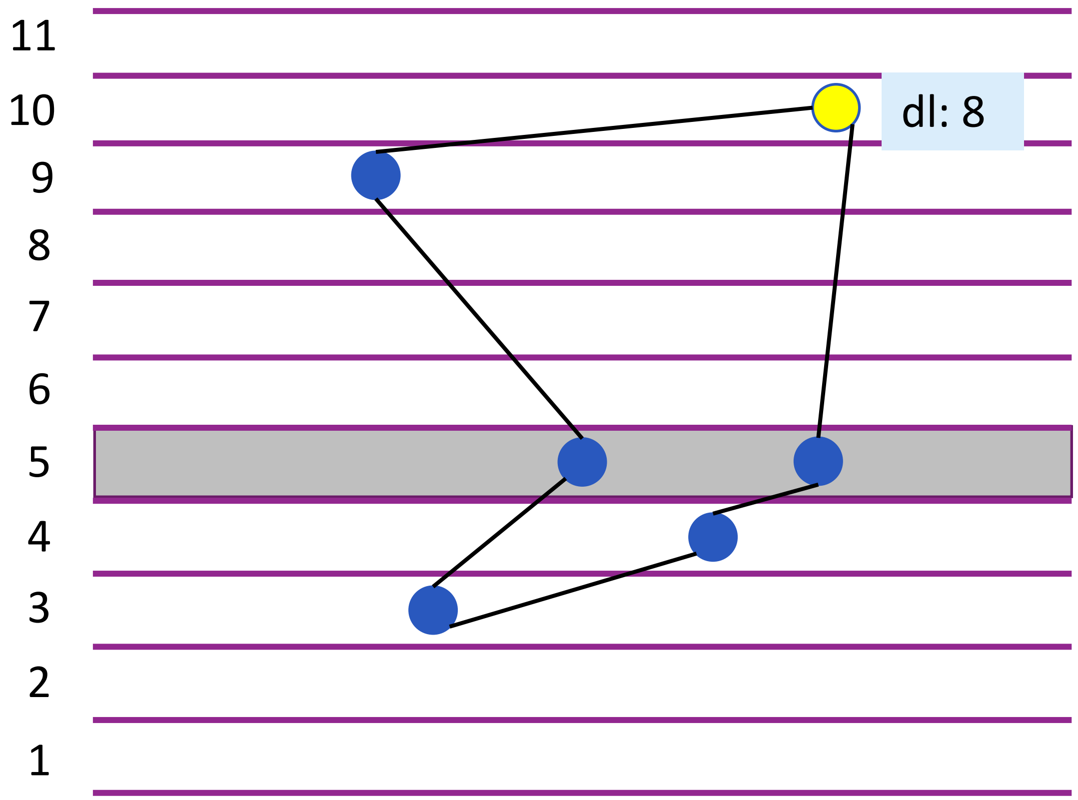
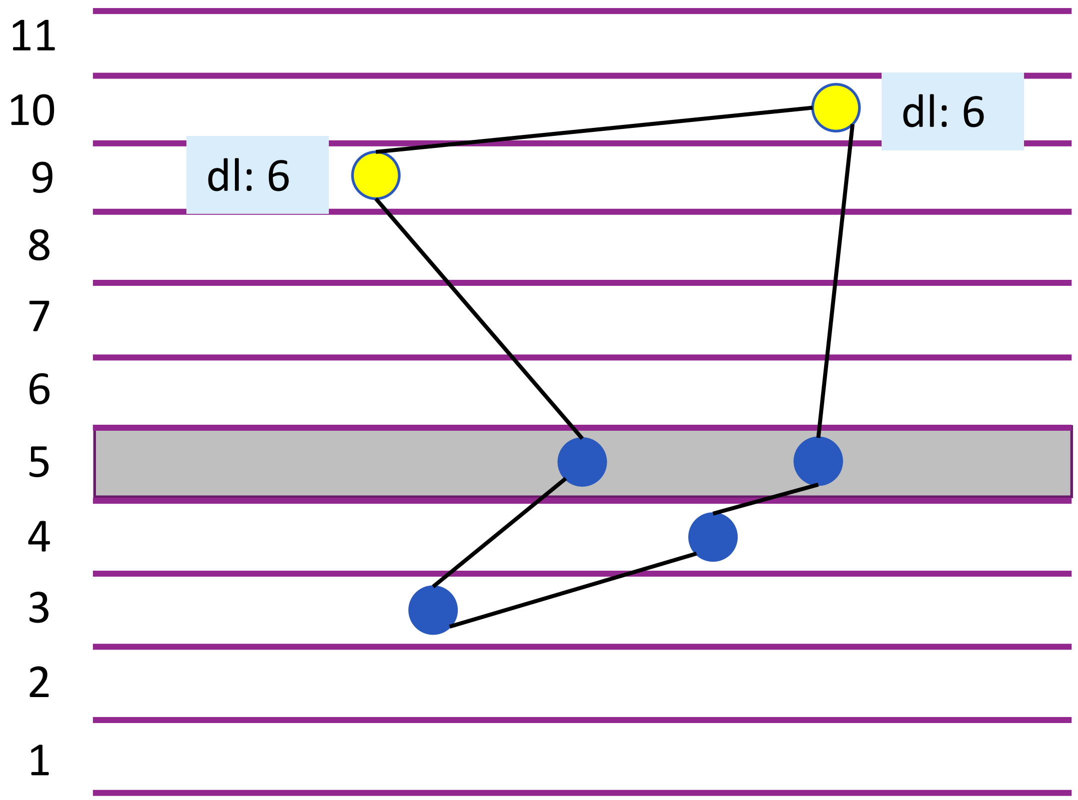


Update desire-levels of adjacent vertices at higher levels

Only lower bound invariant, $(1+\epsilon)^i$, ever violated.

Our Parallel Batch-Dynamic Level Data Structure

Deletions



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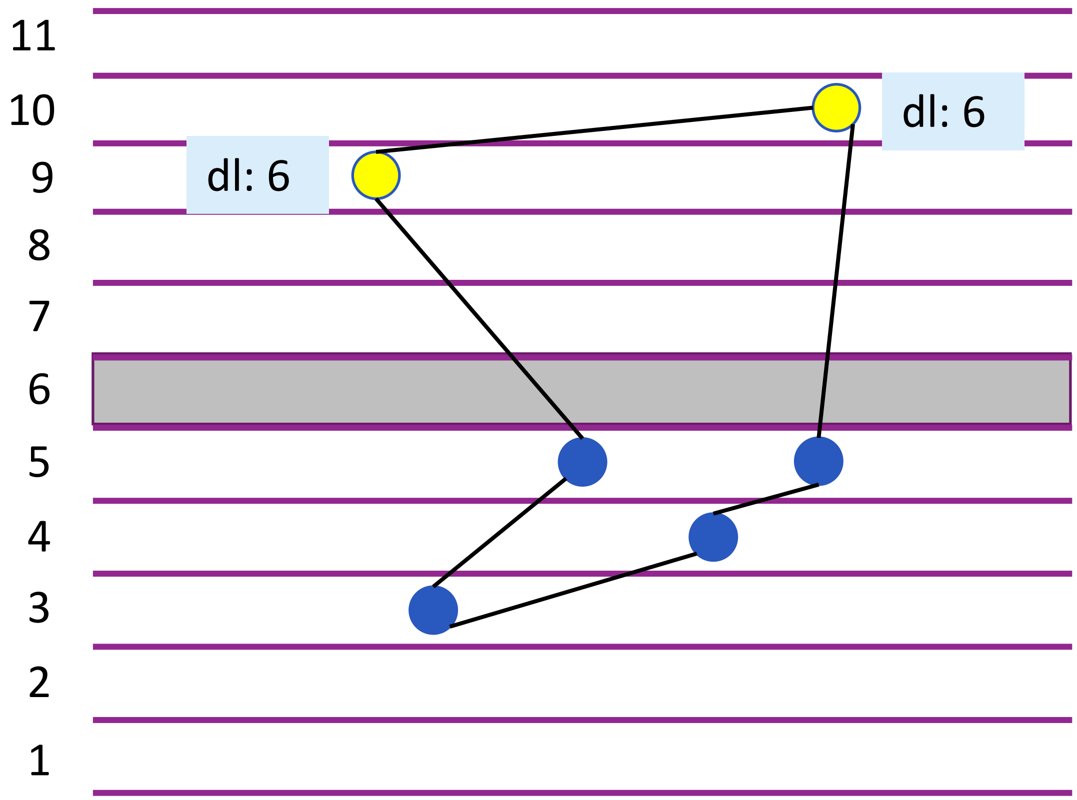

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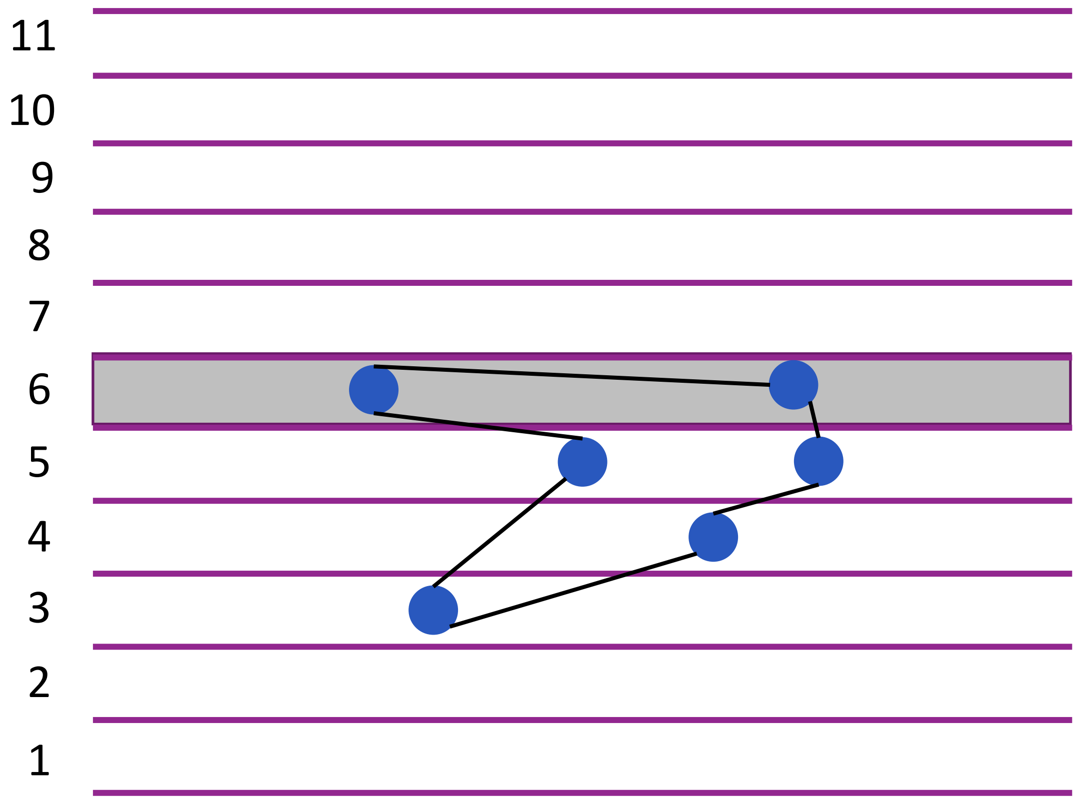

Update desire-levels of adjacent vertices at higher levels

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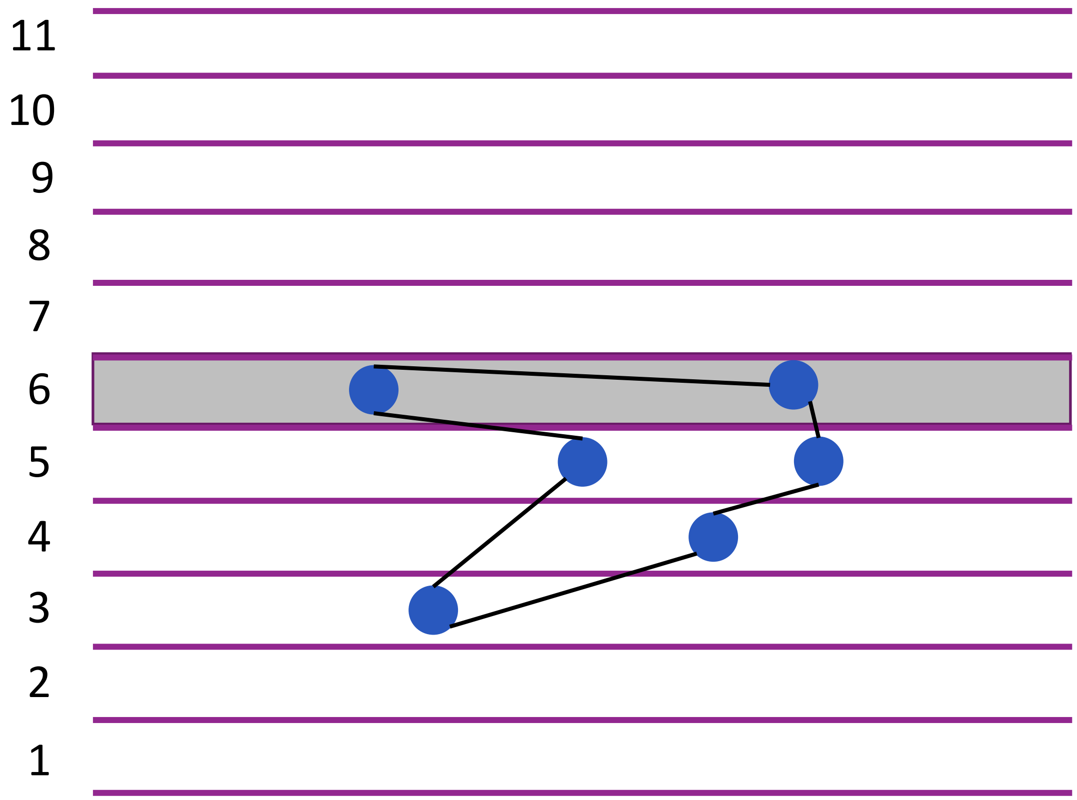
Our Parallel Batch-Dynamic Level Data Structure

No vertices need to be moved to a lower level than the current level!

Calculate *desire-level*: **closest level that satisfies invariants**

Deletions



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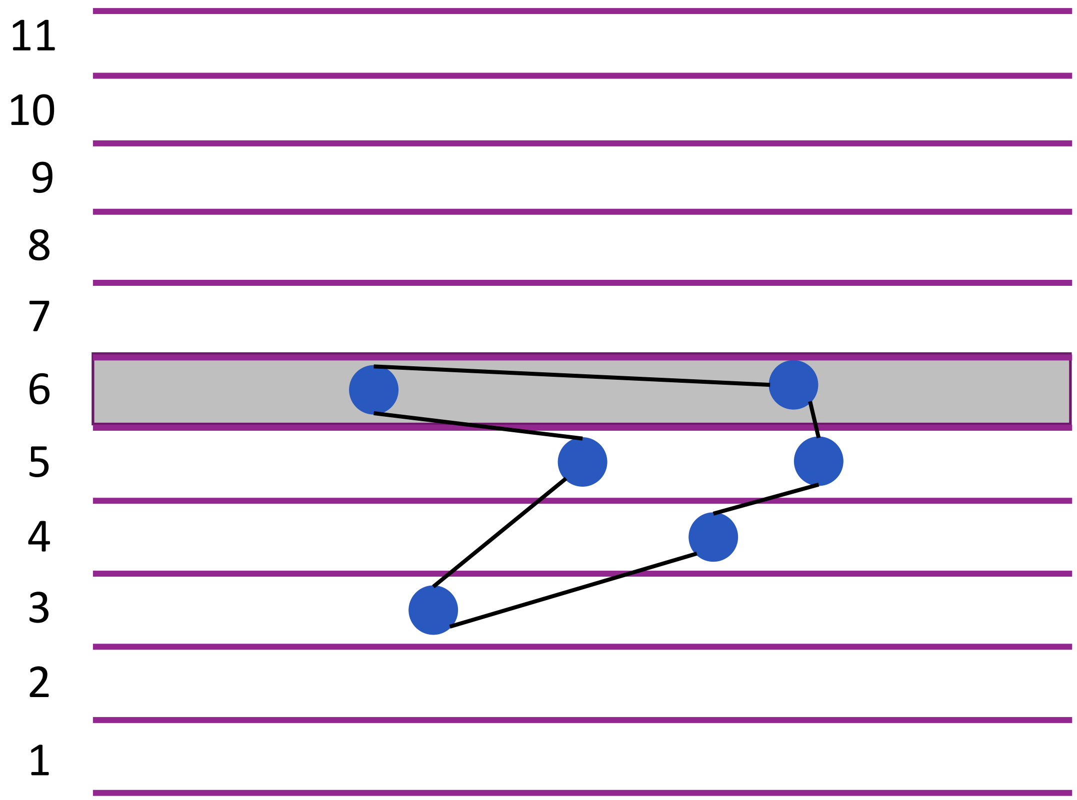

$O(\log^2 n \log\log n)$ depth w.h.p

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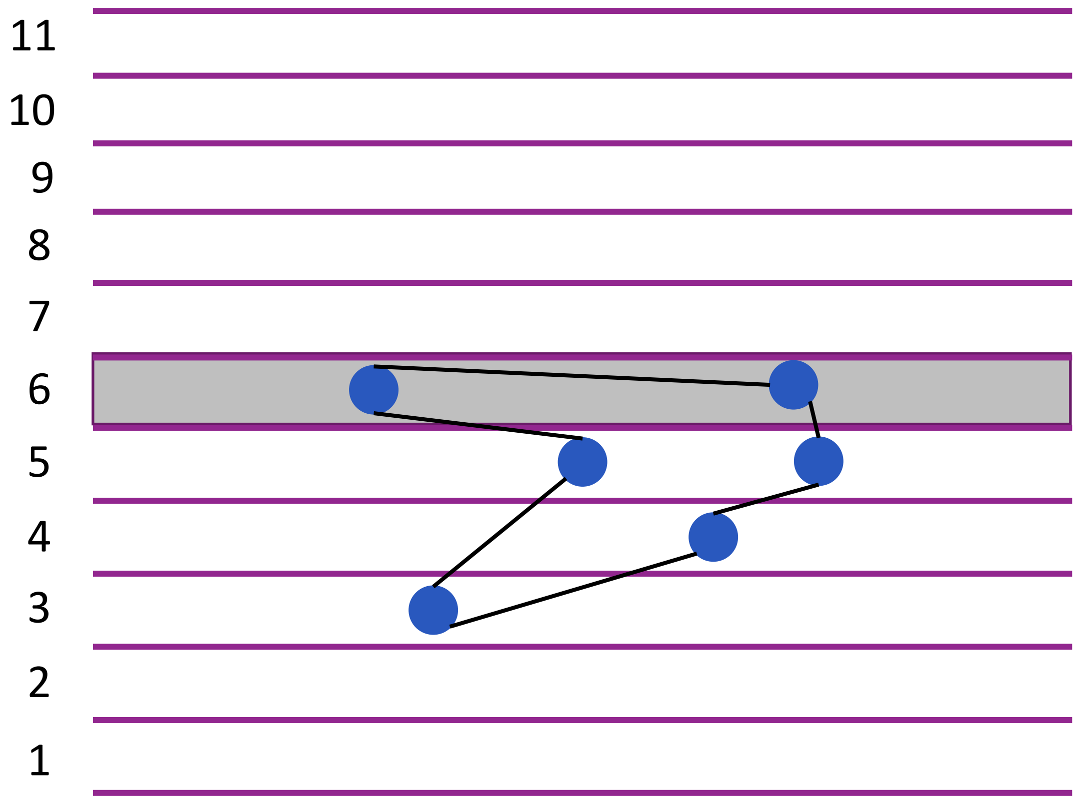

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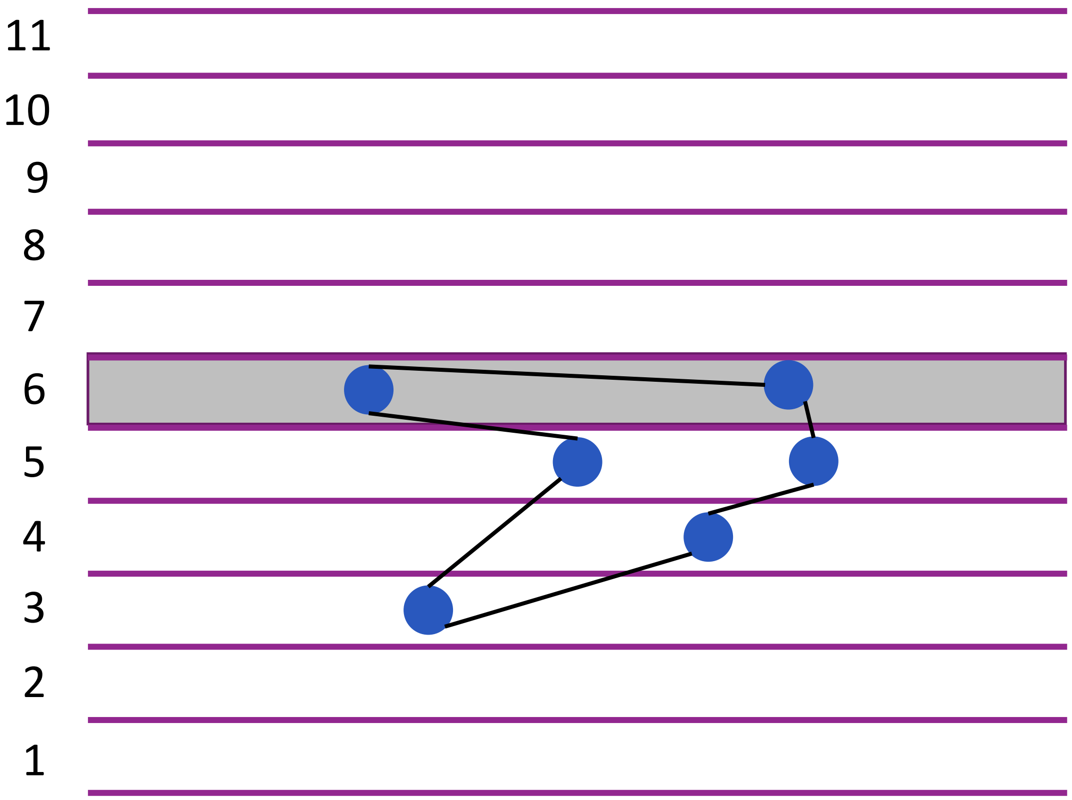
Calculate *desire-level*: **closest level that satisfies invariants**

Work-efficient

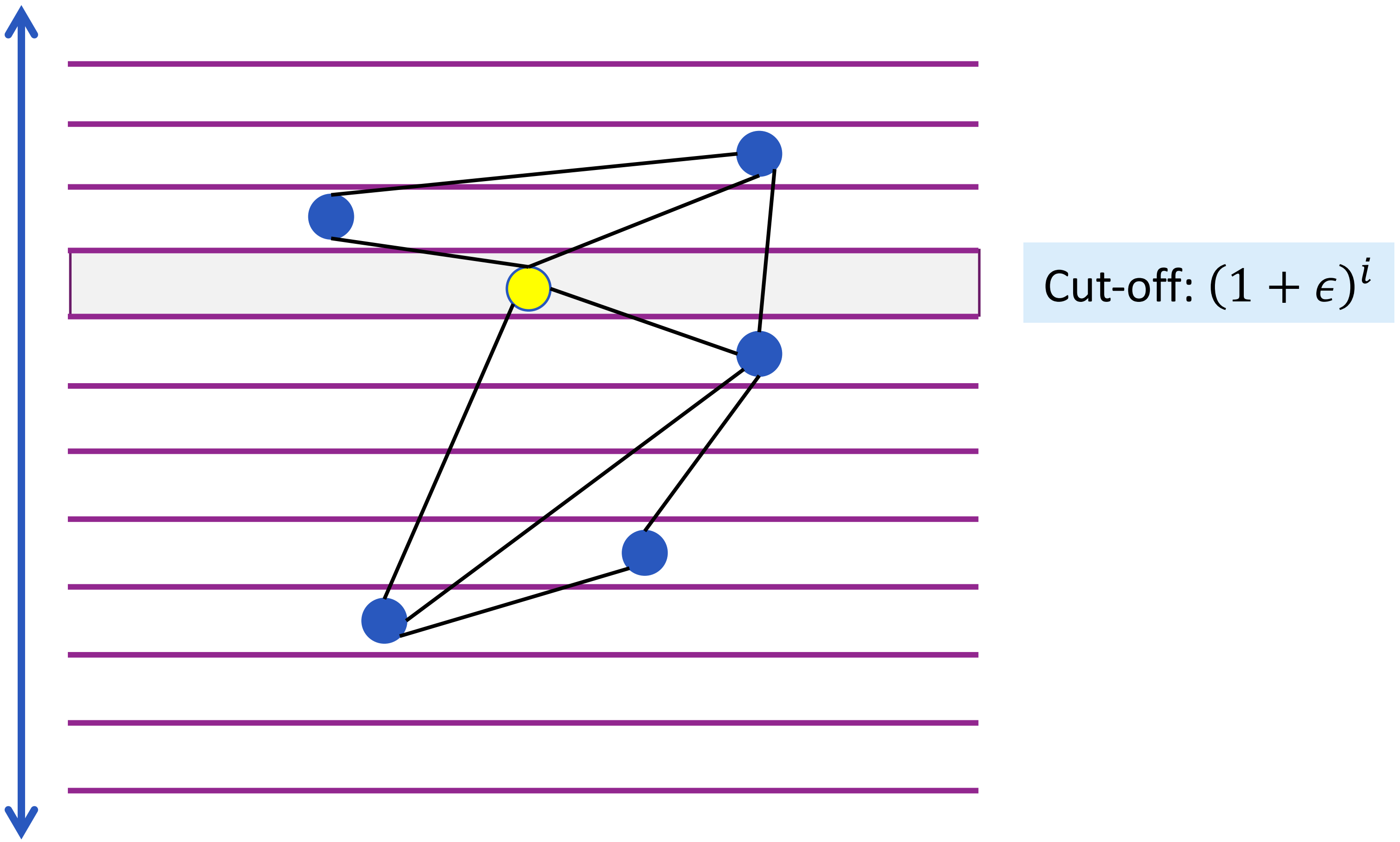


$O(\log^2 n \log\log n)$ depth w.h.p

Update desire-levels of adjacent vertices at higher levels

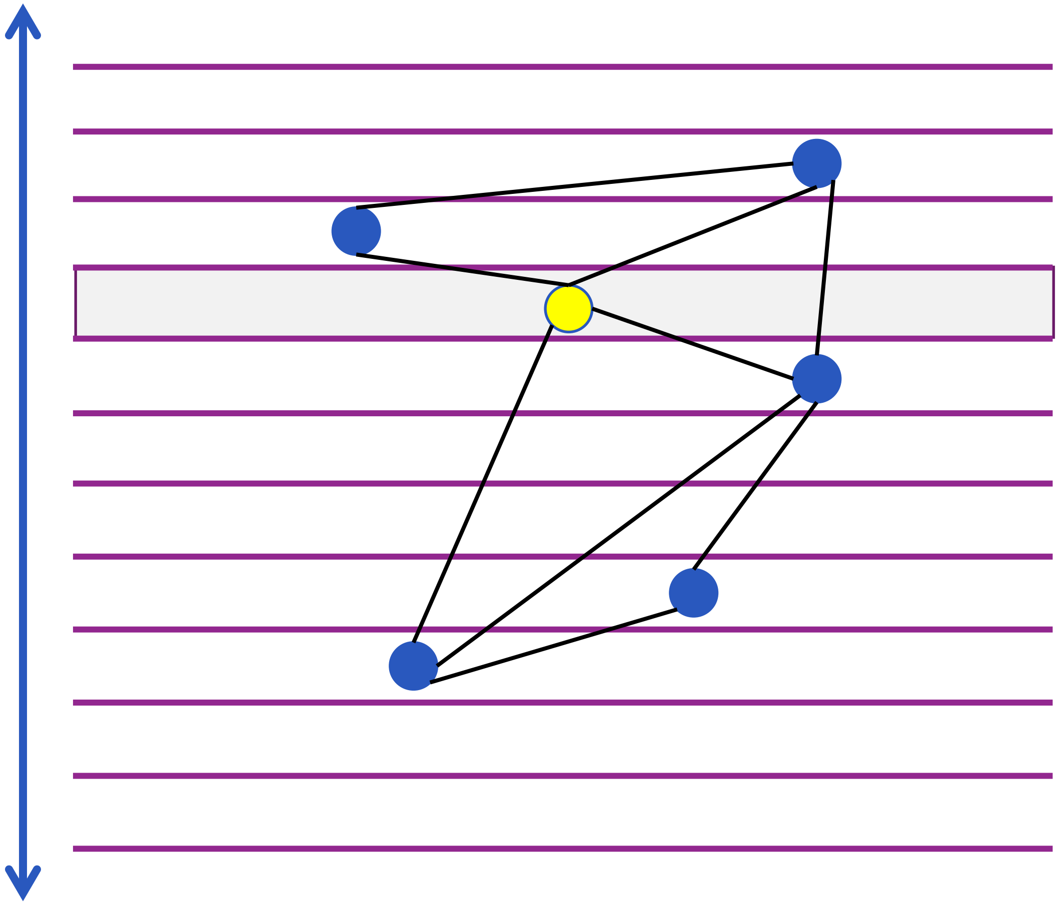
Only lower bound invariant, $(1+\epsilon)^i$, ever violated.

Proof of Our Approximation Factor



Proof of Our Approximation Factor

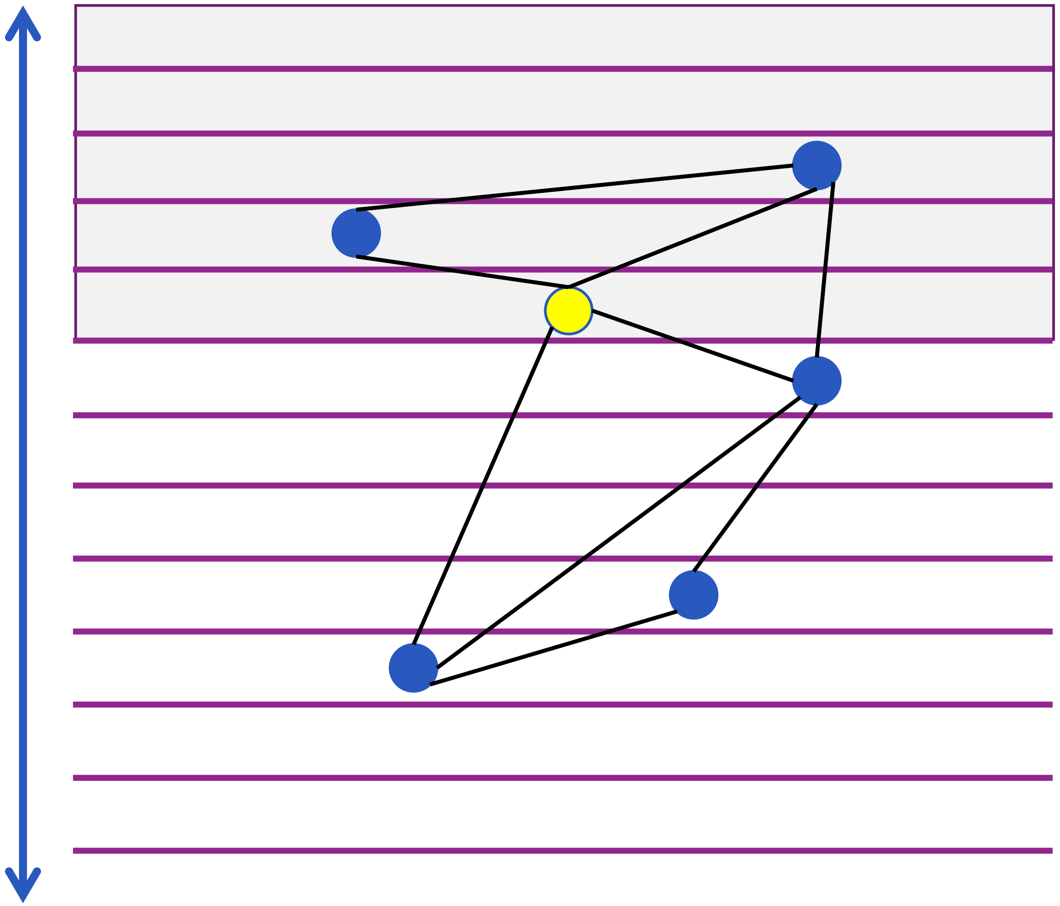
Coreness Estimate:
$(1+\epsilon)^i$



Cut-off: $(1+\epsilon)^i$

Proof of Our Approximation Factor: Upper Bound

Coreness Estimate:
$(1+\epsilon)^i$



Invariant: $\leq 2.1(1+\epsilon)^i$

Proof of Our Approximation Factor: Upper Bound

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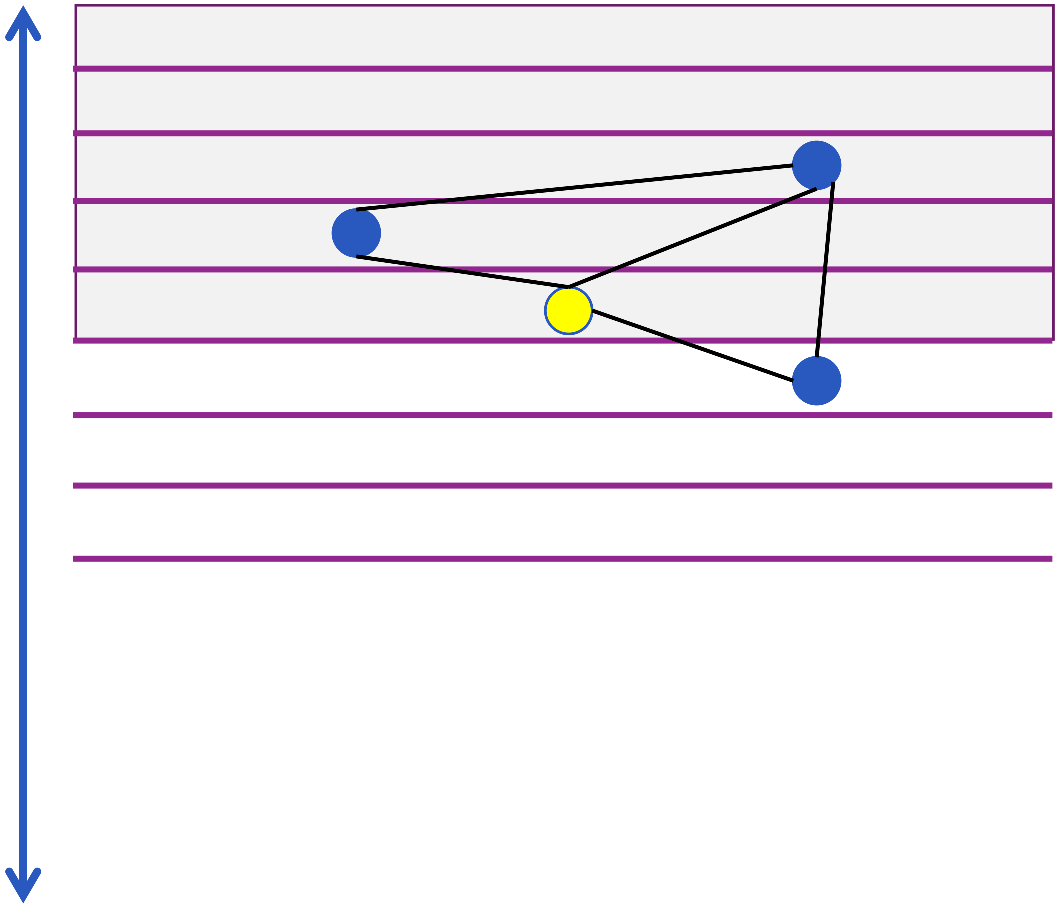

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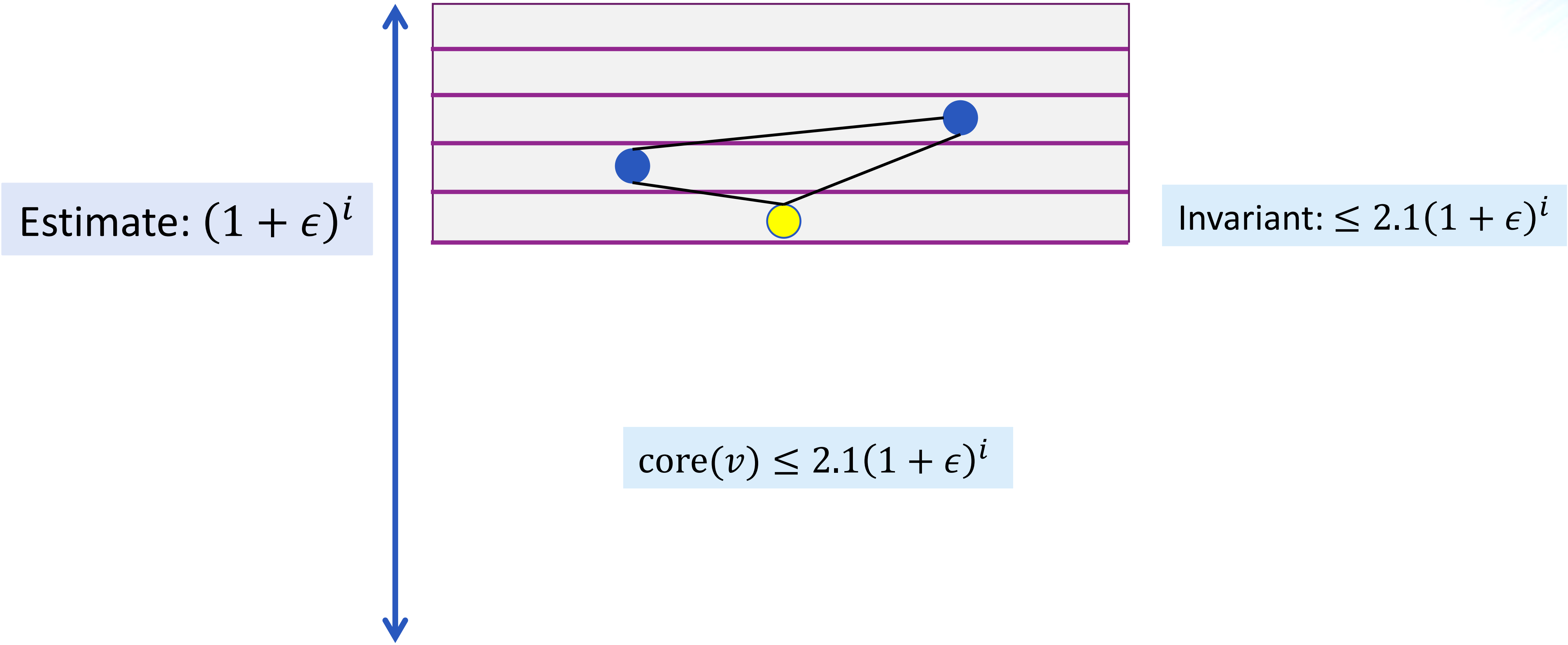

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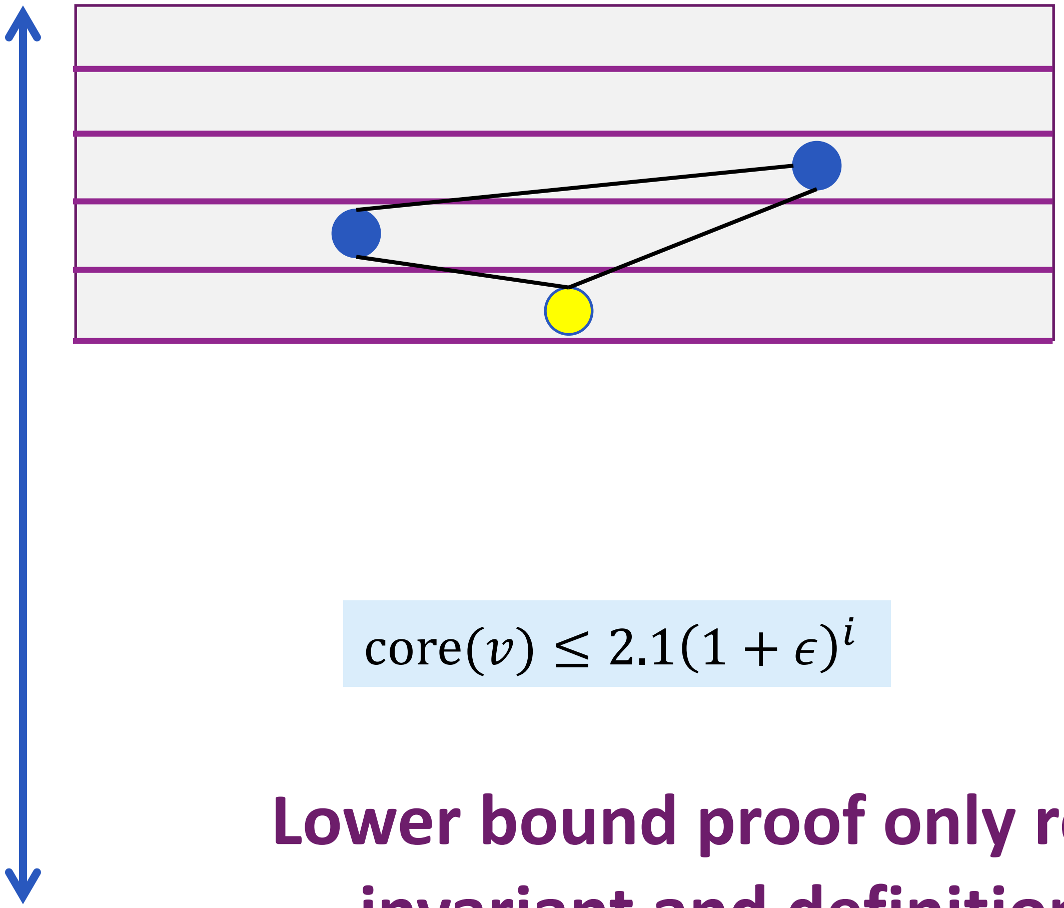

Estimate: $(1+\epsilon)^i$

Invariant: $\leq 2.1(1+\epsilon)^i$

$\text{core}(v) \leq 2.1(1+\epsilon)^i$

Lower bound proof only requires lower invariant and definition of *k-core.*

Tested Graphs

Graphs from Stanford SNAP database, DIMACS Shortest Paths challenge, and Network Repository—including some temporal

Graph	Num. Vertices	Num. Edges	Degeneracy (*k*)
dblp	425,957	2,099,732	101
brain-network	784,262	267,844,669	1200
wikipedia	1,140,149	2,787,967	124
youtube	1,138,499	5,980,886	51
stackoverflow	2,601,977	28,183,518	163
livejournal	4,847,571	85,702,474	329
orkut	3,072,627	234,370,166	253
usa-central	14,081,816	16,933,413	2
usa-road	23,072,627	28,854,312	3
twitter	41,652,231	1,202,513,046	2484
friendster	65,608,366	1,806,067,135	304

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Experiments

- c2-standard-60 Google Cloud instances
  - 30 cores with two-way hyper-threading
  - 236 GB memory
- m1-megamem-96 Google Cloud instances
  - 48 cores with two-way hyperthreading
  - 1433.6 GB memory
- Timeout: 3 hours
- Insertion/Deletion batches generated from random permutation, regular intervals

Graph Based Benchmark Suite:
https://github.com/ParAlg/gbbs

Runtimes/Accuracy Against State-of-the-Art Algorithms

https://github.com/qqliu/batch-dynamic-kcore-decomposition

Sun et al. TKDD: current best **sequential, approx** dynamic algorithm

Hua et al. TPDS: current best **parallel, exact** dynamic algorithm

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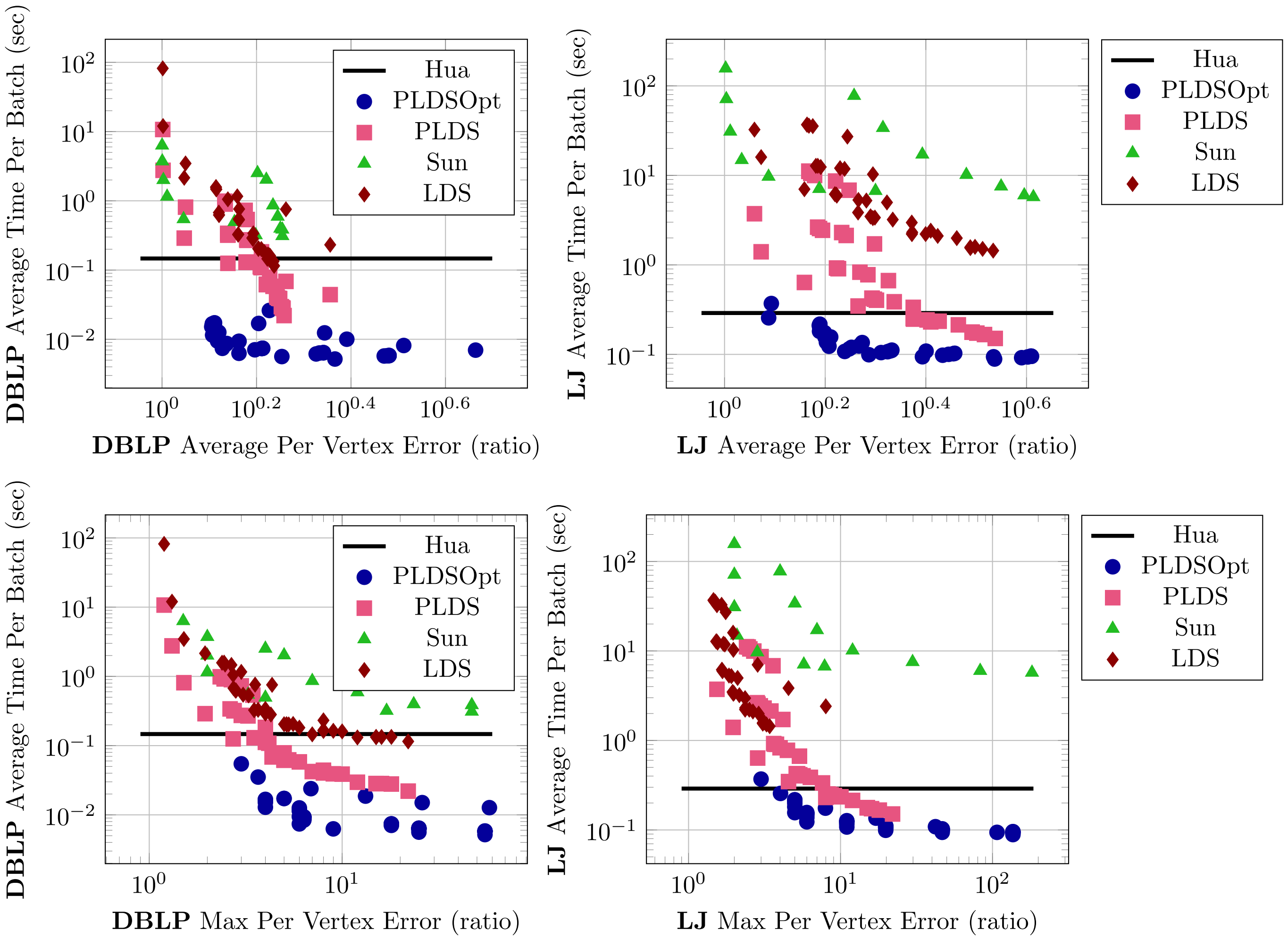

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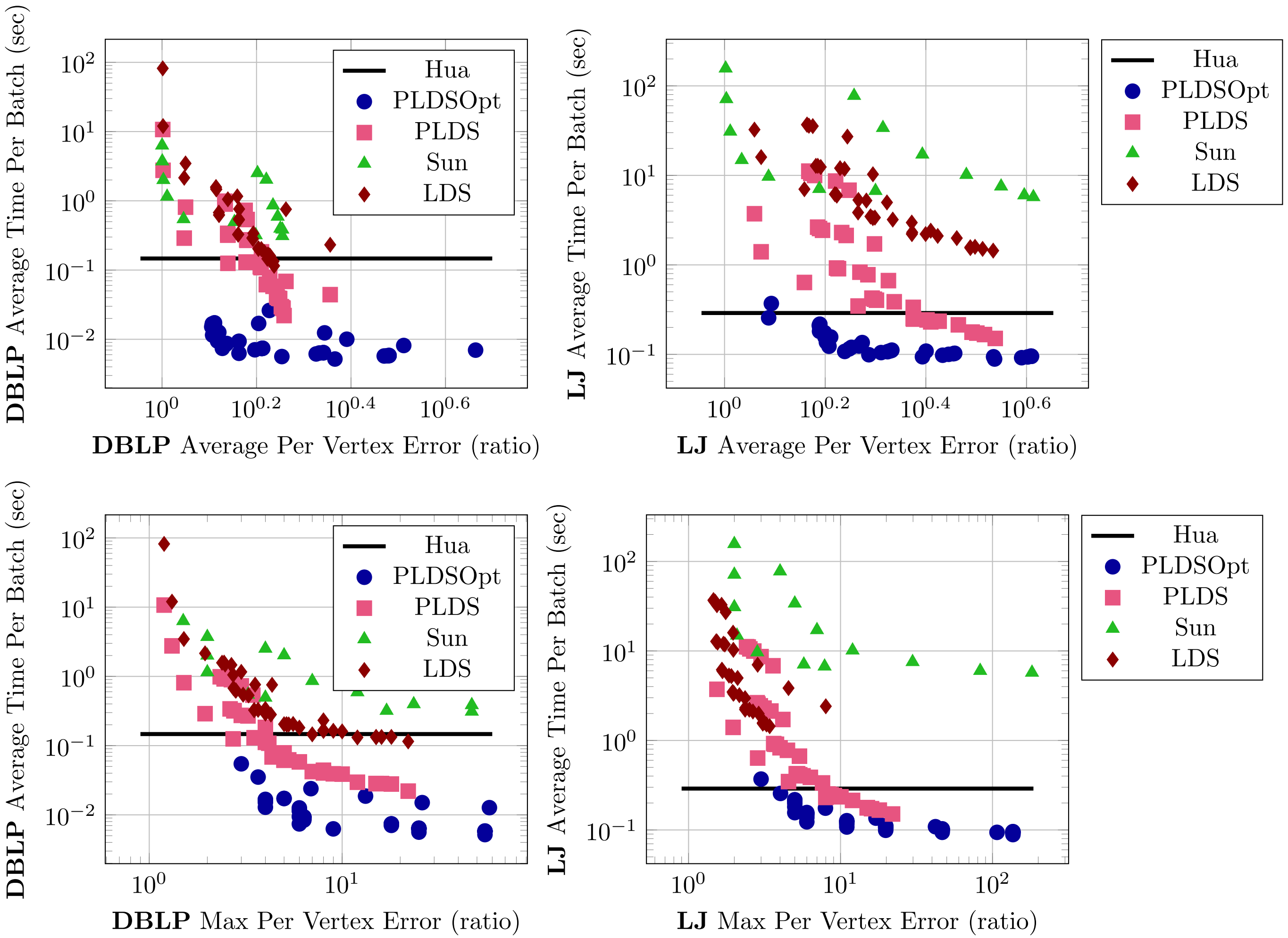

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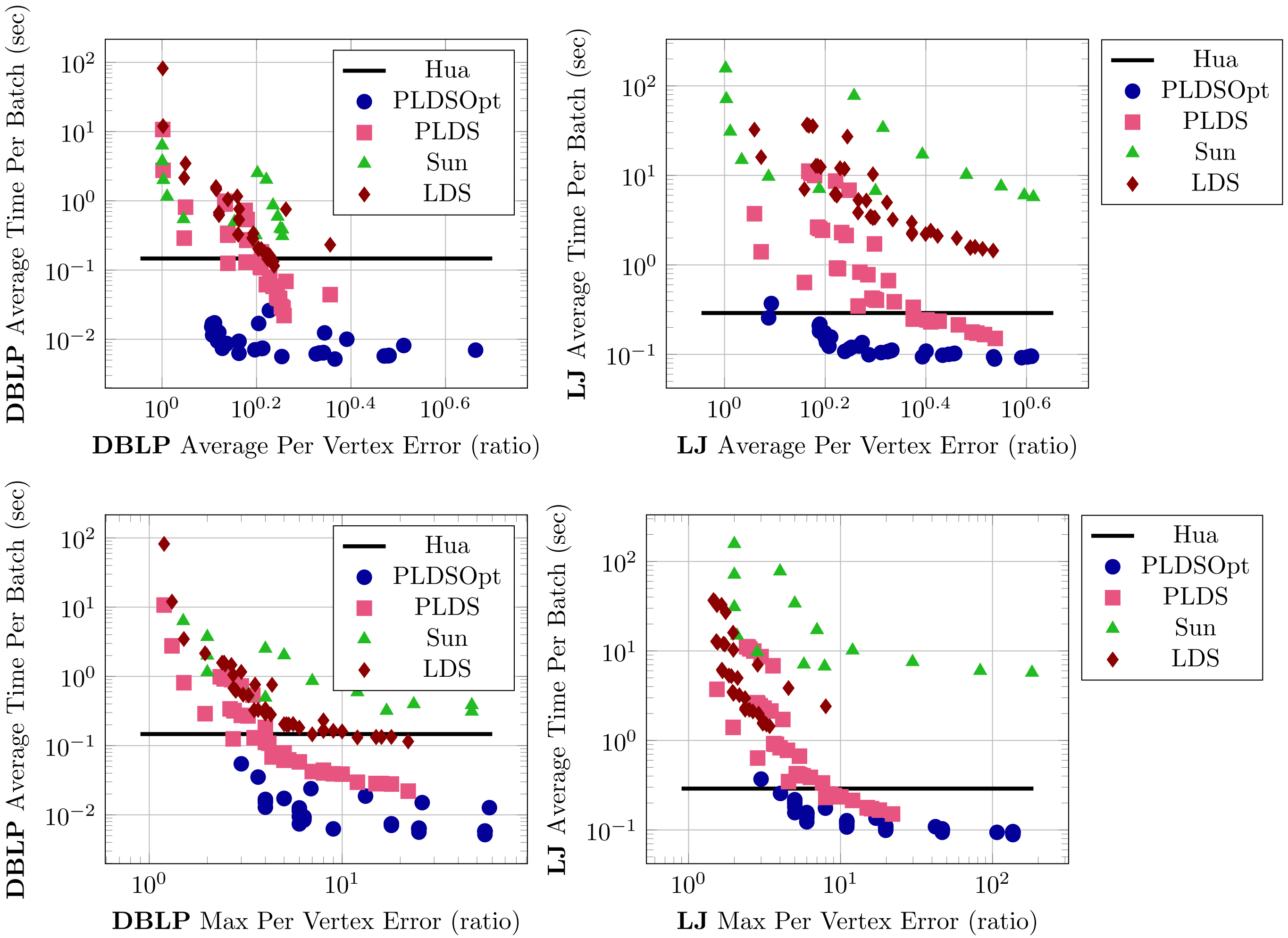

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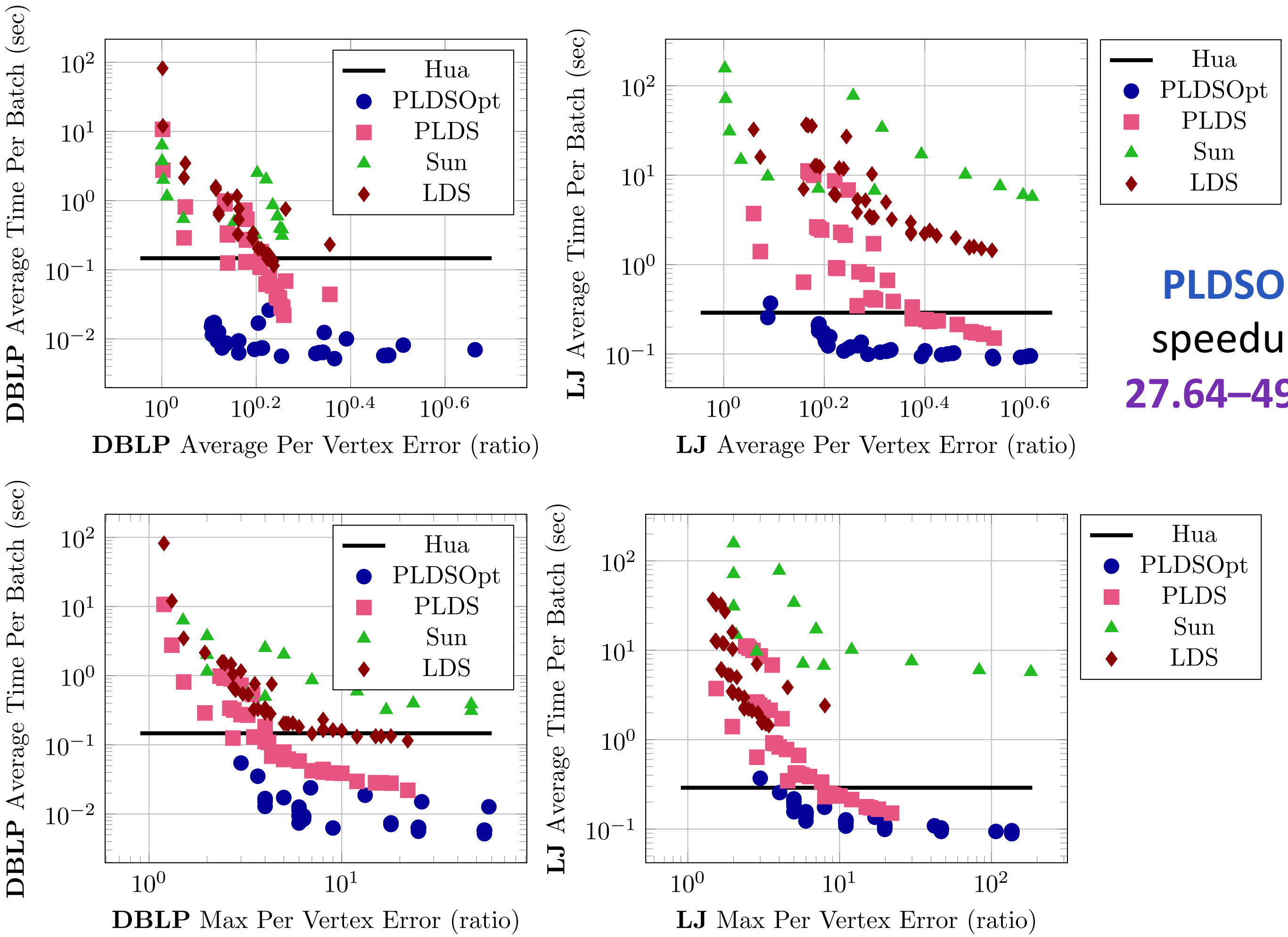

PLDSOpt: 22.35–195.82x speedup over Sun on dblp, **27.64–497.63x** speedup on LJ

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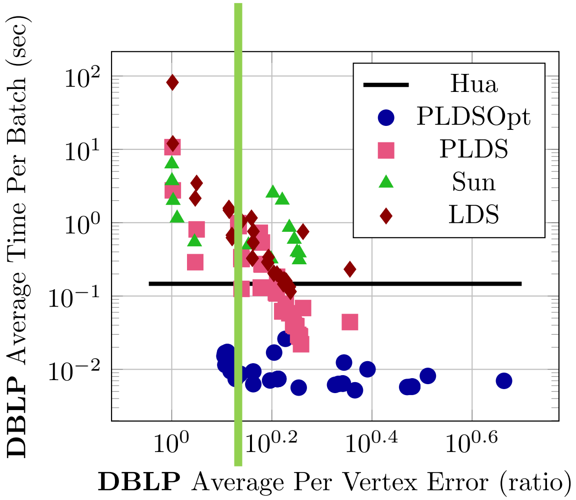
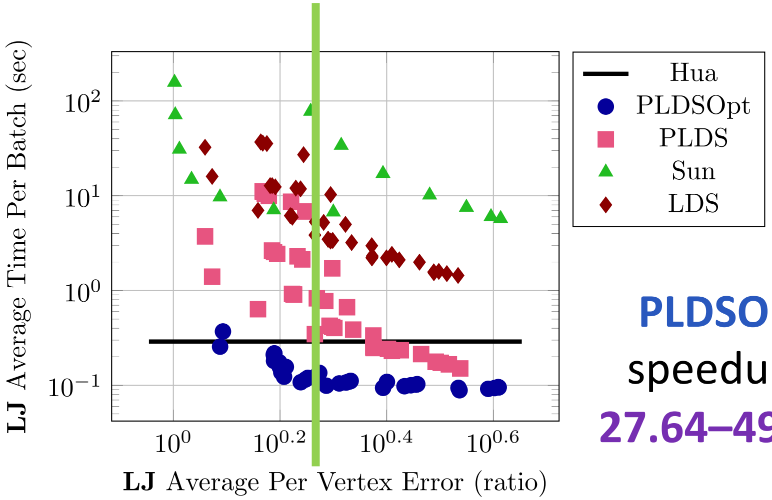
PLDSOpt: 24.5x over Hua



PLDSOpt: 2.68x over Hua



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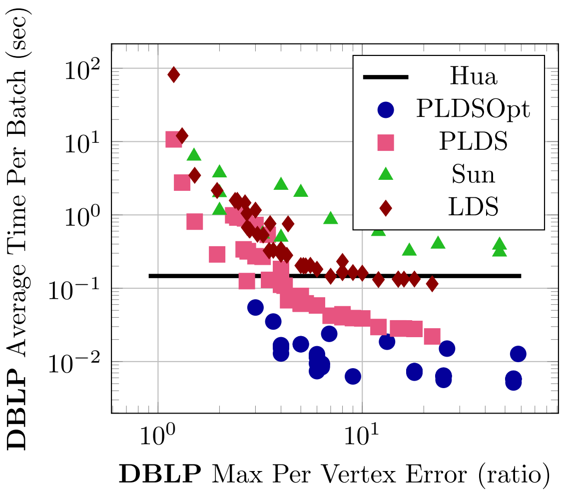

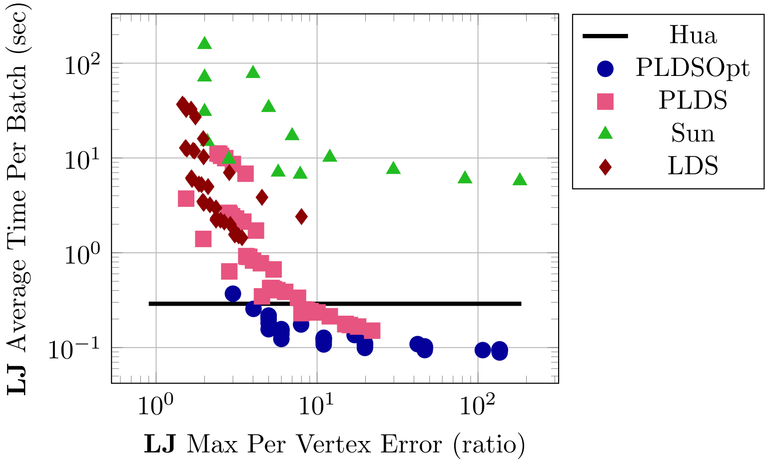

Number of Workers

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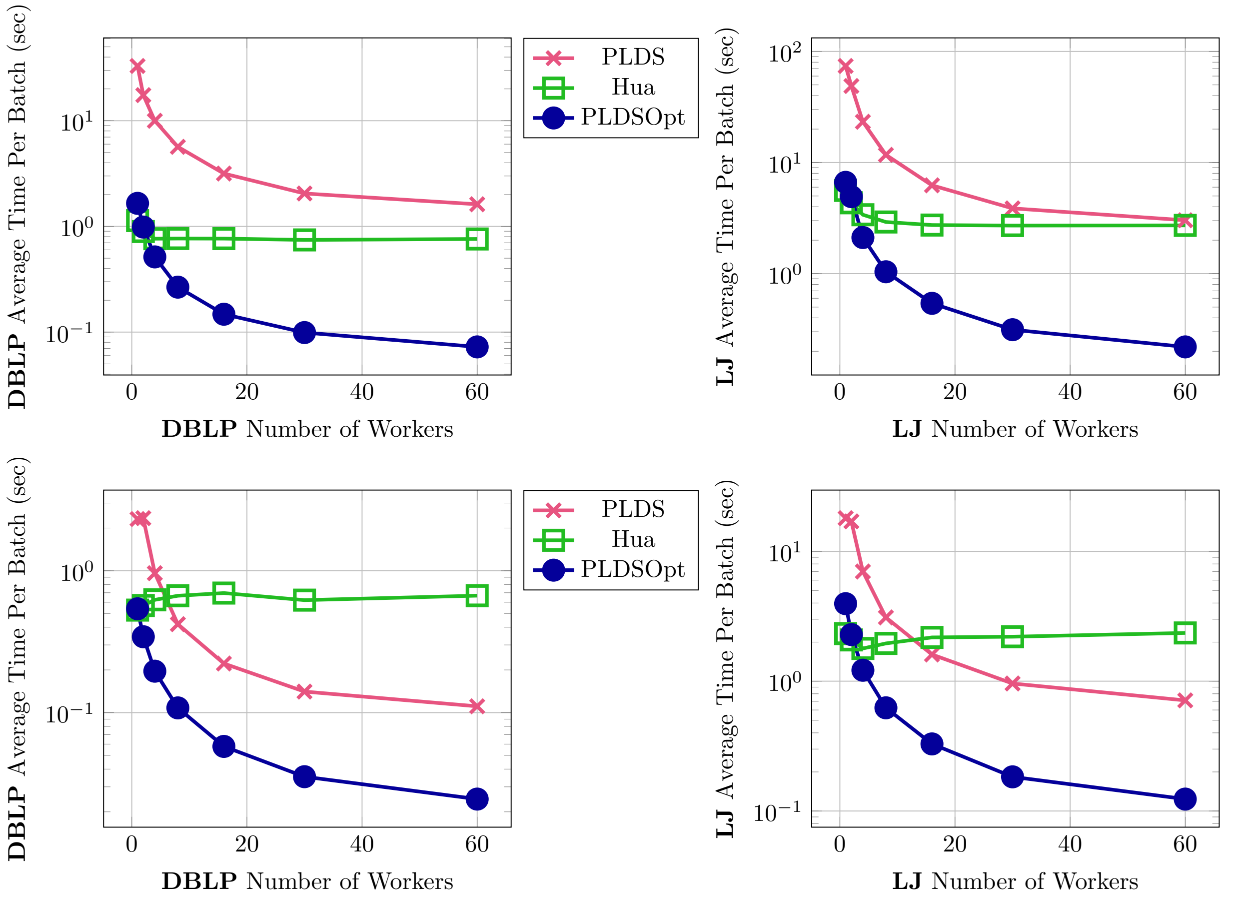

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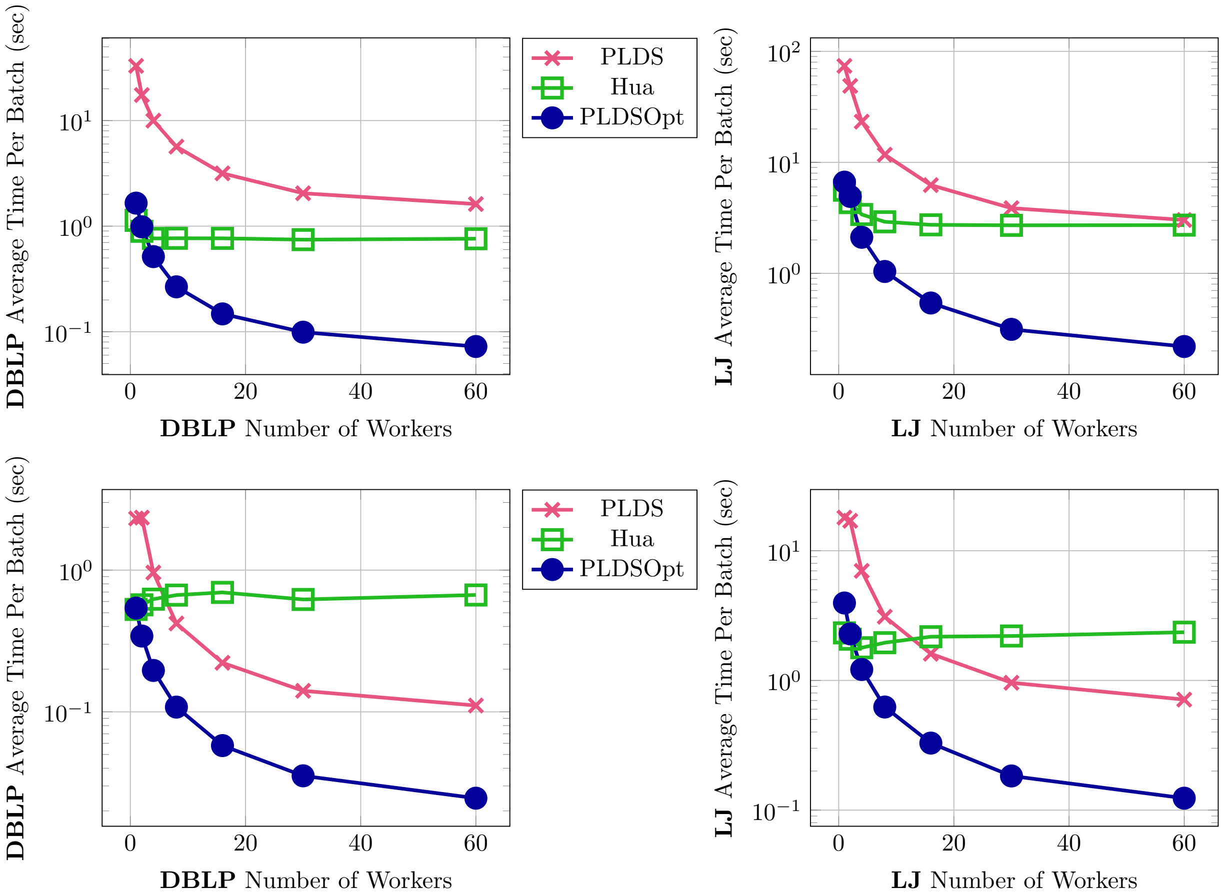

PLDSOpt: 30.28x
self-relative speedup

PLDS: 26.46x

Hua: 2.07x

On Variety of Graphs

Graph Dataset	PLDSOpt Avg.	PLDSOpt Max	PLDS Avg.	PLDS Max	Approx KCore Avg.	Approx KCore Max
dblp	1.9345	3	2.635	4	1.15	3.875
brain	1.834	3	3.363	4.193	1.315	4.305
wiki	1.590	3	1.780	4.172	1.010	3
youtube	1.359	3	1.593	4	1.1283	3.75
stackoverflow	1.826	3	2.272	4.067	1.048	3.875
livejournal	1.660	3	2.321	4.175	4.175	1.165
orkut	1.926	3	3.115	4.175	1.204	4.2
ctr	1.601	3	1.683	3	1.374	3
usa	1.826	3	1.683	3	1.379	3
twitter	2.118*	3*	T.O.	T.O.	T.O.	T.O.
friendster	1.851*	3*	T.O.	T.O.	T.O.	T.O.

Graph Dataset	PLDSOpt Avg.	PLDSOpt Max	PLDS Avg.	PLDS Max	Approx KCore Avg.	Approx KCore Max
dblp	1.187	6	1.507	2	1.236	3.0
brain	1.575	6	1.943	4.186	1.315	5.0
wiki	1.423	4	1.494	4	1.013	3.875
youtube	1.268	4	1.317	4	1.137	3.706
stackoverflow	1.630	6	1.792	4.172	1.045	3.908
livejournal	1.613	6	1.704	4.14	1.167	3.984
orkut	1.681	6	1.913	4.175	1.205	4.2
ctr	1.243	3	1.257	3	1.524	3.0
usa	1.253	3	1.278	3	1.522	3.0
twitter	1.893*	4*	T.O.	T.O.	T.O.	T.O.
friendster	1.685*	3*	T.O.	T.O.	T.O.	T.O.

On Variety of Graphs

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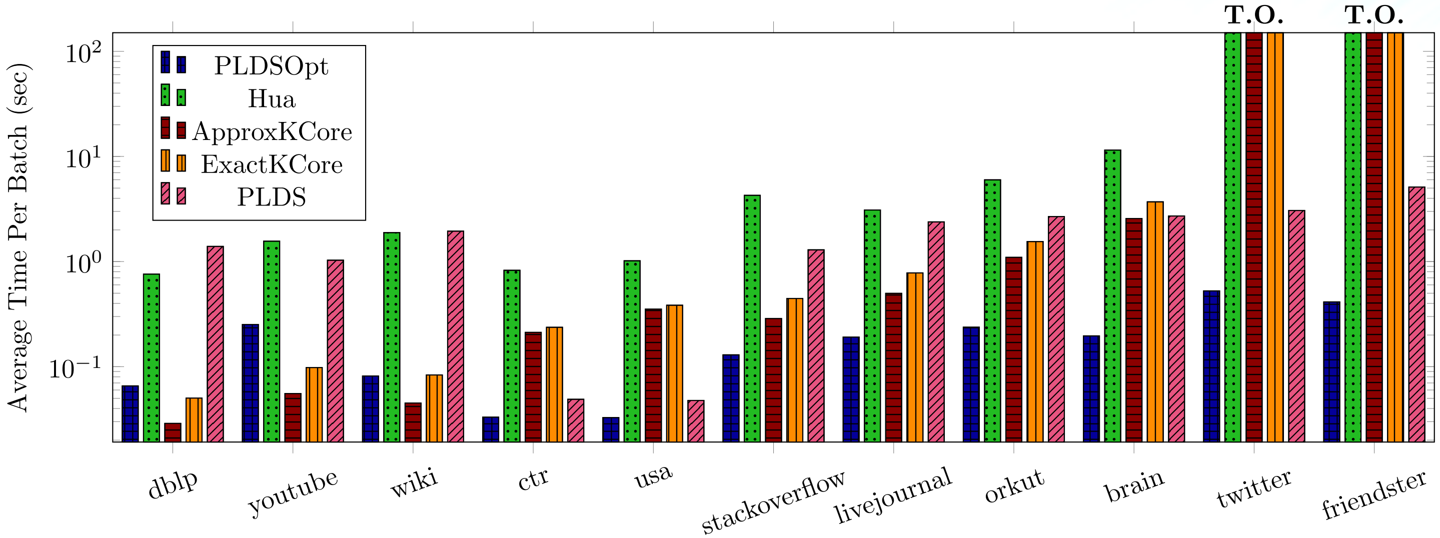

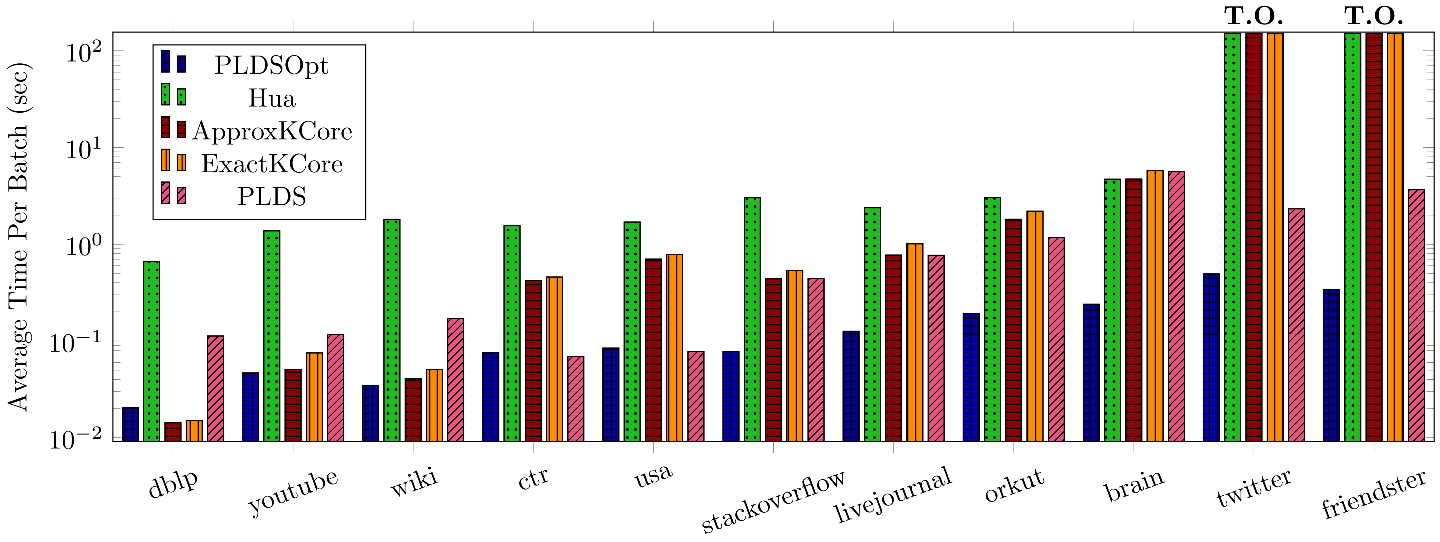

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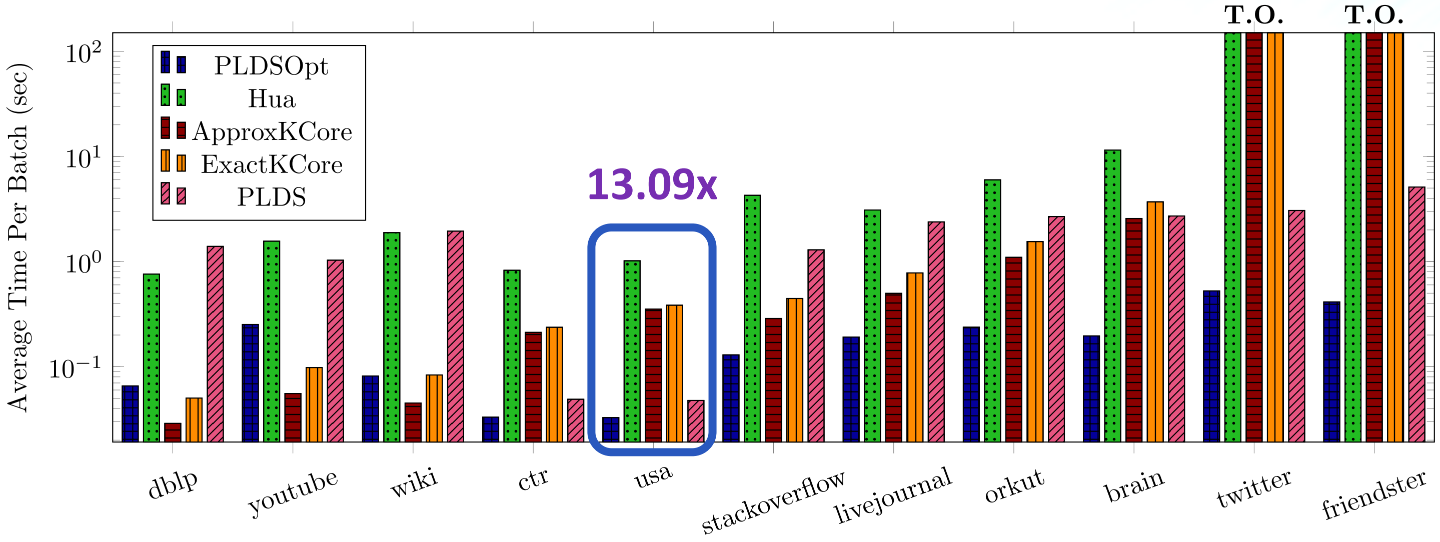

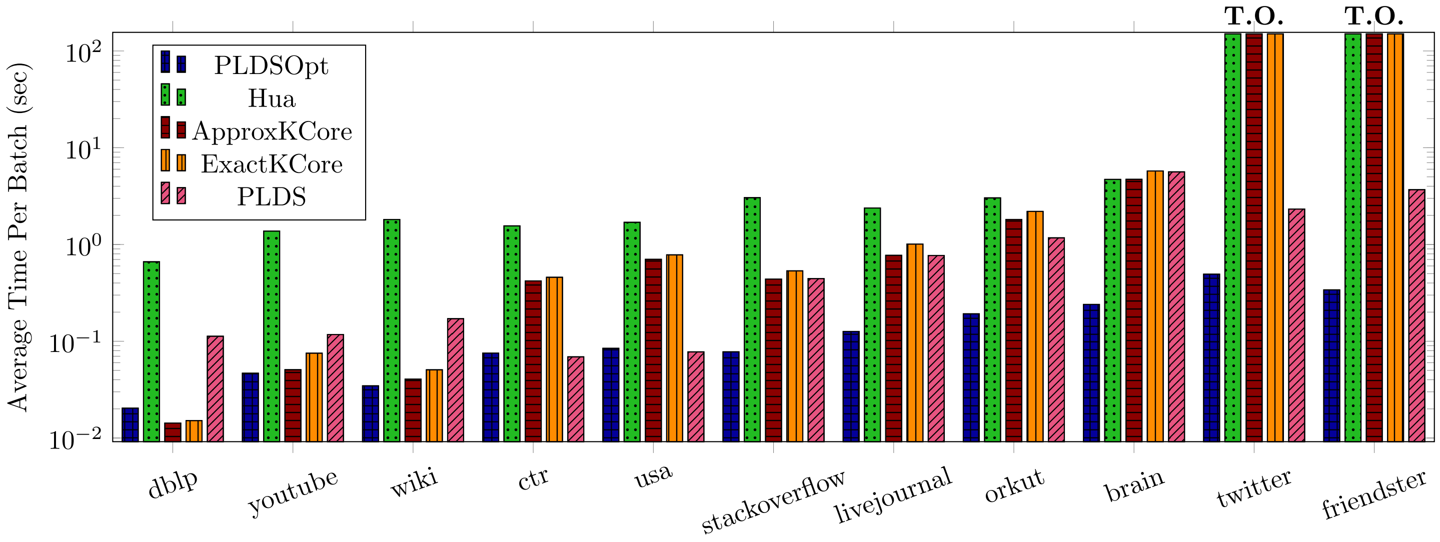

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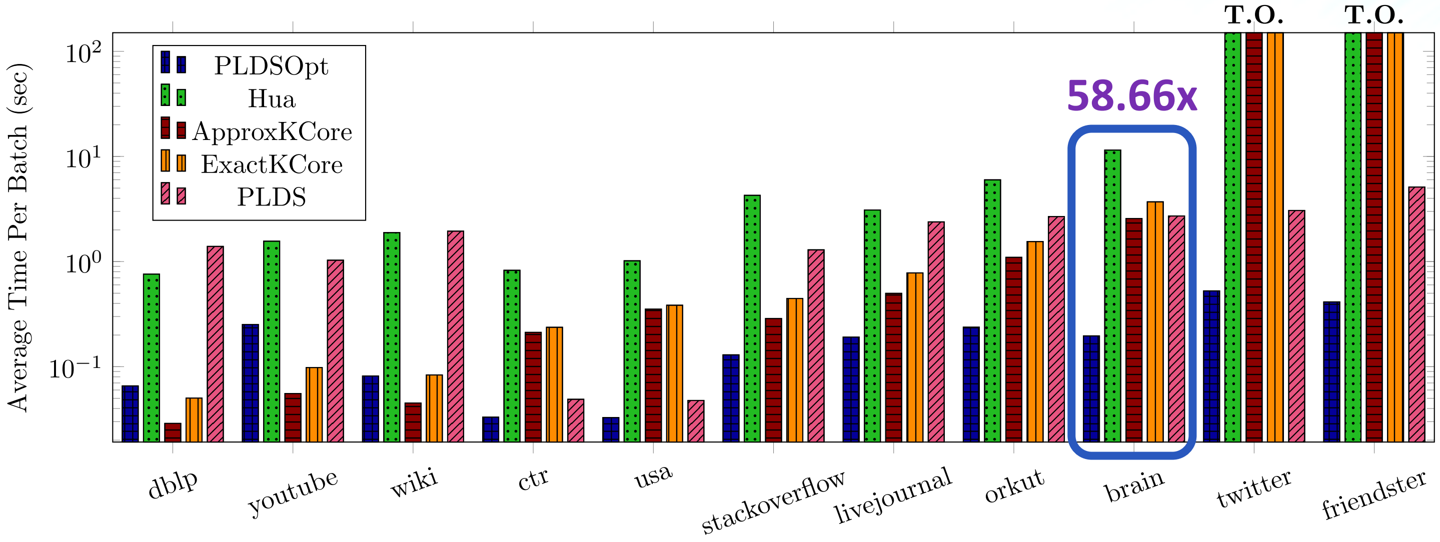

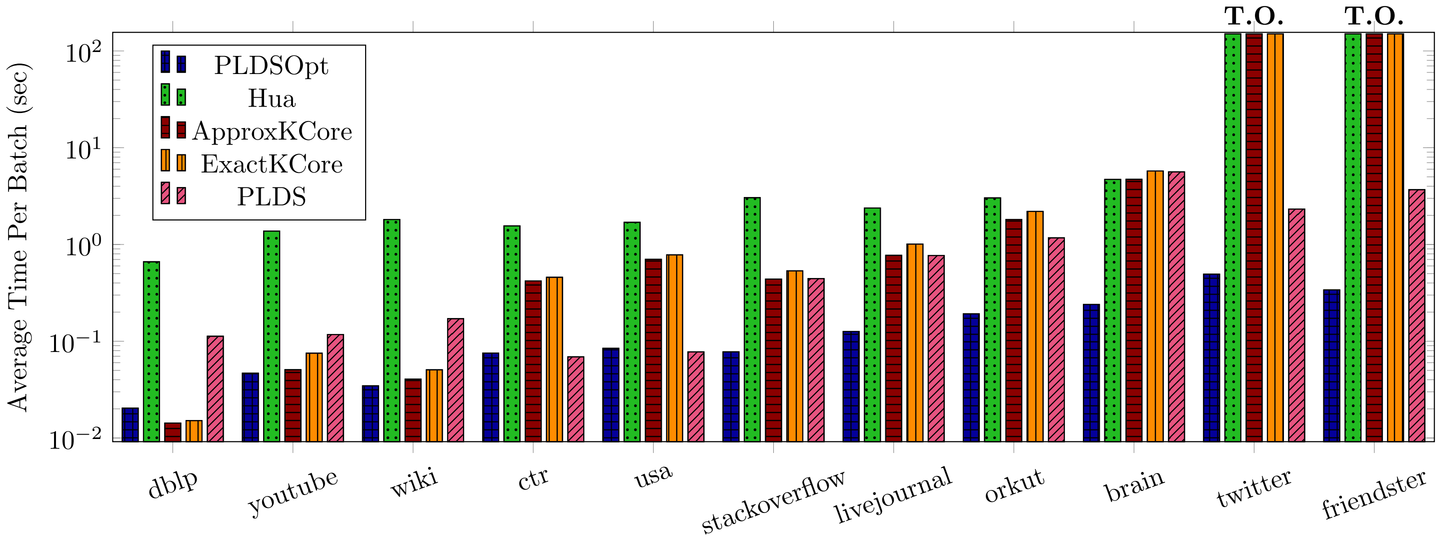

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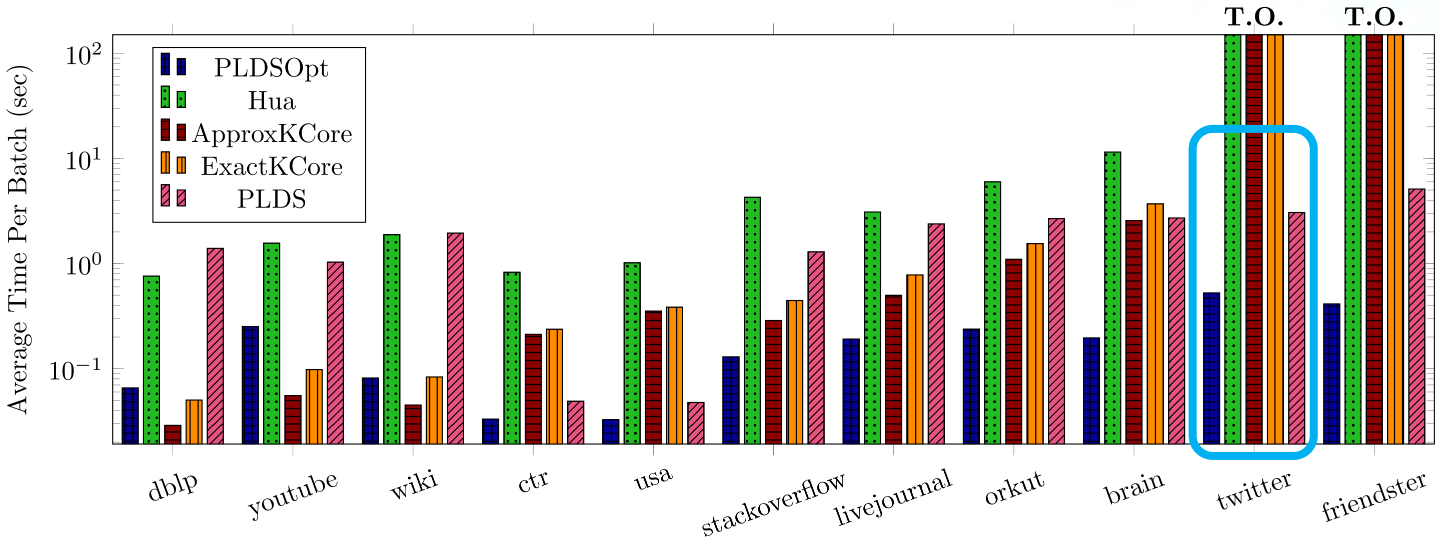
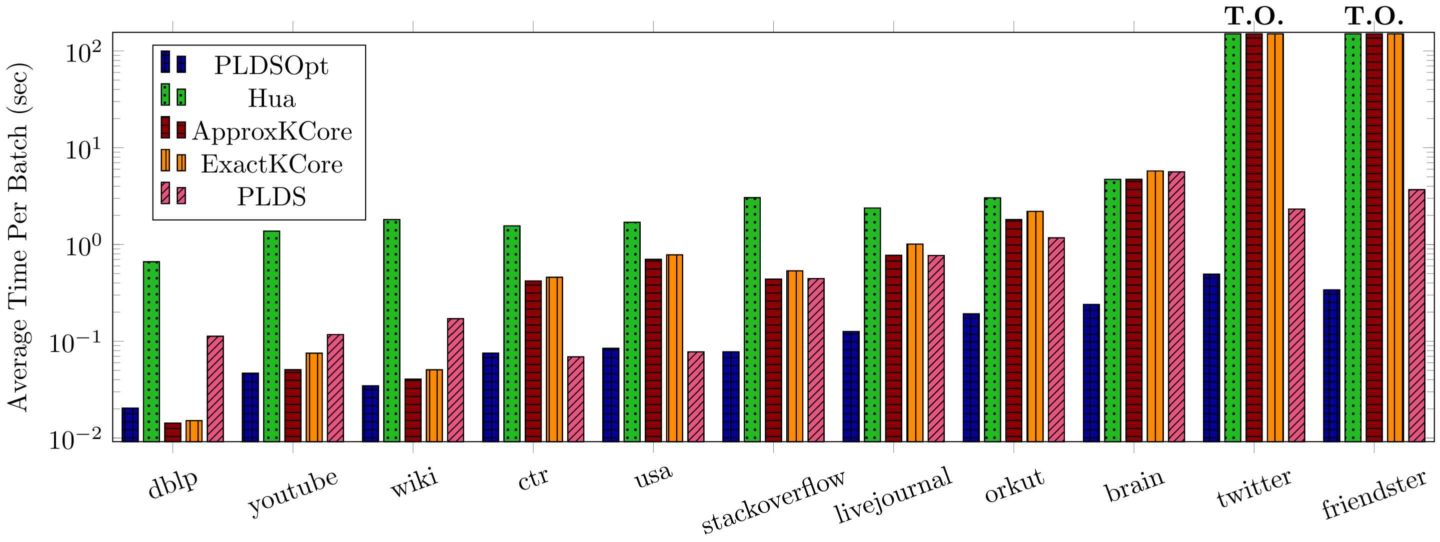


PLDSOpt: 0.53 sec per batch

PLDS: 3.06 sec per batch



Summary of Results

Static Algorithms

Structural Rounding

Structural Rounding: Approximation Algorithms for Graphs Near an Algorithmically Tractable Class

Erik D. Demaine[1], Timothy D. Goodrich[2], Kyle Kloster[2], Brian Lavallee[2], Quanquan C. Liu[1], Blair D. Sullivan[2], Ali Vakilian[1], and Andrew van der Poel[2]

Scheduling with Communication Delay

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MPC Algorithms for Subgraph Counting

Parallel Algorithms for Small Subgraph Counting

Amartya Shankha Biswas[*†], Talya Eden[*‡], Quanquan C. Liu[*], Slobodan Mitrović[*§], Ronitt Rubinfeld[¶]

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Parallel Dynamic k-Core Decomposition

Parallel Batch-Dynamic k-Core Decomposition

Quanquan C. Liu, MIT CSAIL, quanquan@mit.edu; Jessica Shi, MIT CSAIL, jeshi@mit.edu; Shangdi Yu, MIT CSAIL, shangdiy@mit.edu; Laxman Dhulipala, MIT CSAIL, laxman@mit.edu; Julian Shun, MIT CSAIL, jshun@mit.edu

Dynamic Vertex Coloring

Fully Dynamic $(\Delta + 1)$-Coloring in Constant Update Time

Sayan Bhattacharya[*], Fabrizio Grandoni[†], Janardhan Kulkarni[‡], Quanquan C. Liu[§], Shay Solomon[¶]

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Lower Bounds/Constructions

Hardness from Pebbling

Red-Blue Pebble Game: Complexity of Computing the Trade-Off between Cache Size and Memory Transfers

Erik D. Demaine, Quanquan C. Liu

Static-Memory-Hard Hash Functions

Static-Memory-Hard Functions and Nonlinear Space-Time Tradeoffs via Pebbling

Thaddeus Dryja, Quanquan C. Liu, Sunoo Park

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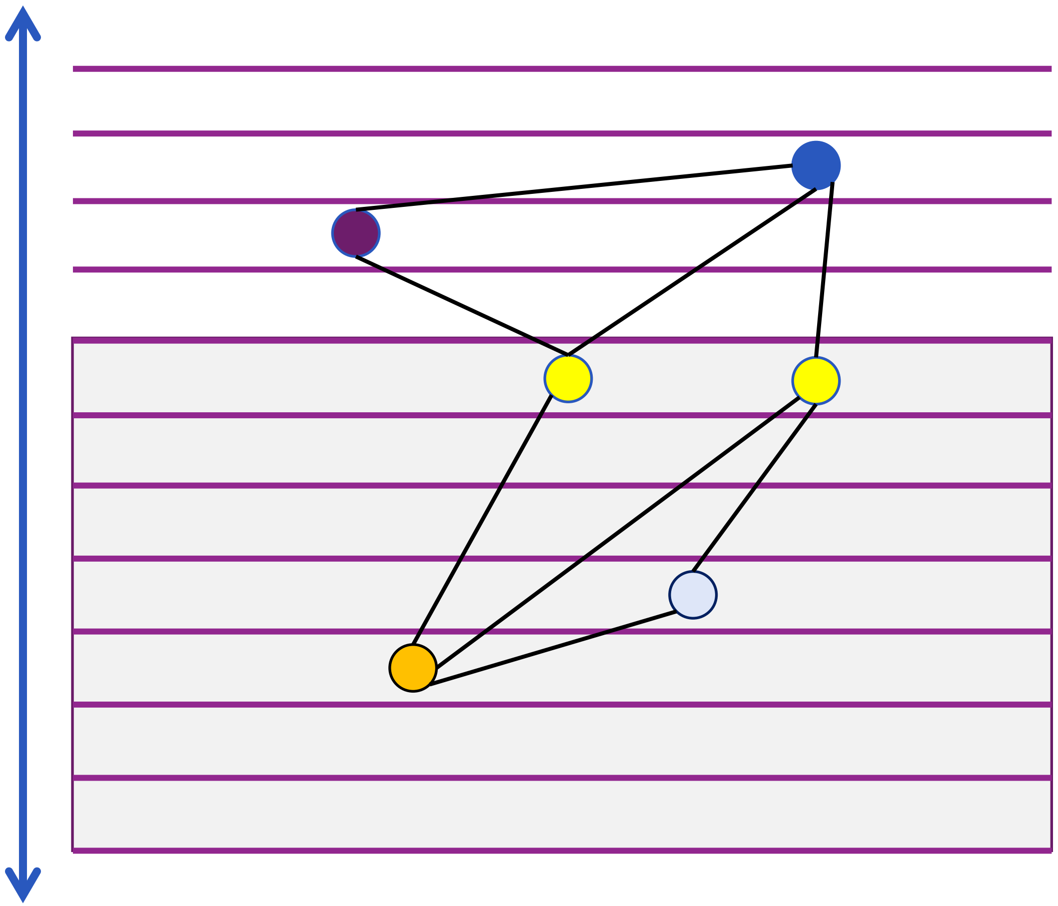
Level Data Structure

Δ := Max Degree



$O(\log \Delta)$

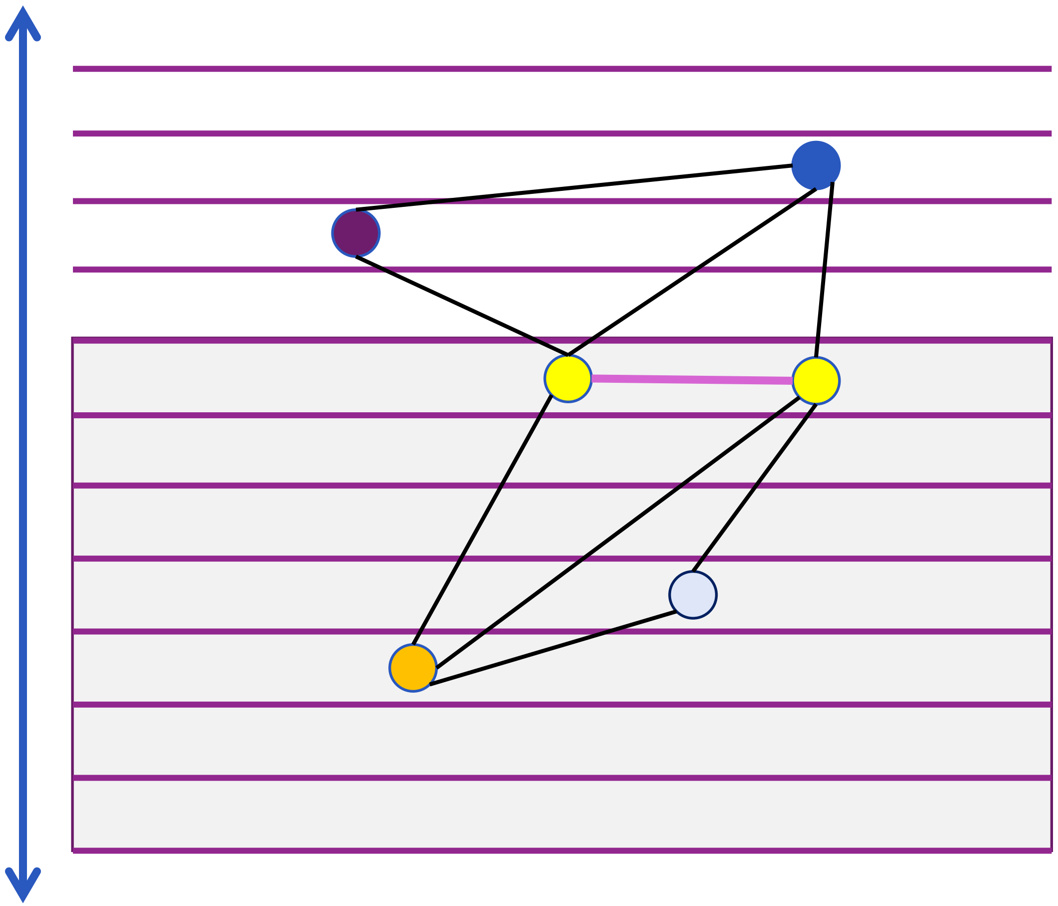
Vertices partitioned into levels

neighbors: $< 3^{\ell(v)+2}$

Level Data Structure

Δ := Max Degree

$O(\log \Delta)$



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= Edge Insertion

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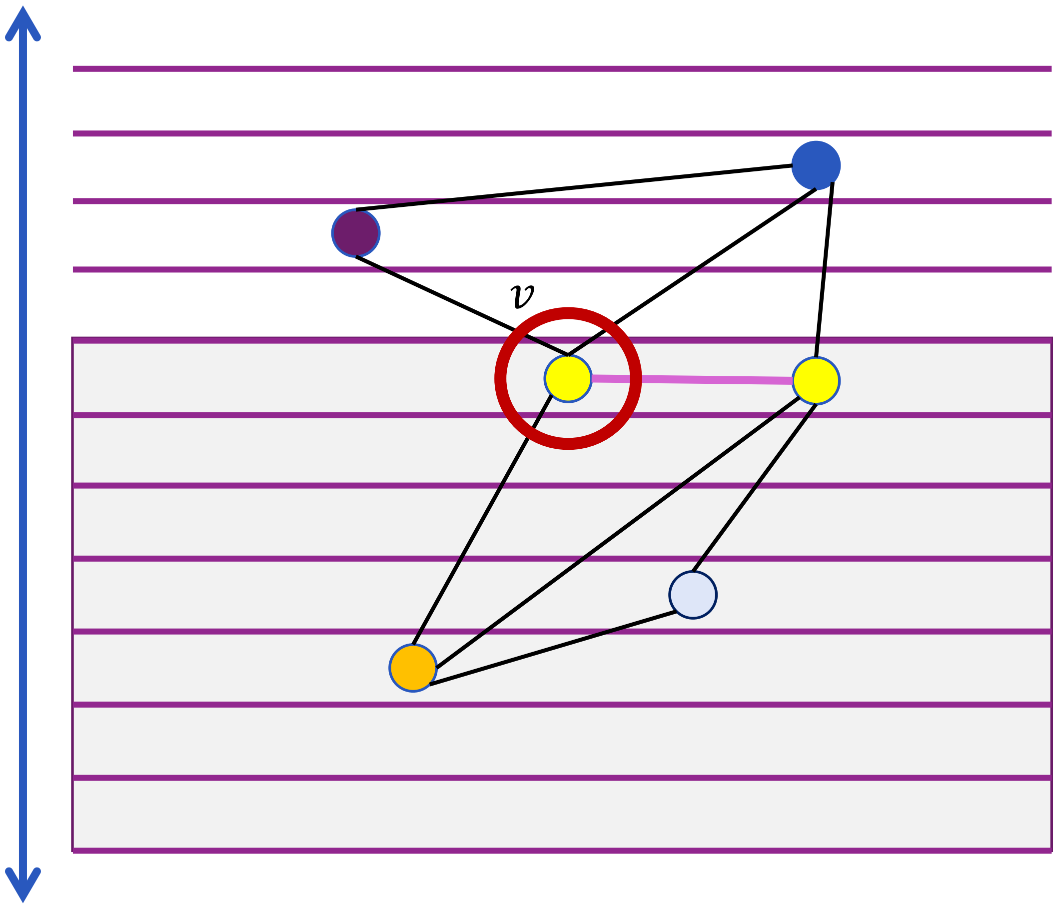

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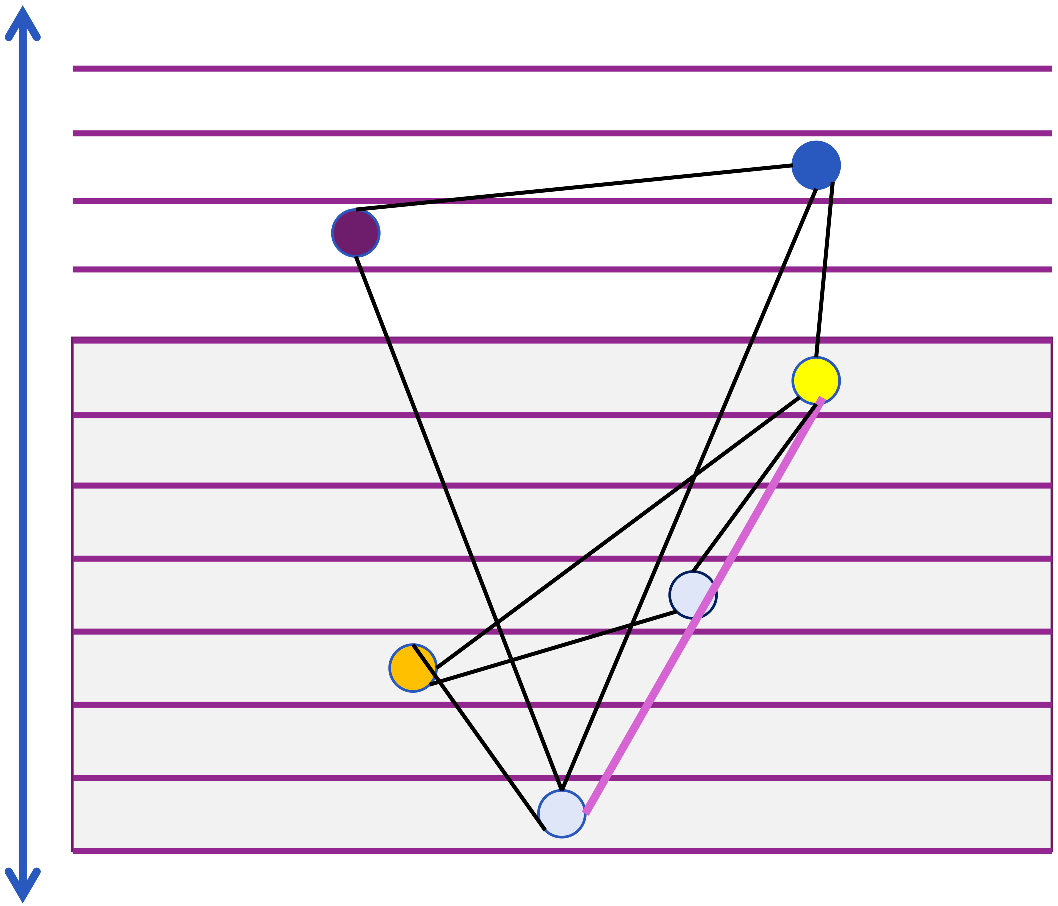

$O(\log \Delta)$

Vertices partitioned into levels

\# neighbors: $< 3^{\ell(v)+2}$

Deterministically recolor, move to bottommost level

Level Data Structure

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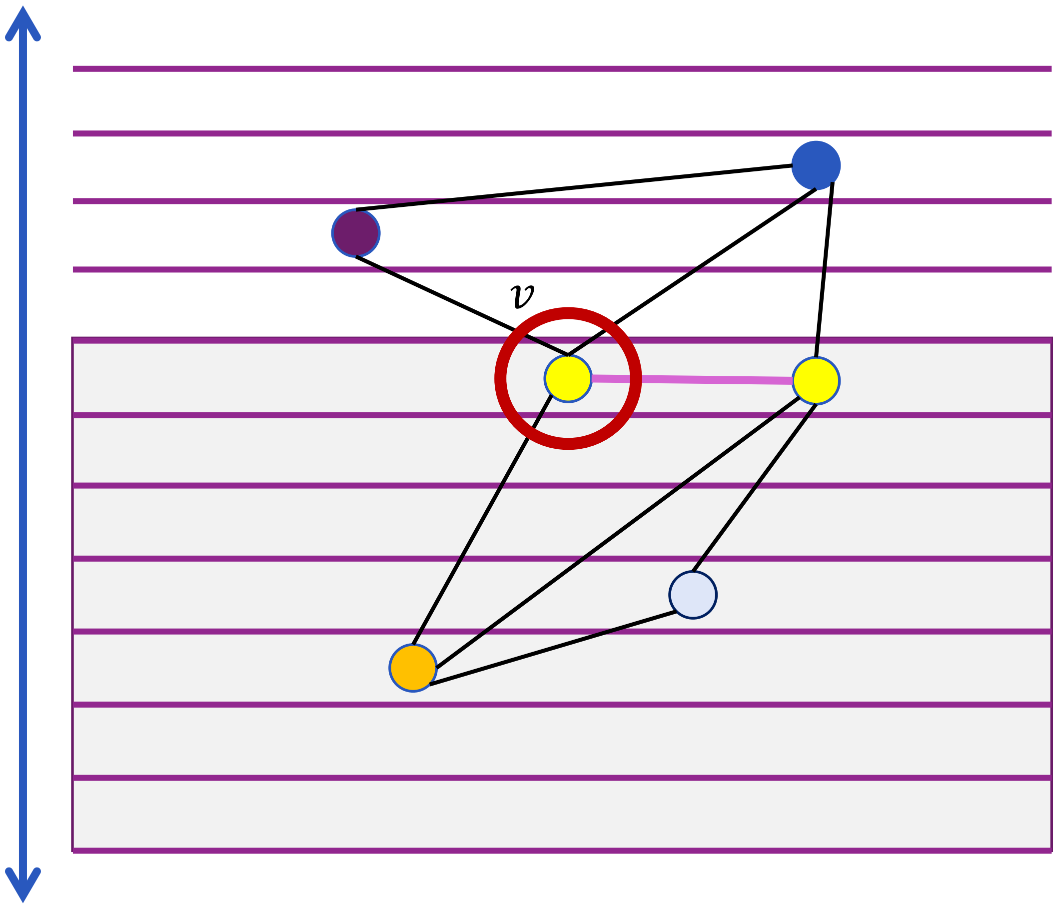

neighbors: $\geq 3^{\ell(v)+2}$

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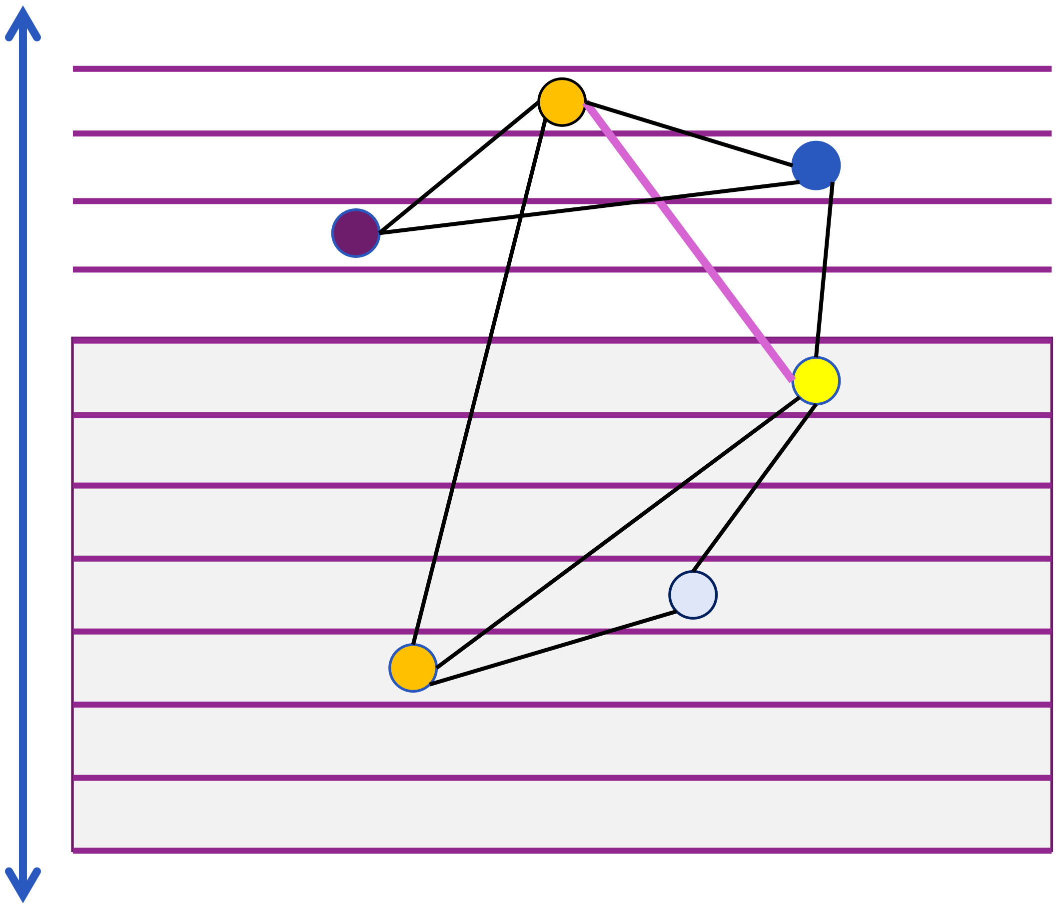

neighbors: $\geq 3^{\ell(v)+2}$

Move to lowest level where # neighbors $< 3^{\ell(v)+2}$ randomly recolor

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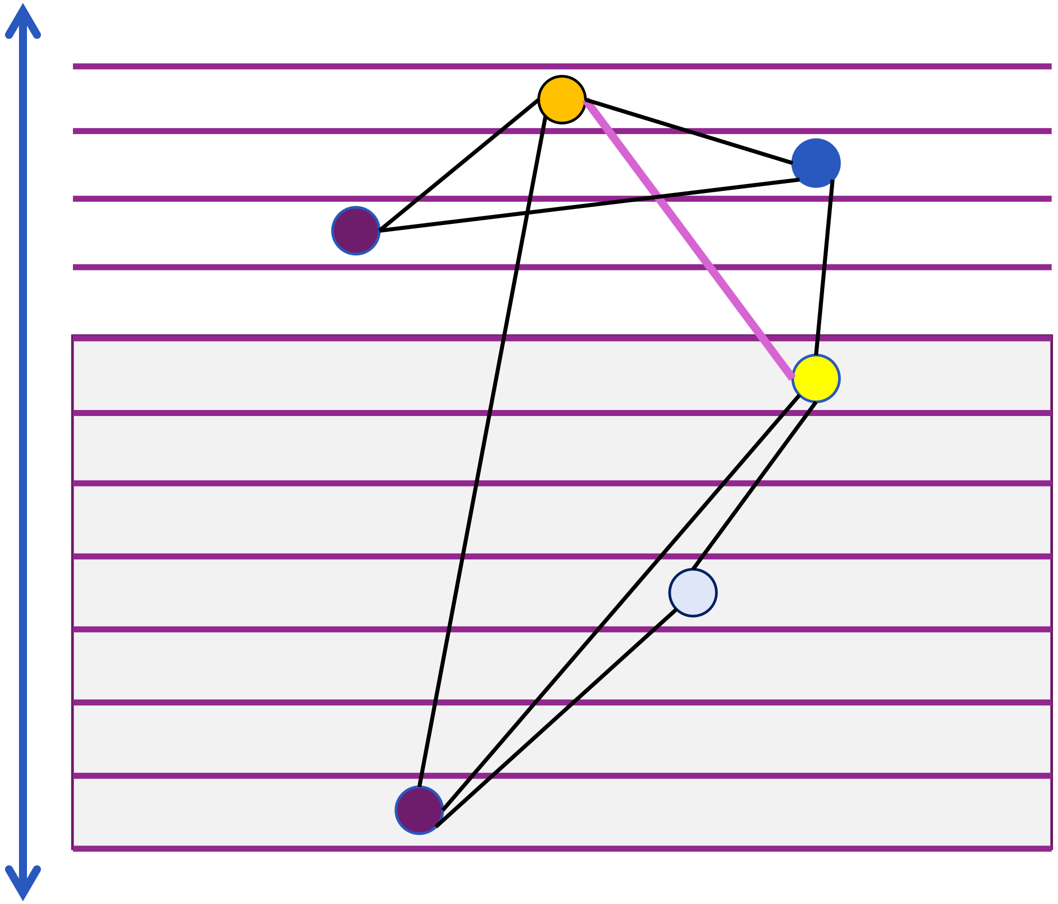

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Recursively resolve new conflicts

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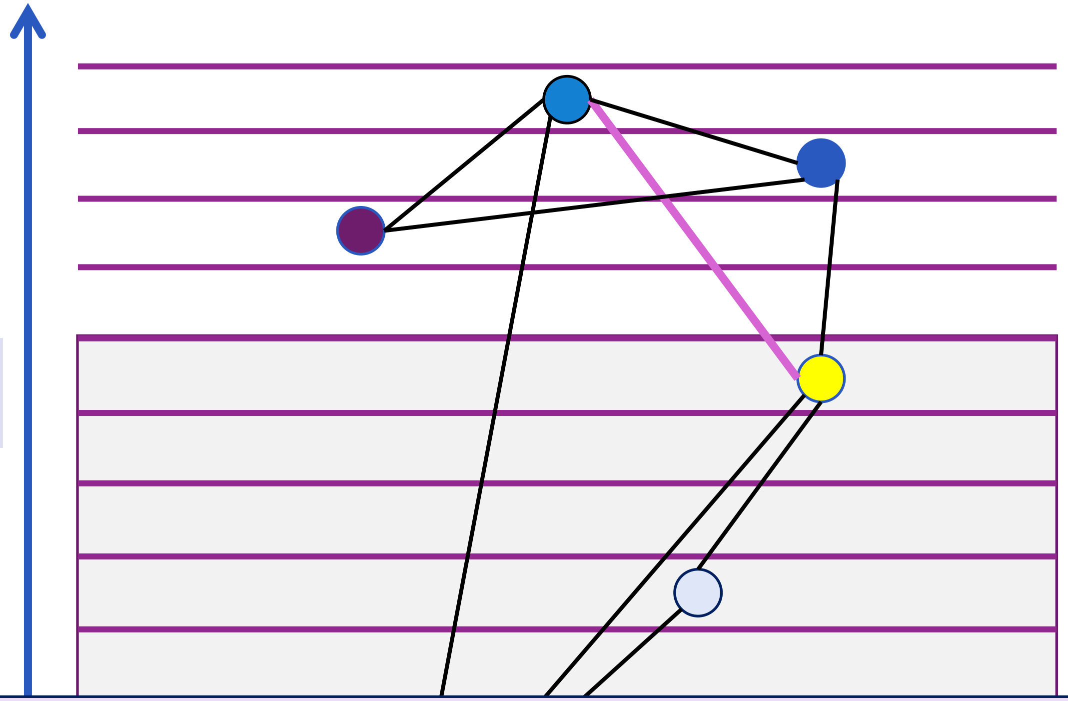


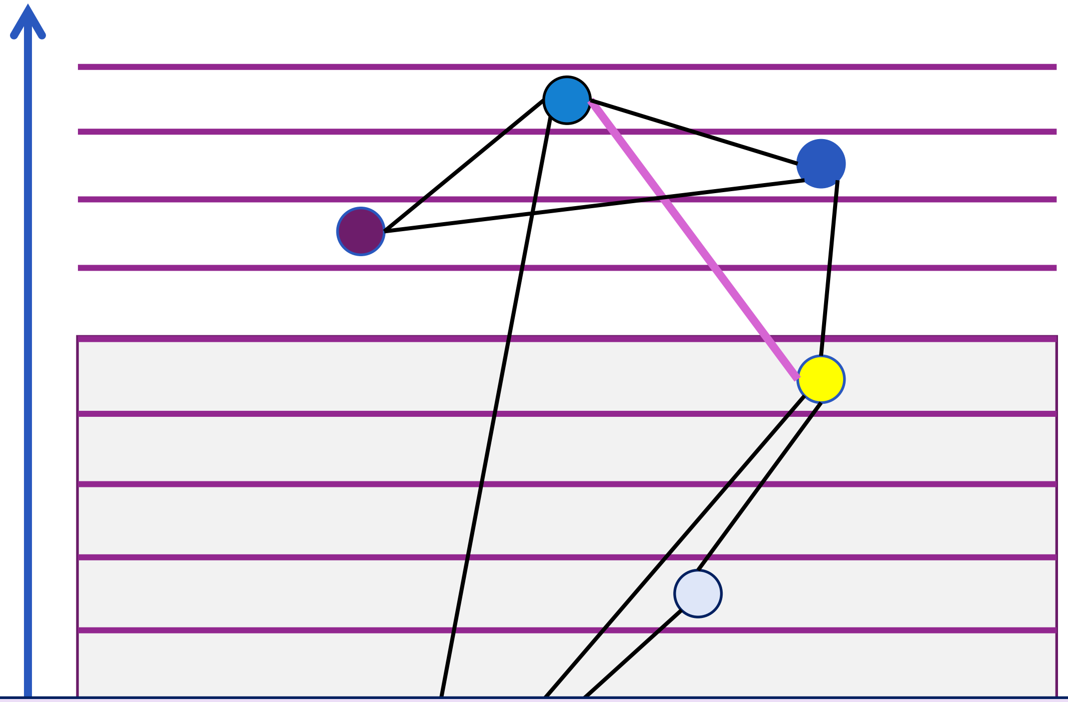
neighbors: $\geq 3^{\ell(v)+2}$

There exists a $O(1)$ amortized update time in expectation and with high probability against an oblivious adversary.

Level Data Structure

Using techniques from PLDS, can probably parallelize in $O(\log \Delta)$ depth

OPEN: more efficient practically?



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Laxman Dhulipala, MIT CSAIL, laxman@mit.edu; Quanquan C. Liu, MIT CSAIL, quanquan@mit.edu; Julian Shun, MIT CSAIL, jshun@mit.edu; Shangdi Yu, MIT CSAIL, shangdiy@mit.edu

Lower Bounds/Constructions

Hardness from Pebbling

Red-Blue Pebble Game: Complexity of Computing the Trade-Off between Cache Size and Memory Transfers

Erik D. Demaine, Quanquan C. Liu

Static-Memory-Hard Hash Functions

Static-Memory-Hard Functions and Nonlinear Space-Time Tradeoffs via Pebbling

Thaddeus Dryja, Quanquan C. Liu, Sunoo Park

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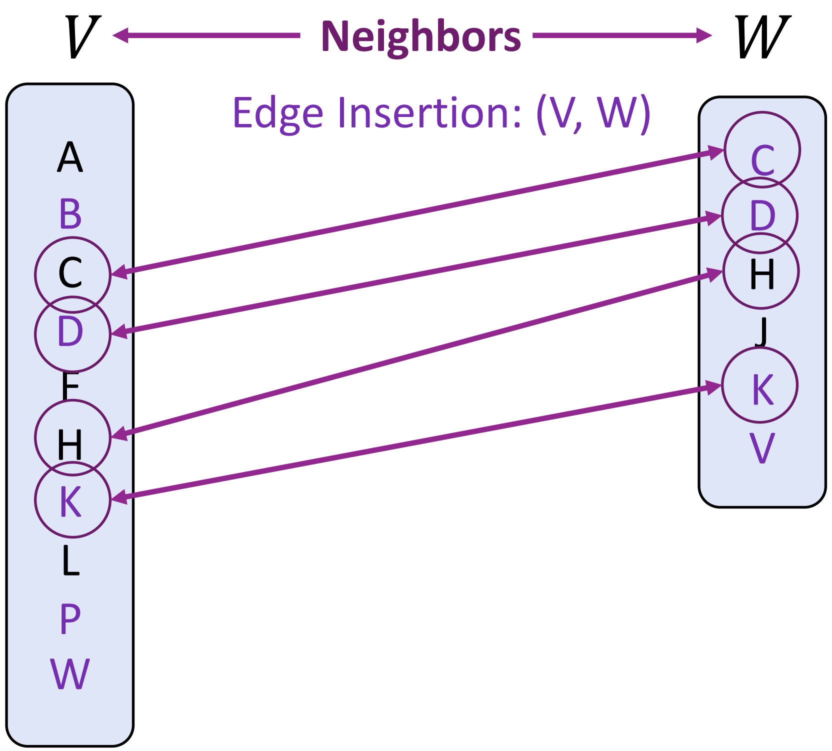
Shangdi Yu

Simple SotA Batch-Dynamic Algorithm

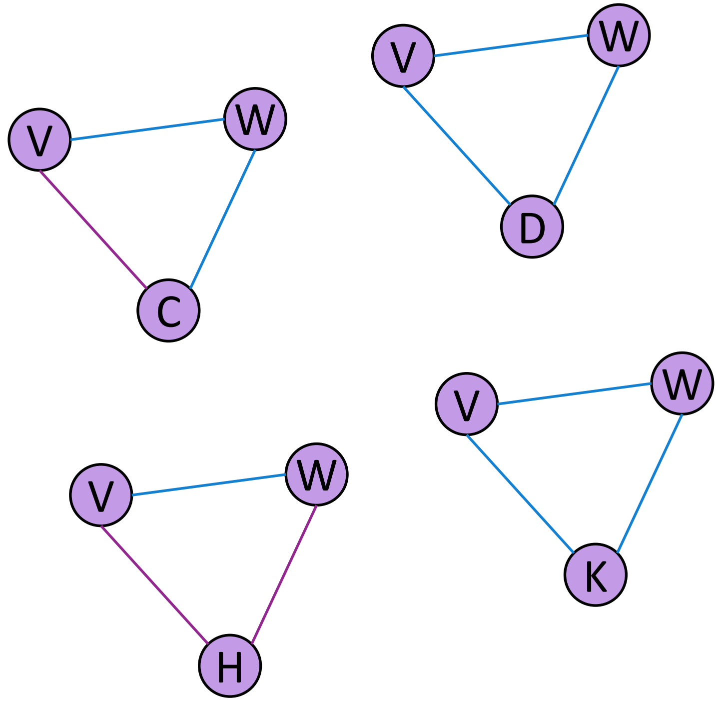
- Makkar, Bader, Green HiPC 2017

Purple: new neighbors
Black: old neighbors

Worst-case: $\Omega(n)$
amortized work per
edge update

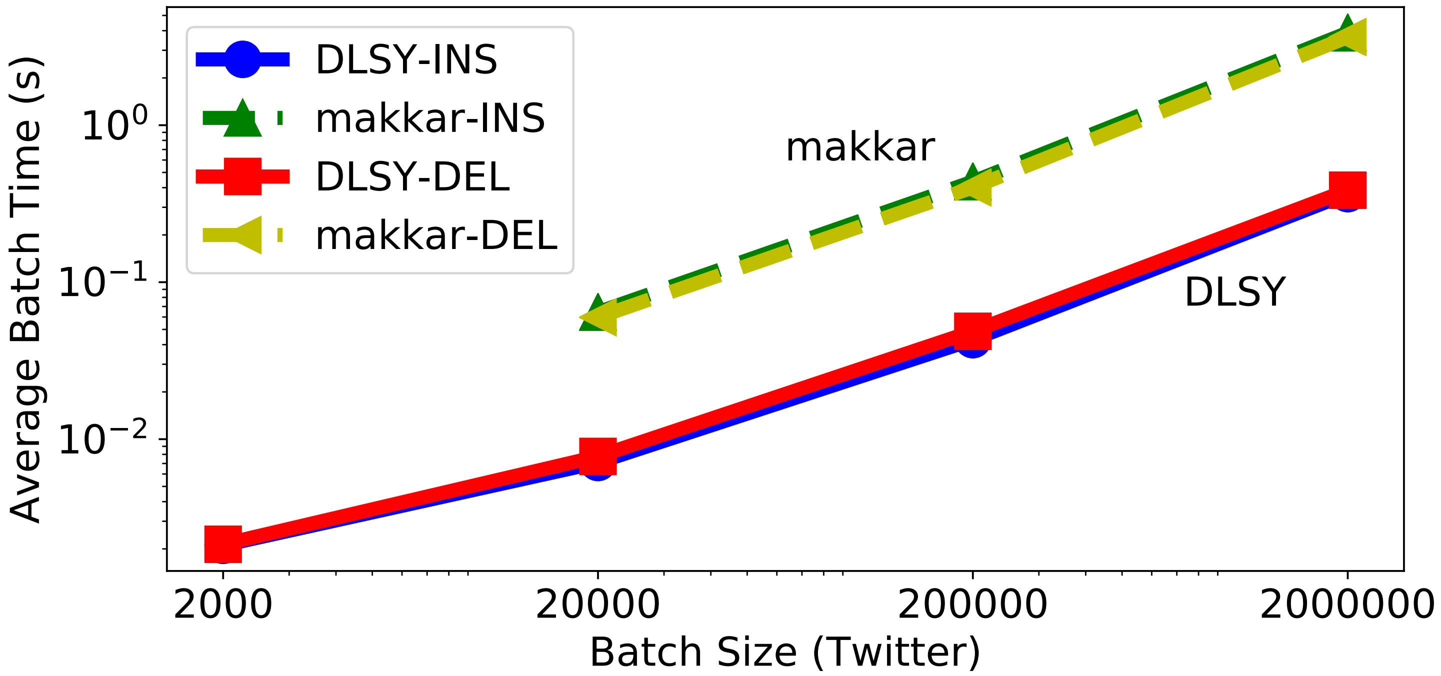


Wedges (two edges that share a vertex)



Batch-Dynamic Triangle Counting

https://github.com/ParAlg/gbbs/tree/master/benchmarks/TriangleCounting/DhulipalaLiuShunYu20



$O(\sqrt{m})$ amortized work and $O(\log n)$ depth

up to 3.31x speedup for all batch sizes

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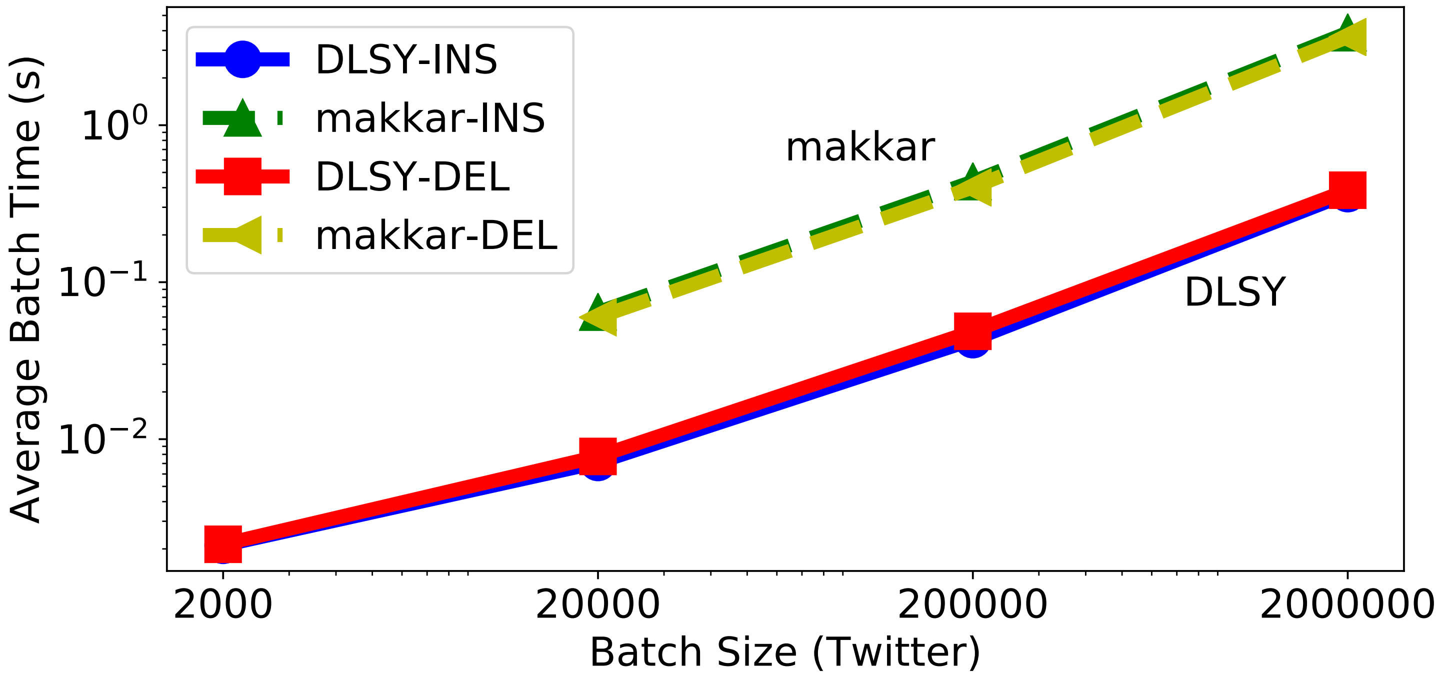

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MPC Algorithms for Subgraph Counting

Parallel Algorithms for Small Subgraph Counting

Amartya Shankha Biswas[*†] Talya Eden[*‡] Quanquan C. Liu[*] Slobodan Mitrović[*§] Ronitt Rubinfeld[¶]

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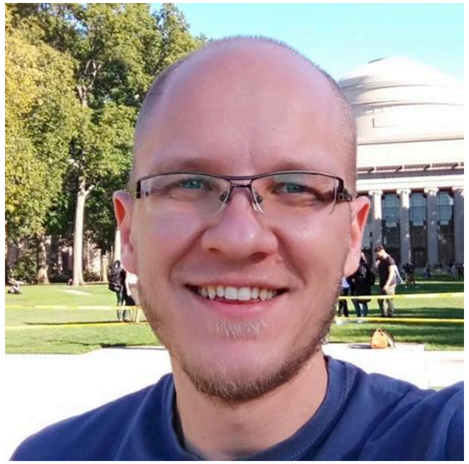
Amartya Shankha Biswas

Talya Eden



Slobodan Mitrović

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MPC Model Definition

- *M* machines
- Synchronous rounds

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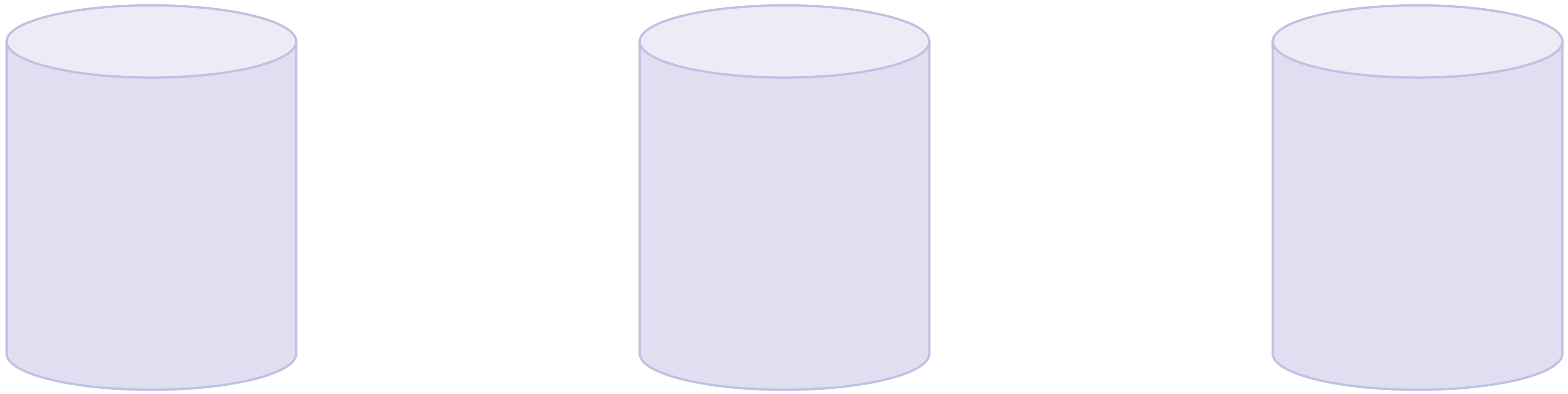

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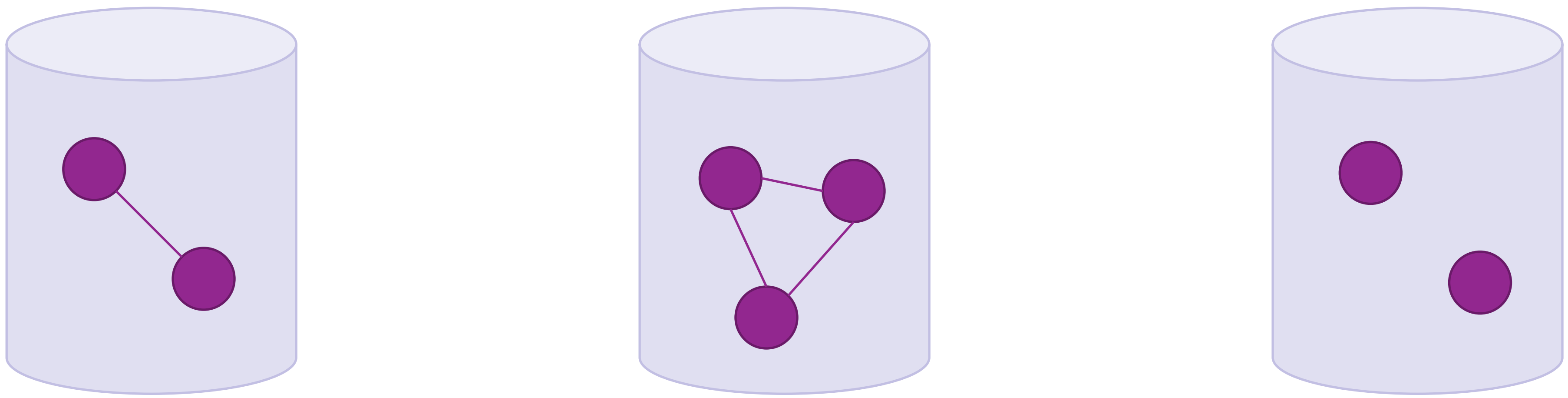

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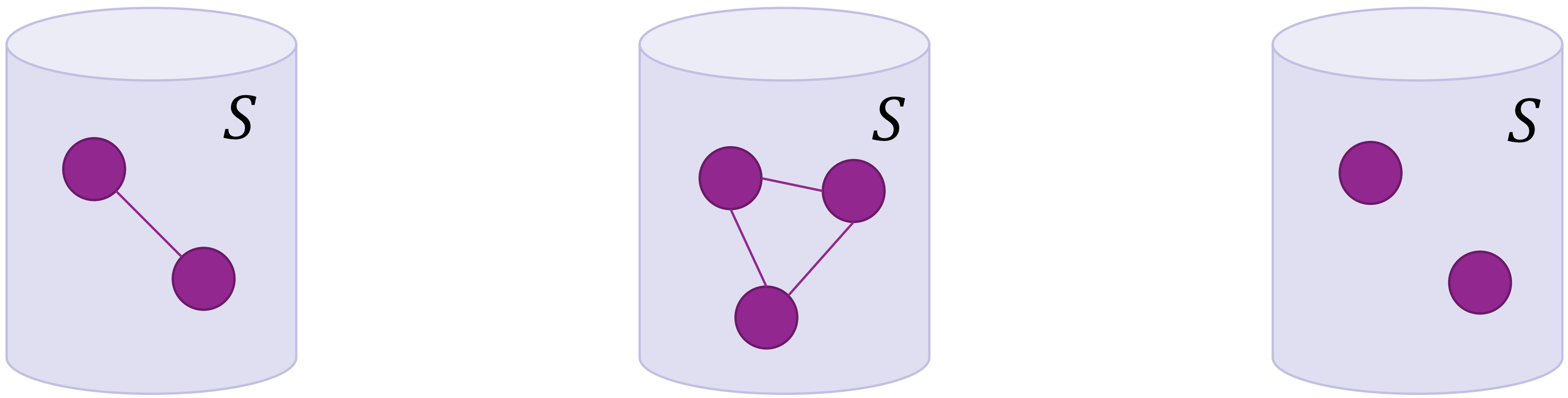

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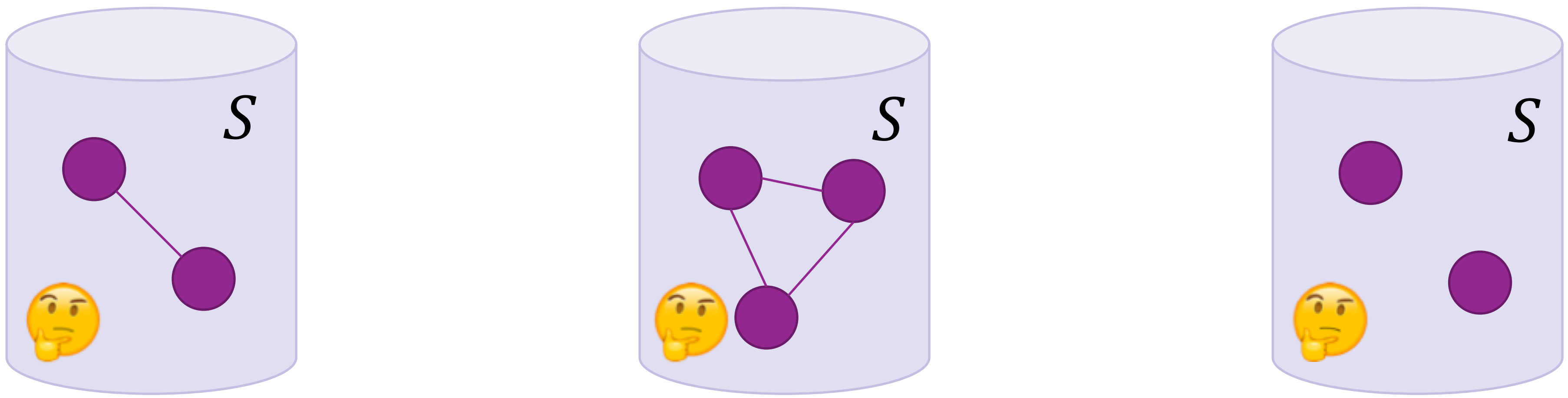

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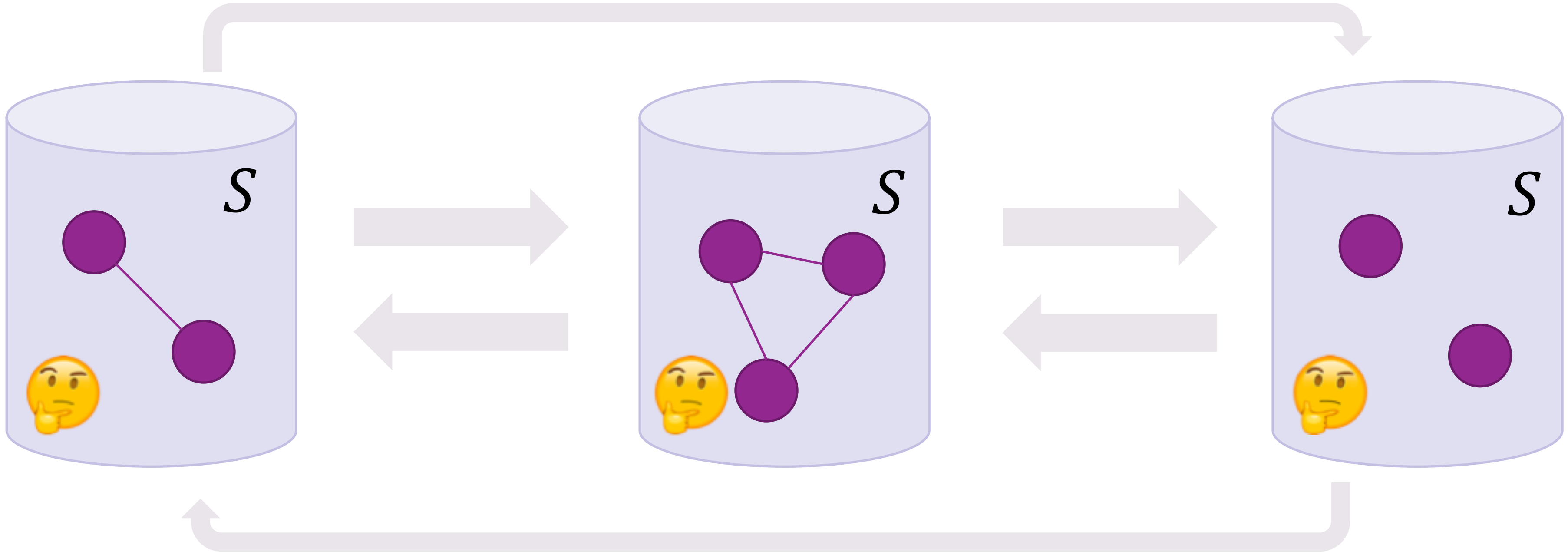

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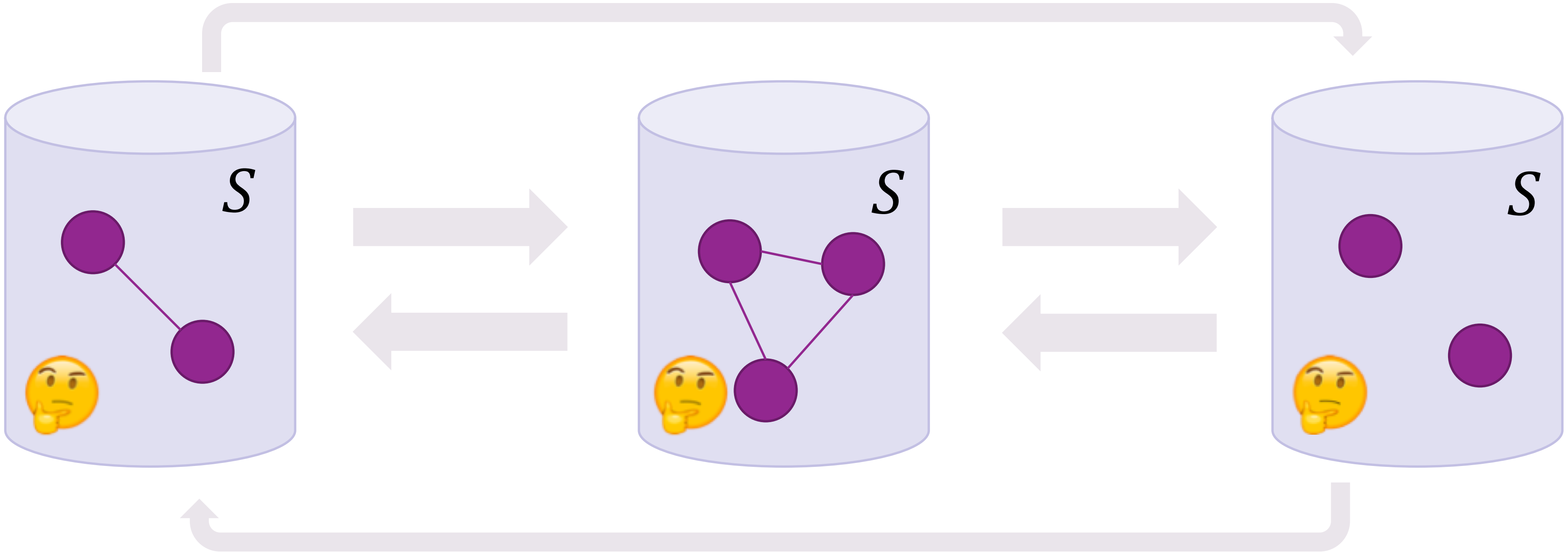
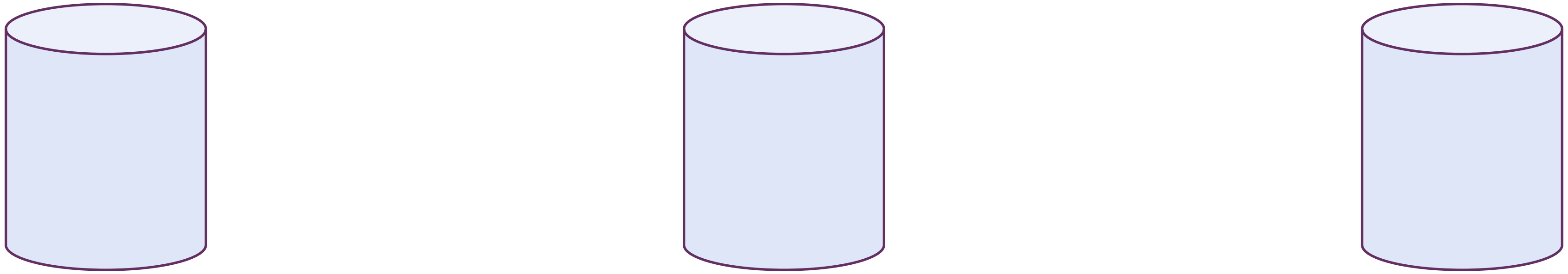
Total Space: $\boldsymbol{M} \cdot \boldsymbol{S}$



Approximate Triangle Counting



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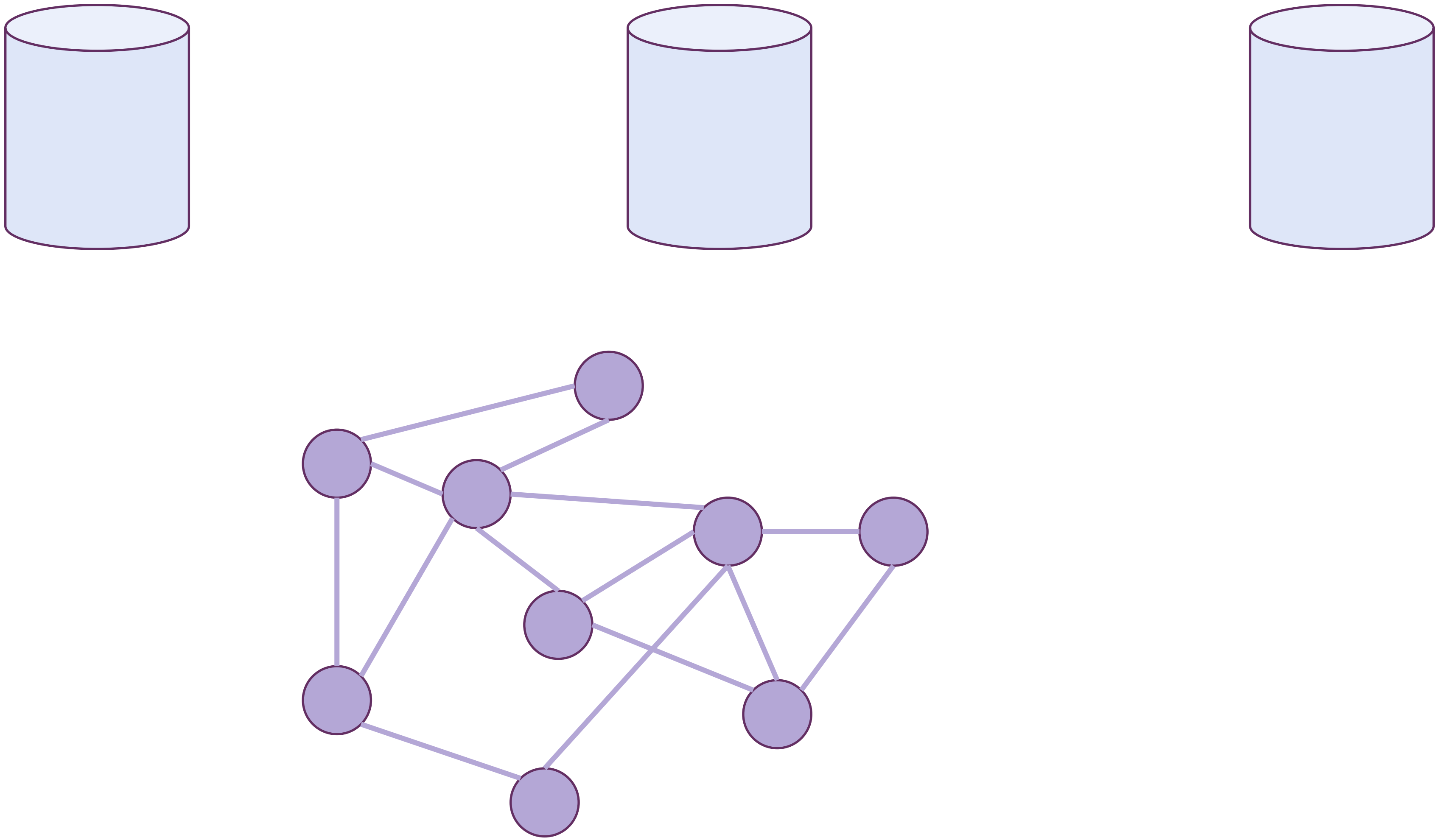

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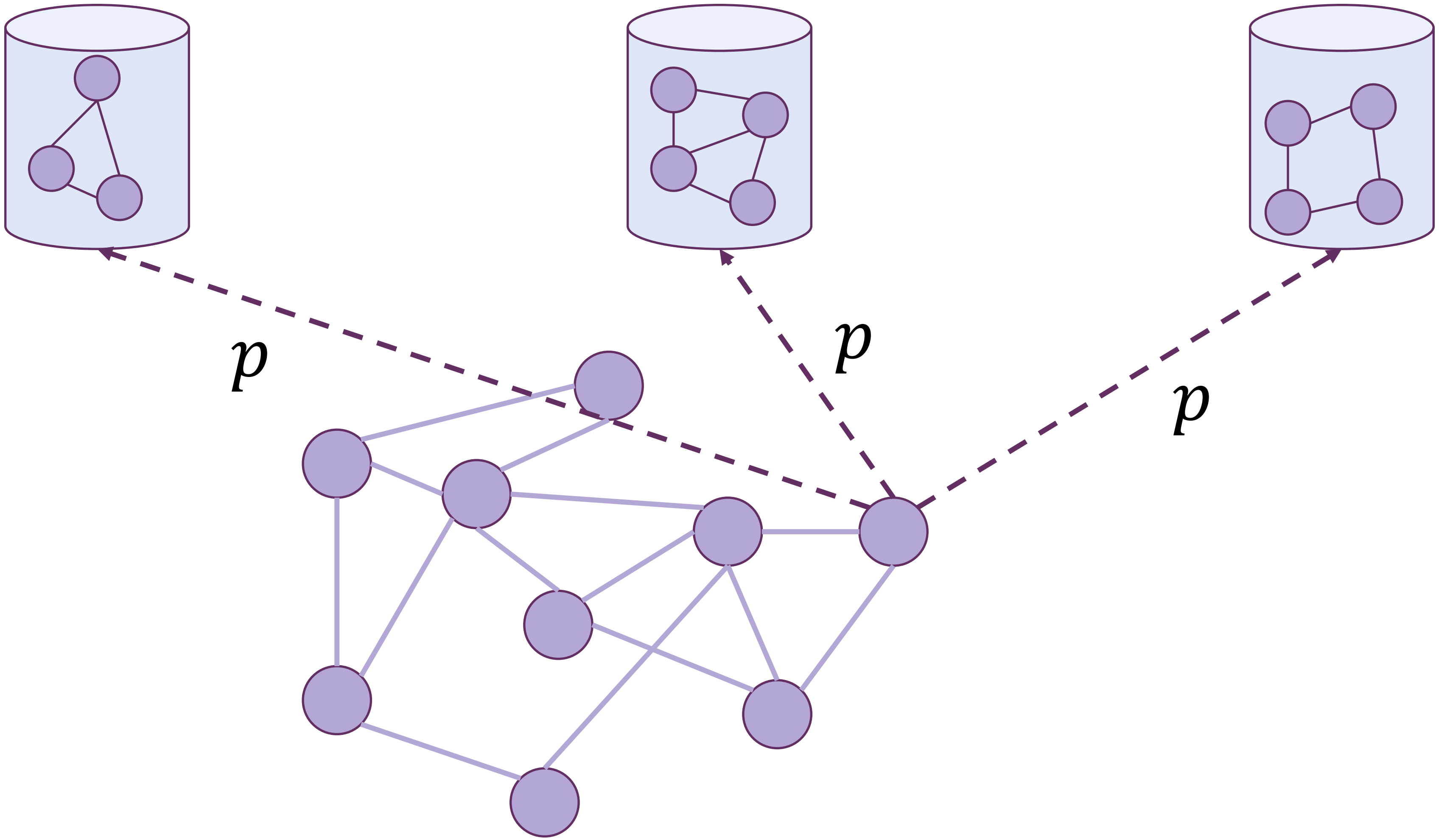

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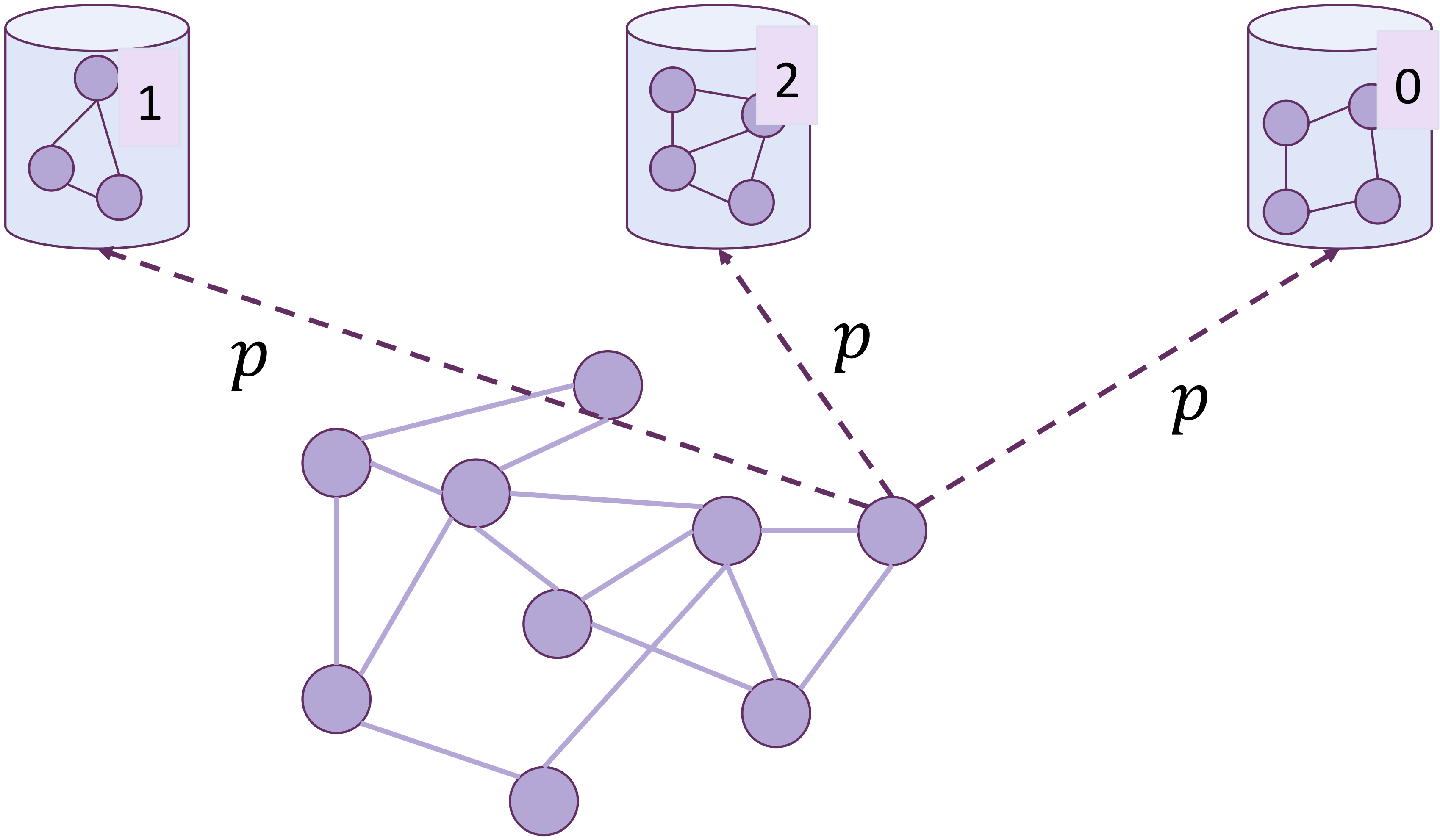

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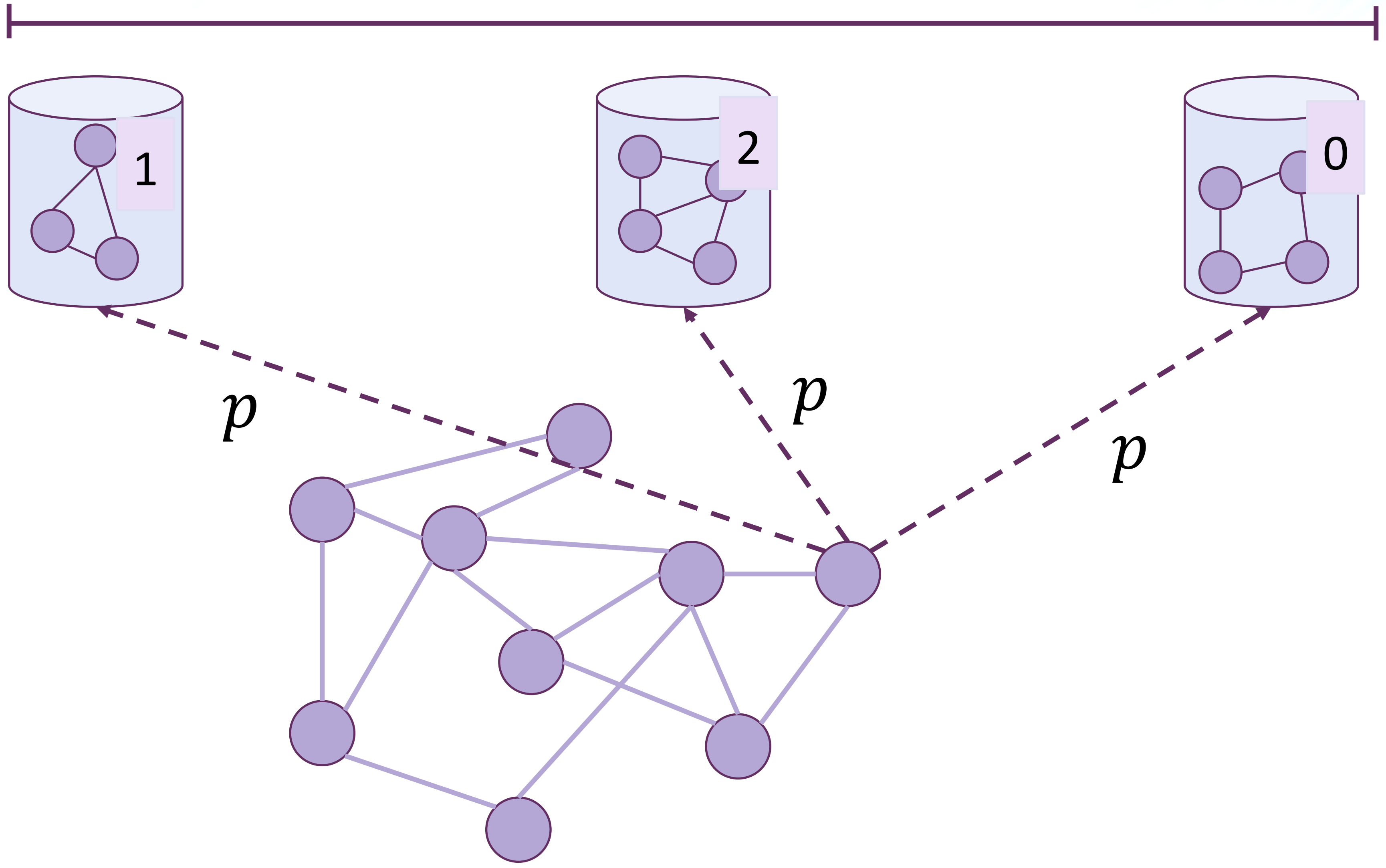

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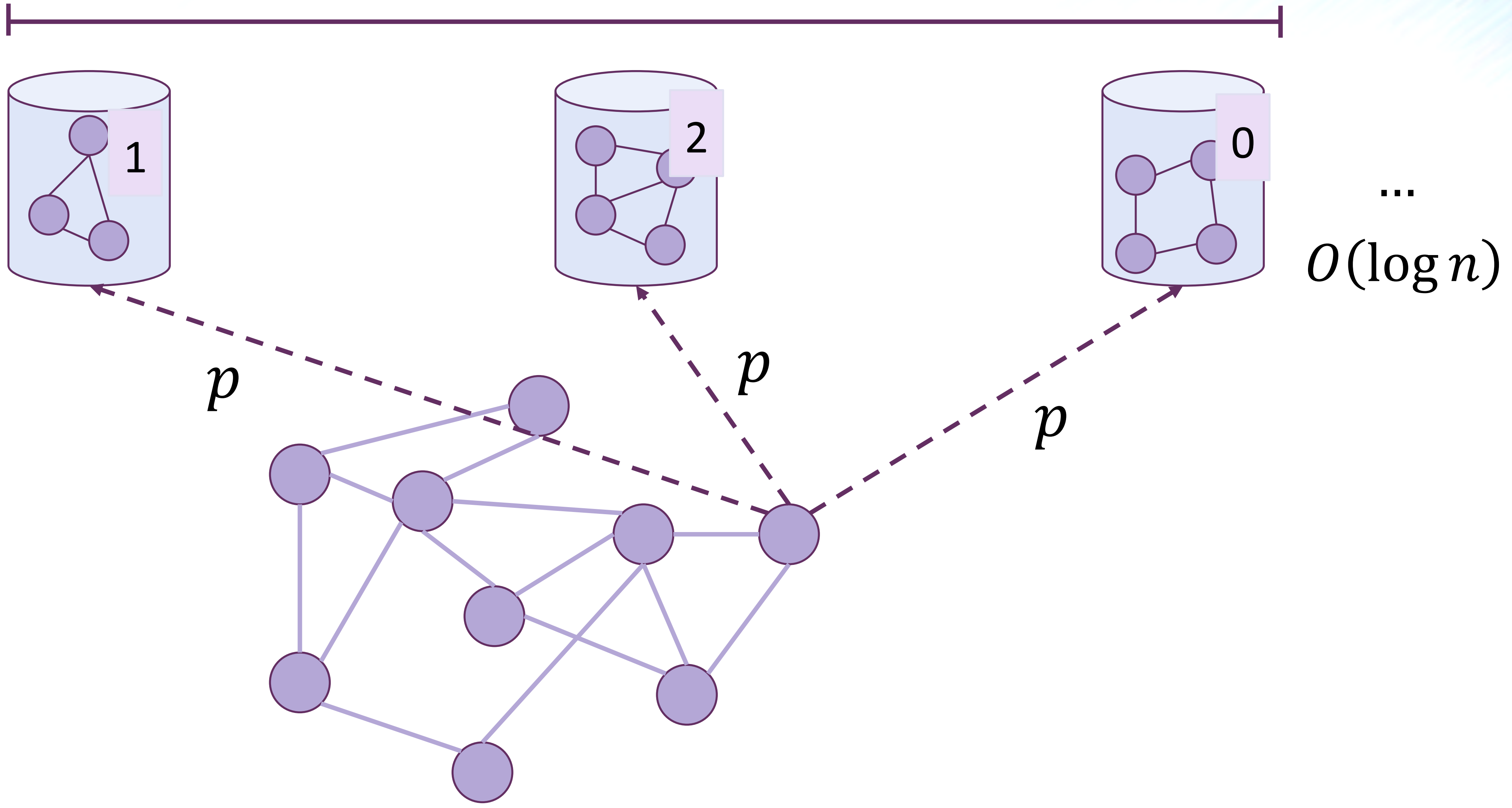

Approximate Triangle Counting

There exists a MPC algorithm that outputs a $(1+\epsilon)$-approximation for the number of triangles if the number of triangles $\boxed{T \geq \sqrt{d_{avg}}}$ and uses $\tilde{O}(m)$ total space and $\widetilde{\Theta}(n)$ space per machine, $O(1)$ MPC rounds.

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[arxiv.org/2002.08299]

Previous: $T \geq d_{avg}$ [Pagh and Tsourakakis '12]

MPC Simulation Experiments

https://github.com/qqliu/mpc-triangle-count-exact-approx-simulations

Graphs from Stanford SNAP Database

File	δ	Partition Approximation	Our Approximation
ego-Facebook	0.5	0.62	1.31
feather-lastfm-social	0.5	5.41	1.08
ca-GrQc	0.5	4.53	1.64
ca-HepPh	0.5	0.66	1.22
ego-Facebook	0.75	0.75	0.97
ca-GrQc	0.75	5.82	0.82
ca-HepPh	0.75	5.90	0.86
musae-twitch (DE)	0.75	0.74	0.95
oregon1_010519	0.75	0.60	0.71

Better approximations on all graphs

Approximation Factor (Same Machine Memory) Compared with PT12

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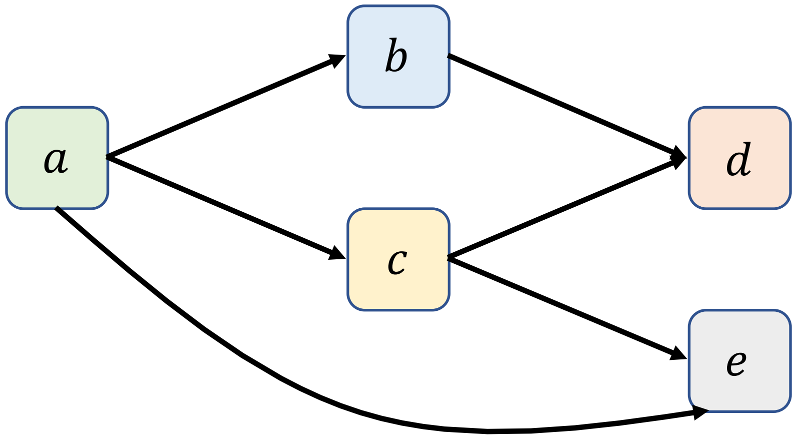
Precedence-Constrained Jobs Modeled as DAG



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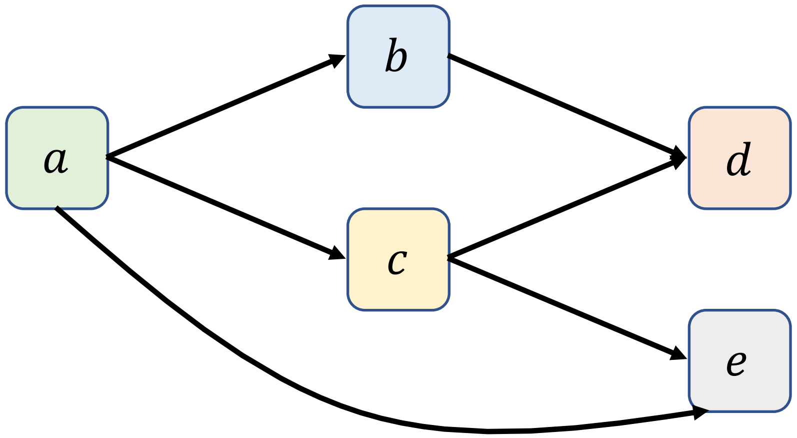

- Nodes scheduled according to some topological sort

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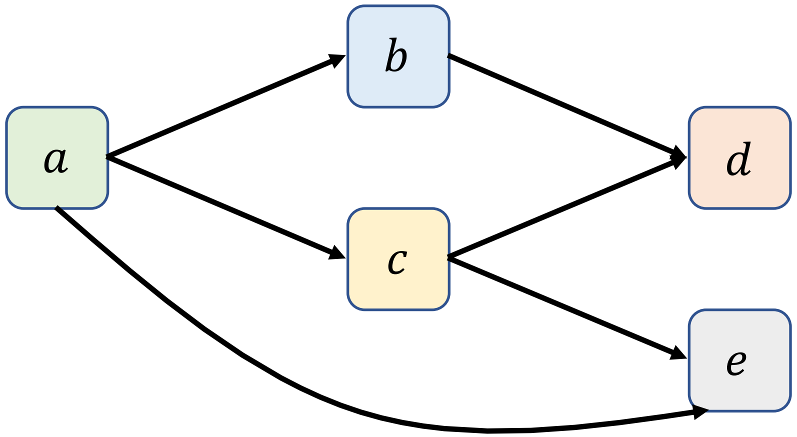

- Nodes scheduled according to some topological sort
- Near-linear time algorithm:
  - Identical machines,
  - Uniform communication delay,
  - Allow duplication of jobs

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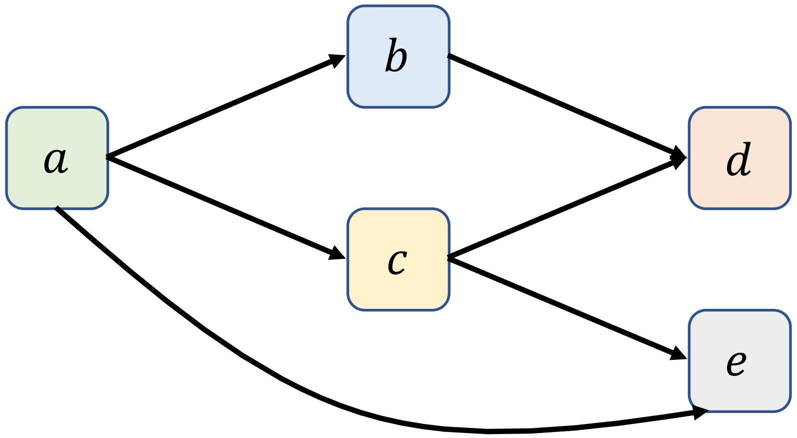

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Main challenge: determine intersection between ancestor sets in $o(n^2)$ time

- Nodes scheduled according to some topological sort
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Precedence-Constrained Jobs Modeled as DAG

Main challenge: determine intersection between ancestor sets in $o(n^2)$ time

Solution: size-estimation via sketching, sampling and pruning, and work charging argument

Open: can we achieve same result *without* duplication?

- Nodes scheduled according to some topological sort
- Near-linear time algorithm:
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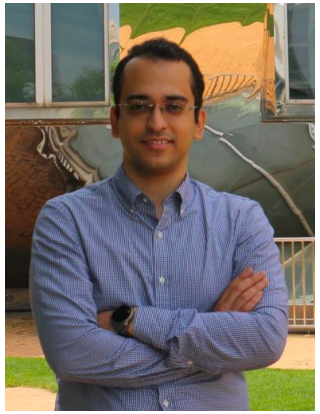


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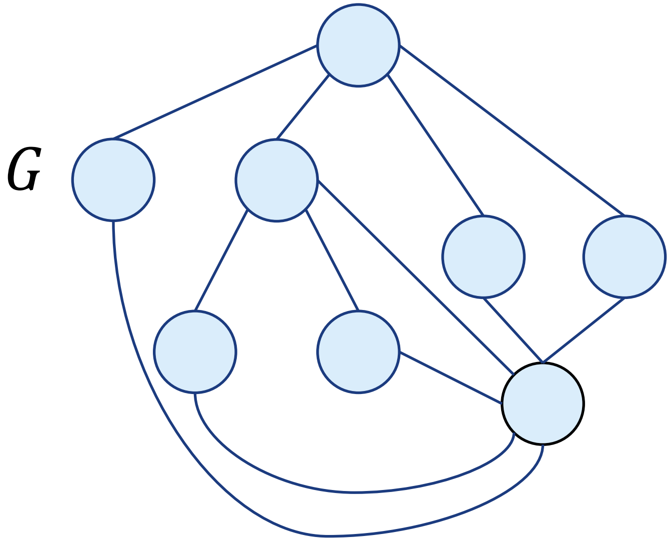
(Minimization) Problem $\mathcal{P}$ with optimal solution $OPT(G)$, e.g. vertex cover

Structural Rounding



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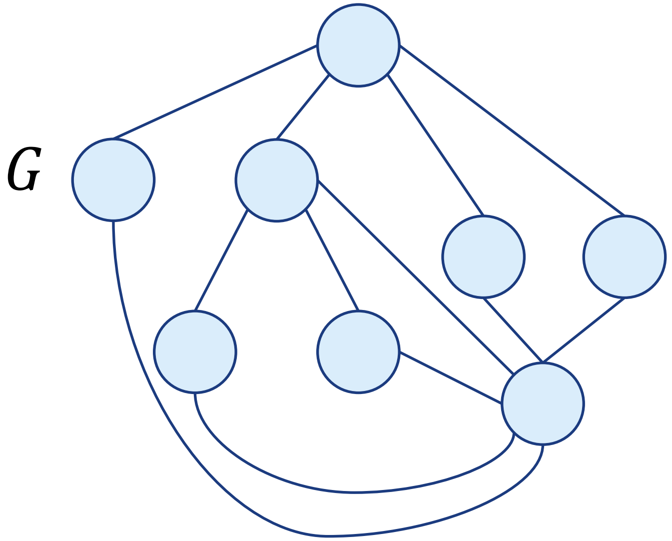
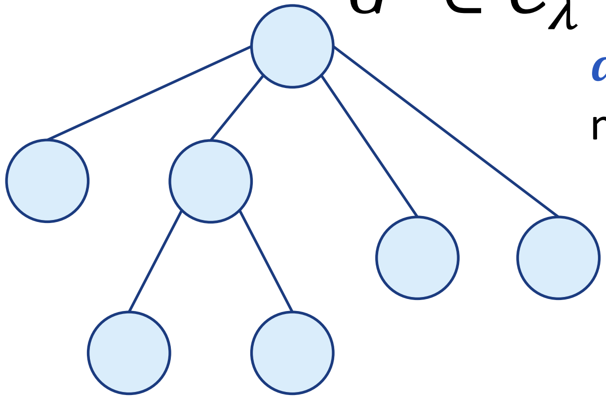


Edit to a Structured Graph Class

(α, β)-approximation



α-approx. min number of edits X

β-approx. on $\mathcal{C}_\lambda$
e.g. $\lambda \leq \beta \cdot \lambda^*$

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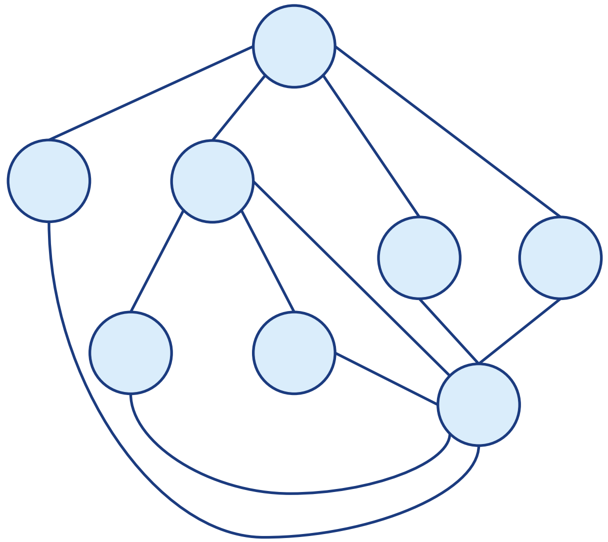
Structural Rounding

G



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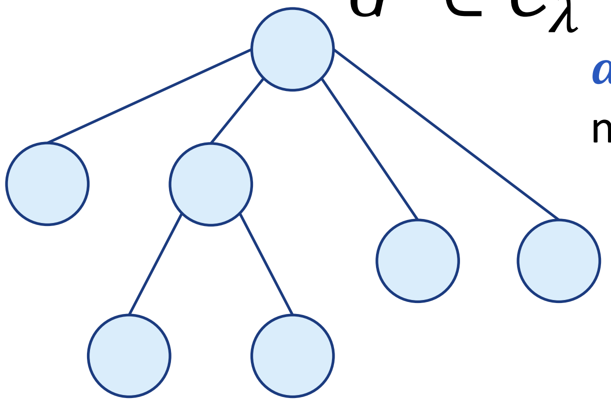
(α, β)-approximation

$G' \in \mathcal{C}_\lambda$



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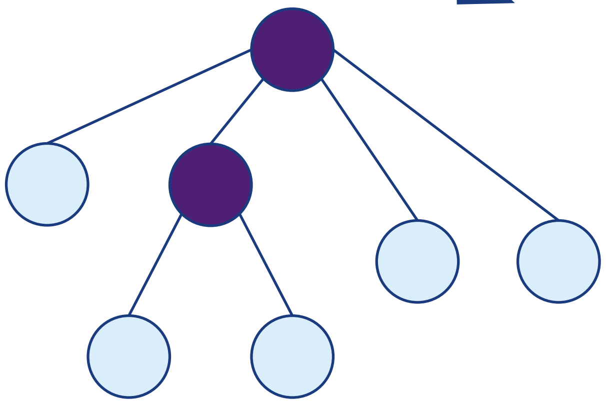

G'

Solve the problem in the edited graph G': $OPT(G')$ where we assume $OPT(G') \leq OPT(G)$

Structural Rounding

(Minimization) Problem $\mathcal{P}$ with optimal solution $OPT(G)$, e.g. vertex cover

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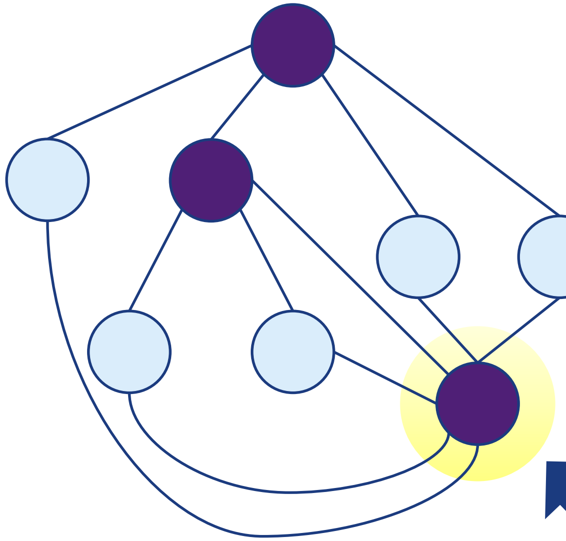

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$(\boldsymbol{\alpha}, \boldsymbol{\beta})$**-approximation**

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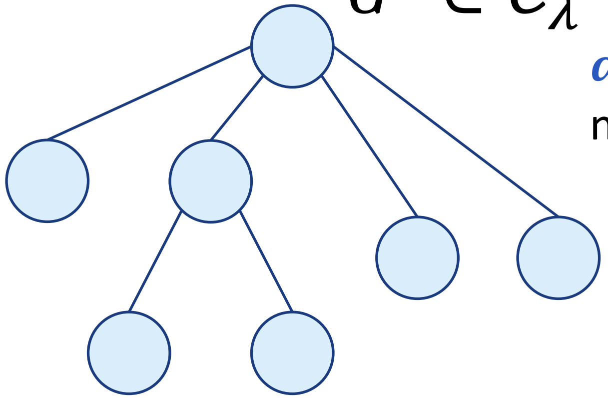

$\boldsymbol{\alpha}$**-approx.** min number of edits X

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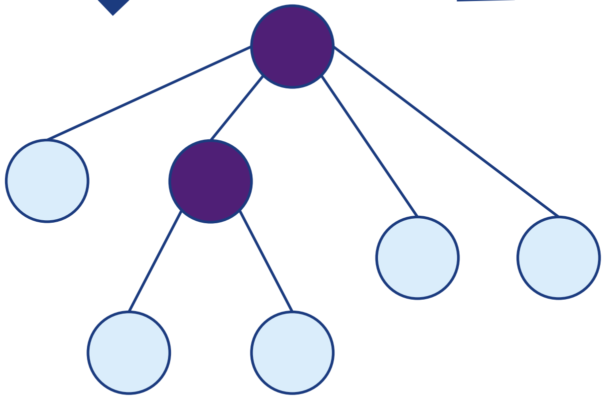

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Lift the solution to the original graph with **loss c** i.e. need to add $c \cdot |X|$ to the solution

$S \leq OPT(G') + c \cdot \alpha \cdot |X|$

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Hardness from Pebbling

Red-Blue Pebble Game: Complexity of Computing the Trade-Off between Cache Size and Memory Transfers

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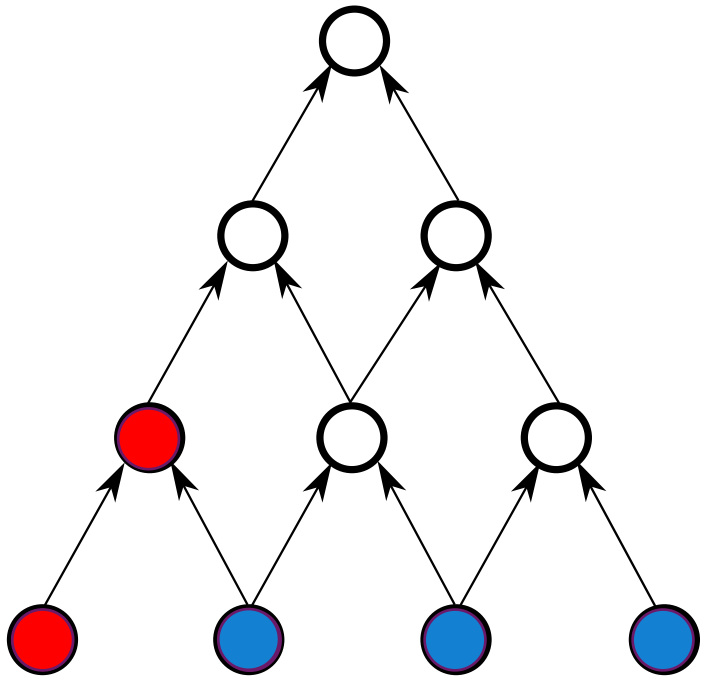
Pebble Games

- Combinatorial game to model:
  - Register allocation
  - Rematerialization/gradient checkpointing in ML
  - Input/output complexity
- Hardness for obtaining optimum schedule in external-memory model [DL18]
  - PSPACE-complete
  - Fixed-parameter intractable



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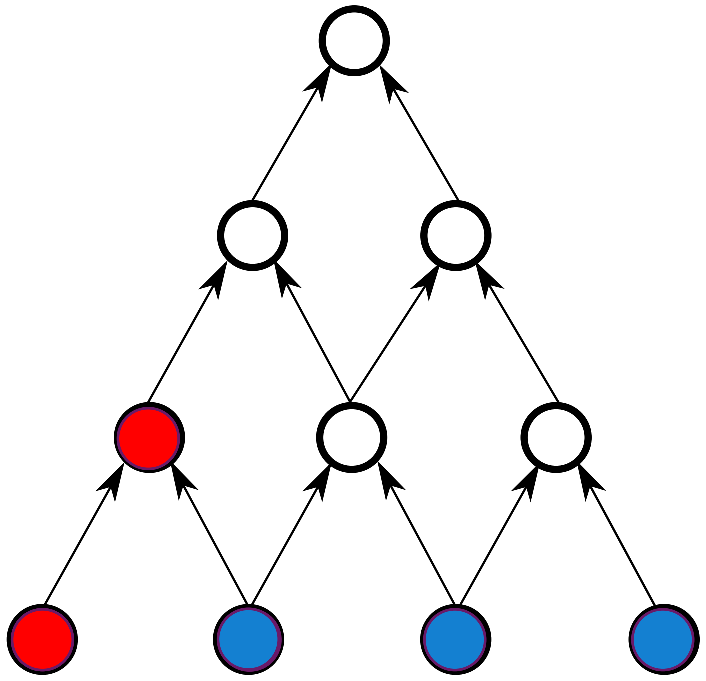
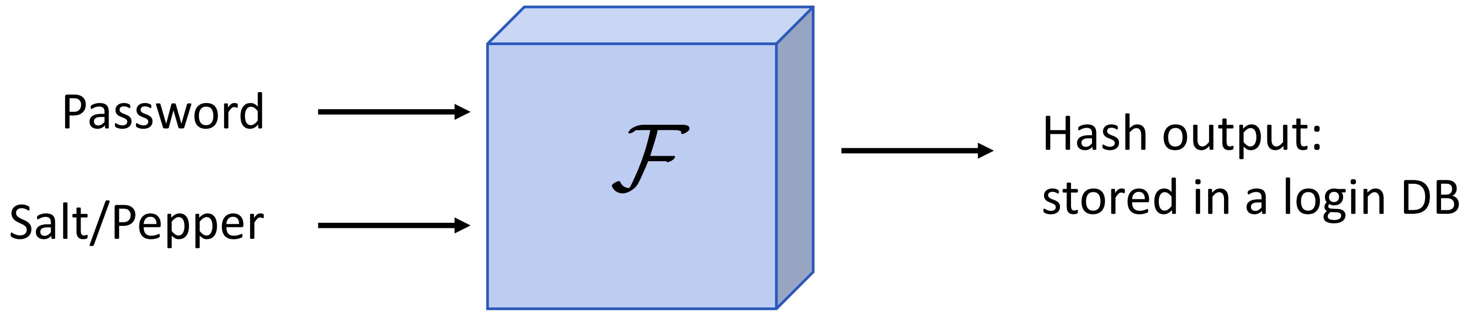


Password Hashing



Honest evaluators need only use $\mathcal{F}$ few times

Adversaries may run $\mathcal{F}$ many times (e.g. large-scale server attacks).

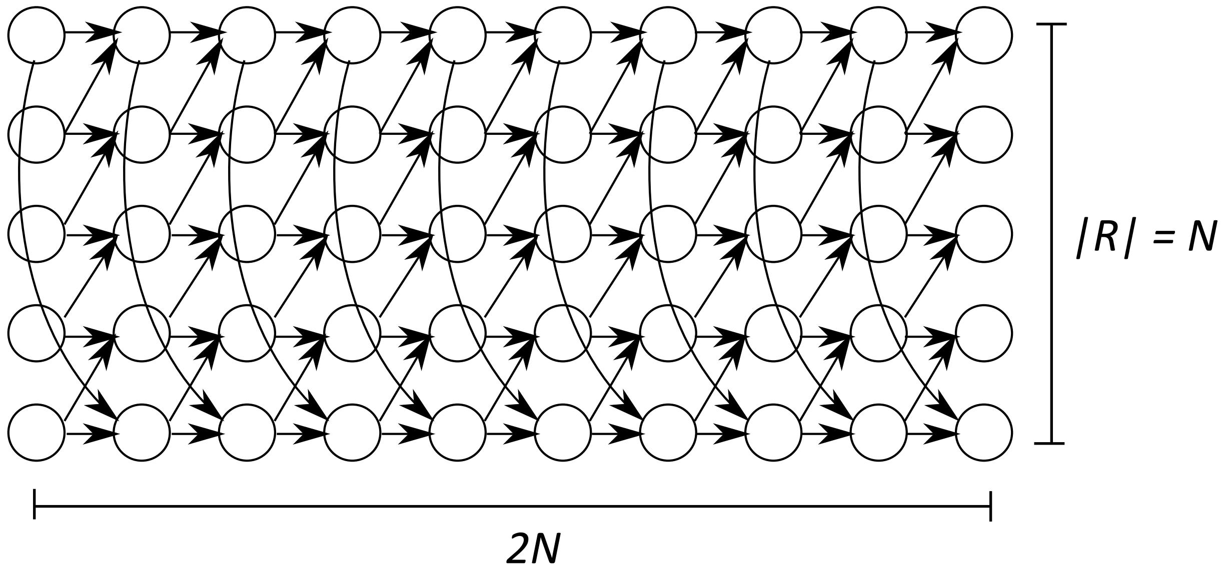
Desirable goal:

Make brute-force attacks hard by making $\mathcal{F}$ hard to compute over many hashes.

(Not implied by traditional hash function guarantees like collision-resistance.)

Candidate Constructions

- Any graph with one target node **doesn't work**
- Need at least enough target nodes so that *number hash outputs* is reasonably large
- Simple construction **cylinder graph** we implemented ($n = N^2$)



Acknowledgements

[Not included—You had to be there...]